ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume VII (XVII), 2008

THE STRESS METHOD IN THE STUDY OF THE STABILITY SYSTEMS OF THE PRISMATIC BARS

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Keywords : prismatic bar, condition equations, critical flexure force.

Abstract: In this work is about the stress method used in the study of the stability systems of the prismatic bars. The stresses, the forces and the moments are unknown and the condition equation is written like a displacement which has to be zero on the unknown stress direction. The condition that the system of the equation (obtained that way) has non-zero solutions for unknown stresses, will give the equation of the critical flexure force.

1. INTRODUCTION

The strength structures of prismatic bars are compound from bars mostly activated at bending and compression.

An example of this type of bars is given in figure number 1. This bar has the following characteristics: l – length, EI – bending constant rigidity, rested of the two ends, by a force F and actuated by the concentrated $M_1^{'}$ and $M_2^{'}$ moments in the end supports.

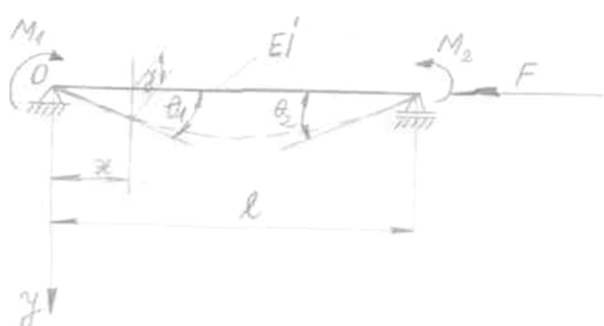


***Fig*.1 Bended bar activated by an external force F**

The bar will be reported to the reference system xOy, where the x-axis will be the same with the bar axis. After the strain, the bar will have the y deflection with x-coordinate and a moment of deflection in x-section.

$$M = M_1 \frac{l-x}{l} + M_2 \frac{x}{l} + Fy \tag{1}$$

The differential equation of the elastic line of the bar will be:

$$EI\frac{d^2y}{dx^2} = -Fy - M_1 + \frac{M_1 - M_2}{l}x \tag{2}$$

With the notation: $$\alpha^2 = \frac{F}{EI}, \tag{3}$$

the equation (3) will be written:

$$\frac{d^2y}{dx^2}+\alpha^2 y=\alpha^2\left(\frac{M_1-M_2}{Fl}x-\frac{M_1}{F}\right) \tag{4}$$

and has the solution:

$$y=C_1\cos\alpha x+C_2\sin\alpha x-\frac{M_1}{F}+\frac{M_1-M_2}{Fl}x \tag{5}$$

Replacing the limits conditions in the deflection relation:

x=0, w=0, x=l, w=0 (6)

it will obtain:

$$C_1-\frac{M_1}{F}=0,\quad C_1\cos\alpha l+C_2\sin\alpha l-\frac{M_2}{F}=0 \tag{7}$$

It calculates the integrating constants C_1 and C_2:

$$C_1=\frac{M_1}{F}\ ,\ C_2=\frac{1}{\sin\alpha l}\left(\frac{M_2}{F}-\frac{M_1}{F}\cos\alpha l\right) \tag{8}$$

Then we calculate the y deflection:

$$y=\frac{M_1}{F}\left(\cos\alpha x-\frac{\cos\alpha l}{\sin\alpha}\sin\alpha x-1+\frac{x}{l}\right)+\frac{M_2}{F}\left(\frac{\sin\alpha x}{\sin\alpha l}-\frac{x}{l}\right) \tag{9}$$

The rotation $\theta=\frac{dy}{dx}$ will be:

$$\theta=\frac{dy}{dx}=\frac{M_1}{F}\left(\frac{1}{l}-\alpha\sin\alpha x-2\frac{\cos\alpha l}{\sin\alpha l}\cos\alpha x\right)+\frac{M_2}{F}\left(\alpha\frac{\cos\alpha x}{\sin\alpha l}-\frac{1}{l}\right) \tag{10}$$

At the ends of the bar, for an x=0 and x=l, the rotation θ_1 and θ_2 will be:

$$\theta_1=\frac{M_1 l}{3EI}\psi(\alpha l)+\frac{M_2 l}{6EI}\varphi(\alpha l)$$

$$\theta_2=\frac{M_1 l}{6EI}\varphi(\alpha l)+\frac{M_2 l}{3EI}\psi(\alpha l) \tag{11}$$

where:

$$\psi(\alpha l)=\frac{3}{\alpha l}\left(\frac{1}{\alpha l}-\frac{1}{tg\alpha l}\right);$$

$$\varphi(\alpha l)=\frac{6}{\alpha l}\left(\frac{1}{\sin\alpha l}-\frac{1}{\alpha l}\right); \tag{12}$$

From the (11) equation we obtain:

$$M_1=\frac{3EI}{l}\left[\frac{2\psi(\alpha l)}{4\psi^2(\alpha l)-\varphi^2(\alpha l)}\theta_1-\frac{\varphi(\alpha l)}{4\psi^2(\alpha l)-\varphi^2(\alpha l)}\theta_2\right]$$

$$M_2=\frac{3EI}{l}\left[\frac{2\psi(\alpha l)}{4\psi^2(\alpha l)-\varphi^2(\alpha l)}\theta_2-\frac{\varphi(\alpha l)}{4\psi^2(\alpha l)-\varphi^2(\alpha l)}\theta_1\right] \tag{13}$$

Making a sign convention for the moments and rotations as well as the positive way it's the clockwise rotation. Therefore: $M_2 \rightarrow -M_2$ and $\varphi_2 \rightarrow -\varphi_2$ (fig. 2).
The (13) relations become:

$$M_1 = \frac{2EI}{l}\left[2R(\alpha l)\theta_1 + S(\alpha l)\theta_2\right]$$

$$M_2 = \frac{2EI}{l}\left[S(\alpha l)\theta_1 + R(\alpha l)\theta_2\right] \tag{14}$$

Where:

$$R(\alpha l) = \frac{3\psi(\alpha l)}{4\psi^2(\alpha l) - \varphi^2(\alpha l)};$$

$$S(\alpha l) = \frac{3\varphi(\alpha l)}{4\psi^2(\alpha l) - \varphi^2(\alpha l)}; \tag{15}$$

The relations (11) and (14) are very important for the stress method and the displacement method in the stability systems of the bars study.

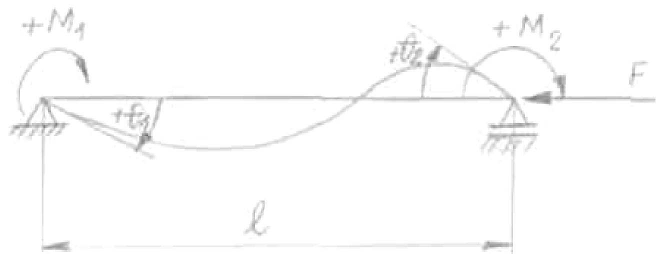
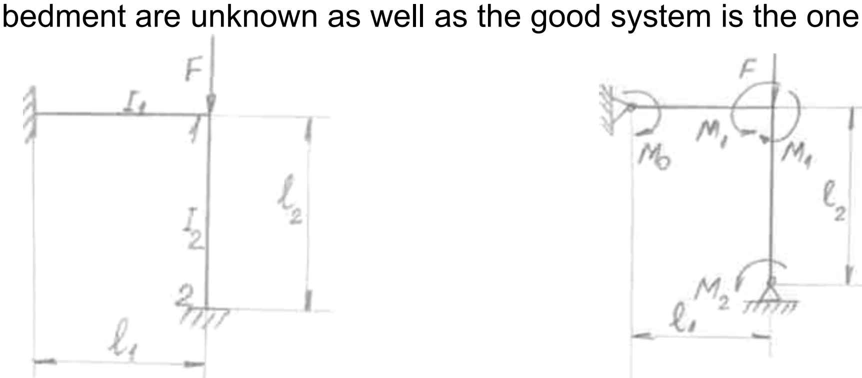


Fig. 2 Sign convention for the moments and rotations

2. THE STRESS METHOD

As we know from the Strength Materials the stresses, the forces and moments are unknown, and the condition equations are written like displacements which have to be zero, on the unknown stress direction. The condition that the system of equations has non-zero solutions, will give us the equation of the critical flexure force. An example is the bars system from the next figure. The system is compound of two bars and is activated by the F force.

In the stress method the system below is three times undetermined. The moments from the embedment are unknown as well as the good system is the one from fig. 3 b.



a) ***b)***

Fig. 3 System activated by an external force F.

The condition equations are in this case:

$$\theta_0 = 0,\ \theta_1^{'} + \theta_1^{''} = 0,\ \theta_2 = 0 \qquad (16)$$

and become:

$$\theta_0 = \frac{l_1}{6EI_1}\left[2\psi_1 M_0 + \psi_1 M_1\right];\quad \theta_1^{'} = \frac{l_1}{6EI_1}\left[2\psi_1 M_1 + \varphi_1 M_0\right];$$

$$\theta_1^{''} = \frac{l_2}{6EI_2}\left[2\psi_2 M_1 + \psi_2 M_2\right];\quad \theta_2 = \frac{l_2}{6EI_2}\left[2\psi_2 M_2 + \varphi_2 M_1\right]; \qquad (17)$$

$$2M_0 + M_1 = 0;\ \frac{l_1}{I_1}M_0 + 2\left(\frac{l_1}{I_1} + \Psi_2\frac{l_2}{I_2}\right)M_1 + \frac{l_2}{I_2}\varphi_2 M_2 = 0;\ 2\Psi_2 M_2 + \varphi_2 M_1 = 0. \qquad (18)$$

The determinant of the algebraically system with unknown M_0, M_1 and M_2, must be zero (to obtain non-zero solutions) and result:

$$3\frac{l_1}{I_1}\psi_2 + \frac{l_2}{I_2}(4\psi_2^2 - \varphi_2^2) = 0 \qquad (19)$$

Finally, we obtain $\alpha_2 = \frac{F}{EI_2}$, resulting that way the critical force F.

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