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THEORETICAL CONTRIBUTION AND ORIGINAL EXPERIMENTS FOR DETERMINING ALL MATHEMATICAL RELATIONS FOR CALCULATING SYMMETRICAL THERMAL TENSIONS

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Abstract
The warm rolling cylinders are under stress – composite tension – caused by the mechanical tensions, on one hand and tensions produced by the temperature fields, on the other hand. Such tensions vary and are complicated. They also have excessively high values which are caused by the impact of the cylinders with the incandescent rolling product.
Our paper work is going to describe how, during laboratory tests, the composite tensions harm the rolling cylinders of a rolling mill. These tests are actually some original concepts made according to the principle of symilar that allows us to extrapolate the results we have obtained in case of the industrial rolling mills.

1. Introduction

Rolling cylinders admit complete variable stress due to all tensions produced within the temperatures fields. These tensions have excessively high values due to the impact amongst the cooled off cylinders (water cooled off) and the incandescent half-manufactured material. Our paper work refers to the determination of thermal tensions within different machine elements, as they have been less studied by specialised literature. All research has been extended by the authors of these works about warm rolling cylinders through original mathematical relations. All research has been made throughout many years and they have been financed through two research grants and several scientific works published into volumes, magazines, and presented into conferences. This paper work describes the method for determining through analytical calculation all thermal tensions within the warm rolling cylinders.

2. Theoretical elements

In order to establish the calculation relation, the whole of a rolling cylinder is assimilated to a certain body that is stressed by a complex system of forces, according to fig. 1.

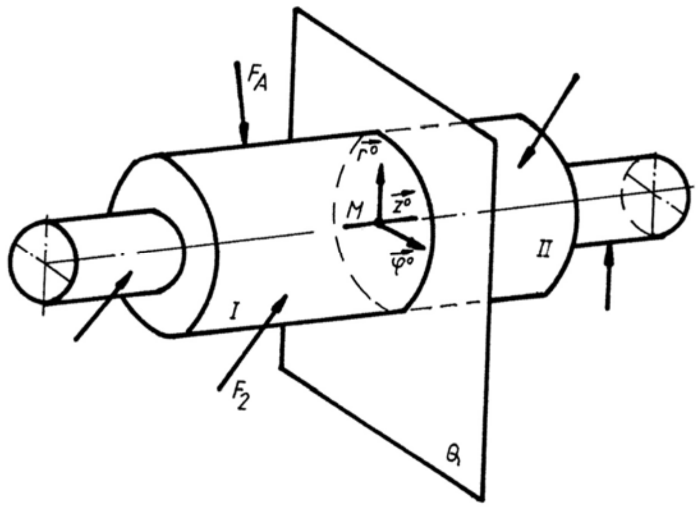

Fig.1 A rolling cylinder subject to stress by a complex system of forces

The point O of the coordinates r, φ, z, refers to making up a thriedral and a system of coordinates where vectors $\vec{r}^{\,o}, \vec{\varphi}^{\,o}, \vec{z}^{\,o}$, who are represented in fig. 1. The tension on a certain point of the cylinder is determined by the tension elements that are connected to the system of coordinating axes.

Through a lot of elementary surfaces that cross point O, we use for the calculation only those that overlap with the vectors $\vec{r}^{\,o}, \vec{\varphi}^{\,o}, \vec{z}^{\,o}$, and the vectors of the tensions in case of these surfaces; they are called $\vec{\sigma}_r, \vec{\sigma}_\varphi, \vec{\sigma}_z$. Each of these vectors could be turned into elements on the coordinate axes and we can obtain an old system of vectors, based on relations (1).

$$\begin{aligned} \vec{\sigma}_r &= \sigma_{rr}\vec{r}^{\,o} + \tau_{r\varphi}\vec{\varphi}^{\,o} + \tau_{zz}\vec{z}^{\,o} \\ \vec{\sigma}_\varphi &= \sigma_{\varphi r}\vec{r}^{\,o} + \tau_{\varphi\varphi}\vec{\varphi}^{\,o} + \tau_{\varphi z}\vec{z}^{\,o} \\ \vec{\sigma}_z &= \sigma_{zr}\vec{r}^{\,o} + \tau_{z\varphi}\vec{\varphi}^{\,o} + \tau_{zz}\vec{z}^{\,o} \end{aligned} \qquad (1)$$

Thus, tensions σ_{rr}, $\sigma_{\varphi\varphi}$, σ_{zz}, represent the normal tensions for the elementary surfaces of the coordinate axes. The values of τ have different indexes and they represent the tangential tensions on the normal elementary surfaces, on the coordinate axes. In such case, the first index refers to the direction of the normal axis on the elementary surface, meanwhile the second index refers to the direction of the tangential direction. The vectors of normal and tangential tensions referring to three-dimensional co-ordinates, [2] are described in fig. 2.

In fig. 2, axes X-Y represent the coordinates of the radial section of the rolling cylinder. On the six surfaces of the volume element which has been detached from the cylinder, there are some normal tensions and perpendicular tensions on the plane surface of the element we consider, and they are different:

In case of any calculation meant for normal tensions, (+) is used to refer to stretching, meanwhile (-) refers to compression. As far as tangential tensions are concerned, they are considered positive if their direction is alike with the positive direction of the co-ordinating axes, and if the normal tensions refer to stretching.

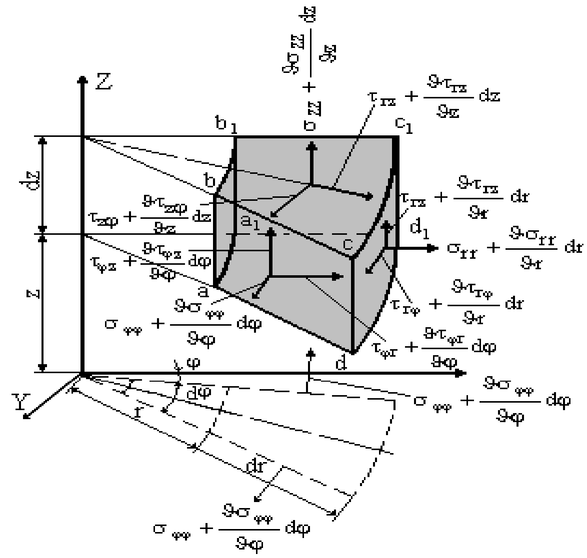

Fig.2. Vectorial representation of the normal and tangential tensions, which appear in an elementary volume of cylinders

σ_{rr} – radial normal tension, appearing in radical direction, perpendicularly on quadrangle abb_1a_1 and cc_1dd_1; σ_{zz} – axial normal tension, appearing in OZ axe, perpendicularly on quadrangle bb_1cc_1 and aa_1dd_1;$\sigma_{\varphi\varphi}$ – circumferential normal tension, appearing perpendicularly on quadrangle abcd and $a_1b_1c_1d_1$; τ_{zr} – tangential tension, appearing in radical direction in quadrangle bb_1cc_1 and aa_1dd_1;τ_{rz} – tangential tension, appearing parallel with OZ axe, in quadrangle $abb_1 a_1$ and cc_1dd_1;τ_{φ} – tangential tension, appearing in radical direction in quadrangle abcd and $a_1b_1c_1d_1$; $\tau_{r\varphi}$ – tangential tension, appearing in quadrangle abb_1a_1 and cc_1dd_1;$\tau_{\varphi z}$ – tangential tension, appearing parallel with OY axe, in quadrangle abcd and $a_1b_1c_1d_1$; $\tau_{z\varphi}$ – tangntial tension, appearing in quadrangle bb_1cc_1 and aa_1dd_1;

Tangential tensions could be positive if their direction is inverted to that of the positive direction of the co-ordinating axes, and if the normal tensions on these sides are inverted to the positive direction of their corresponding coordinating axes. Nevertheless, we should remember that all tensions produced by radial symmetrical temperature fields should use the „s" index, and they are called symmetrical tensions.

3. Mathematical relations for determining the symmetrical thermal tension

The variation of the symmetrical radial temperature fields and the average temperature variations $\bar{t}$ cause some variable tensions; their values are specific to the warm rolling cycle.

In his paper work „The Basis of Thermal-resilience", [1], paragraph 4.6 – Thermal tensions within cylinders and disks through plane and axial symmetrical fields, the author, Kovalenko A.D, analyses all thermal tensions within an empty long body, whose radial section is a cylindrical ring - whose outside radium is r_2 and inner radium is r_1. If we consider the stress of a symmetrical radial field, such cylinder causes a plane deformation which is axially symmetrical; there are some radial and circumferential temperature fields involved. They work on the radial plane section and the specific radium is $\rho = \frac{r}{r_2}$; tensions are expressed as in relation (2), (3).

$$\sigma_r = C_1 + \frac{C_2}{\rho^2} - \frac{\alpha_{T_1} E_1}{\rho^2} \int_{\rho_1}^{\rho} (T - T_0)\rho d\rho \tag{2}$$

$$\sigma_\theta = C_1 + \frac{C_2}{\rho^2} - \alpha_{T_1} E_1 (T - T_0) + \frac{\alpha_{T_1} E_1}{\rho} \int_{\rho_1}^{\rho} (T - T_0)\rho d\rho \tag{3}$$

These relations have been formulated by Kovalenko A.D. after he has established some limit conditions on the outside surface of the ring (a cylinder bore): $\sigma_r = 0$ for $\rho = \rho_1$ and $\rho = 1$. After we have performed the integration and determining the constant values C_1 and C_2, the relations (2) and (3) are rewritten in the equations (4) and (5).

$$\sigma_r = \frac{\alpha_{T_1} E_1}{\rho^2} \left[\frac{\rho^2 - \rho_1^2}{1 - \rho_1^2} \int_{\rho_1}^{1} (T - T_0)\rho d\rho - \int_{\rho_1}^{\rho} (T - T_0)\rho d\rho \right] \tag{4}$$

$$\sigma_\theta = \frac{\alpha_{T_1} E_1}{\rho^2} \left[\frac{\rho^2 - \rho_1^2}{1 - \rho_1^2} \int_{\rho_1}^{\rho} (T - T_0)\rho d\rho - \int_{\rho_1}^{\rho} (T - T_0)\rho d\rho - (T - T_0)\rho^2 \right] \tag{5}$$

After [1], paragraph 4.2, the radial movement is determined based on the study about deformation in case of plane tension, referred to in relation (6).

$$u_r = \frac{\alpha_{T_1} r_2}{\rho} \left[(1 - \nu) \int_{\rho_1}^{\rho} (T - T_0)\rho d\rho + \frac{(1 - \nu)\rho^2 + (1 + \nu)\rho_1^2}{1 - \rho_1^2} \int_{\rho_1}^{1} (T - T_0)\rho d\rho \right] \tag{6}$$

As far as axial tension is concerned σ_z within the empty cylinder, we should determine [1], the following relation.

$$\sigma_z^{`} = \frac{2\alpha_T E \nu}{(1 - \nu)(1 - \rho_1^2)} \int_{\rho_1}^{1} (T - T_0)\rho d\rho - \frac{\alpha_T E}{1 - \nu}(T - T_0) \tag{7}$$

Axial tensions occur within the cylinder, on the side surfaces we consider to be fixed, referring to the movement $u_z = 0$ when the side surfaces of the disk are tension free; in order to determine the axial tensions σ_z, we should add tension – that is a rule when there are some dilatation blocking out; $E \cdot \varepsilon_z$, (ε_z= constant). In this case, deformation ε_z should be considered the resulting element of the tension, so that it stresses if the surfaces of the cylinder disk (cylinder bore) and its value reaches 0, according to relation (7).

$$\varepsilon_z = \frac{2\alpha_T}{(1 - \rho_1^2)} \int_{\rho_1}^{1} (T - T_0)\rho d\rho \tag{8}$$

We refer to the meaning of E_1, ν_1, α_{T1}:

$$E_1 = \frac{E}{1 - \nu^2}; \quad \nu_1 = \frac{\nu}{1 - \nu}; \quad \alpha_{T_1} = \alpha_T (1 - \nu) \tag{9}$$

- where: ν - transversal contraction value;

Considering the equations we have already referred to - relations (6), (7), (8) and (9) -, the relation for determining axial tensions is comprised in relation (10).

$$\sigma_z = \sigma_z^{`} + E\varepsilon_z = \frac{\alpha_T E}{(1-\nu)}\left[\frac{2}{1-\nu}\int_{\rho_1}^{1}(T-T_0)\rho d\rho - (T-T_0)\right] \tag{10}$$

We should mention that for the relations (2)…(8), we have used the elements used in specialised literature.

If we refer strictly to warm rolling cylinders who are not empty, and we refer to the similar features between relations (9) and the fact that $r_1 = 0$, we have $\rho_1 = 0$, and we can rewrite the relations (11), (12), (13) in a proper way, so we can use it for numerical calculations:

$$\sigma_r = \frac{\alpha_T E}{(1-\nu)}\left[\int_0^1 (T-T_0)\rho d\rho - \frac{1}{\rho^2}\int_0^{\rho}(T-T_0)\rho d\rho\right] \tag{11}$$

$$\sigma_\theta = \frac{\alpha_T E}{(1-\nu)}\left[\int_0^1 (T-T_0)\rho d\rho + \frac{1}{\rho^2}\int_0^{\rho}(T-T_0)\rho d\rho - (T-T_0)\right] \tag{12}$$

$$\sigma_z = \frac{\alpha_T E}{(1-\nu)}\left[2\int_0^1 (T-T_0)\rho d\rho - (T-T_0)\right] \tag{13}$$

Thus, we have:

T – uniform temperature within the radial section of the cylinder; T_0 – initial temperature of the rolling cylinder, then time is τ = 0; α_T – liniary dilatation value, α; E – longitudinal resilience module; ν - transversal contraction value; $(T-T_0)$ – it represents the value of average temperatures $\bar{t}$ that have been determined in our study about the thermal process within warm rolling cylinders.

In case of rolling cylinders, inside radium is r_1 = 0; and $\rho_1 = \frac{r}{r_1} = 0$, there are some undetermined elements who should be solved out by the method of l`Hospital [1], [2] according to relations (14) and (15).

$$\lim_{\rho \to 0} \frac{1}{\rho^2}\int_0^{\rho}(T - T_o)\rho\, d\rho = \frac{1}{2}(T - T_o)_{\rho=0} \tag{14}$$

$$\lim_{\rho \to 0} \frac{1}{\rho}\int_0^{\rho}(T - T_o)\rho\, d\rho = 0 \tag{15}$$

Relations (11), (12), (13) could be used to rewrite the final relations (16), (17), (18) for a numerical calculation of the thermal tensions produced within radial symmetrical temperature fields within the warm rolling cylinders.

$$\sigma_{rr}^{s}(\rho,\tau) = \frac{E\alpha}{1-\mu}\left[\int_0^1 \bar{t}(\rho,\tau)\rho d\rho - \frac{1}{\rho^2}\int_0^{\rho}\bar{t}(\rho,\tau)\rho d\rho\right] \tag{16}$$

$$\sigma_{\varphi\varphi}^{s}(\rho,\tau) = \frac{E\alpha}{1-\nu}\left[\int_0^1 \bar{t}(\rho,\tau)\rho d\rho + \frac{1}{\rho^2}\int_0^{\rho}\bar{t}(\rho,\tau)\rho d\rho - \bar{t}(\rho,\tau)\right] \tag{17}$$

$$\sigma_{zz}^{s}(\rho,\tau) = \frac{E\alpha}{1-\nu}\left[2\int_0^1 \bar{t}(\rho,\tau)\rho d\rho - \bar{t}(\rho,\tau)\right] \tag{18}$$

These relations: $\bar{t}(\rho,\tau)$ - average temperature and the integral of the function that describes the temperature fields, represented by the exponential curves of the temperature diagrams; ρ - specific radium within the radial cylinder section, $\rho = \frac{r}{R}$.

As far as time τ is concerned, which influences temperature, this is a parameter who indicates the thermal tensions; it influences every stage of the rolling process – while the average temperature is different.

In order to comply with limit conditions, in case of a detached circular ring, and to the principles of thermo-resilience - in our case it refers to the surface of the warm rolling cylinders, when r = R and $\rho = \frac{R}{R} = 1$, according to [2], [3], the result is that radial tensions $\sigma^{s}_{rr}(\rho,\tau)$ are null, and:

$$\sigma^{s}_{rr}(\rho,\tau) = 0 \tag{19}$$

$$\sigma^{s}_{\varphi\varphi}(\rho,\tau) = \sigma^{s}_{zz}(\rho,\tau) \tag{20}$$

In the same time, in case of conditions on the surface of the cylinders, axial tensions $\sigma^{s}_{zz}(\rho,\tau)$, which are calculated according to the relation (18), are equal to the sum of radial tensions – relation (16), and with circumferential tensions – relation (17), the result is equal (21).

$$\sigma^{s}_{zz}(\rho,\tau) = \sigma^{s}_{rr}(\rho,\tau) + \sigma^{s}_{\varphi\varphi}(\rho,\tau) \tag{21}$$

Thus, limit conditions referring to the cylinder surface comply with Saint-Venant [2] principle, and the thermal-resilience principles amongst tensions [1]; they support the method and the research for determining that all tensions produced by the radial symmetrical tension fields are true, and relations (16), (17), (18) could be used for numerical calculations of such types of thermal tensions within the warm rolling cylinders; we could use temperature diagrams.

4. Numerical determining of the symmetrical thermal tension

Thermal tensions produced within radial asymmetrical temperature fields should be numerically calculated for each isochrone diagram which is specific to temperature variation fields; they are possible if using oscillograms belonging to temperature diagrams used for experimental rolling process. The values of those tensions produced by symmetrical temperature fields have been determined after we had solved out the numerical calculations of the relations: (16), (17), (18); and we have obtained the values of the integrals of such relations due to Simpson formula, after [2].

$$\bar{t} = \frac{1}{2\pi}\int_0^{2\pi} t(r,\varphi,\tau)d\varphi = \frac{1}{2\pi}\cdot\frac{b-a}{6n}\{f(a)+f(b)+4[f(x_1)+f(x_3)+\ldots+f(x_{2n-1})] \text{ -- } 2[f(x_2)+f(x_4)+\ldots+f(x_{2n-2})]\} \text{ pentru } n = 1,2,3. \tag{22}$$

- where: t – variable temperature described by the exponential curve on the surface or the radial section to different levels of within the rolling cylinders; r – radium within the cylinder section; τ - the time we need to perform the rolling process; it is a parameter who indicates that temperature *t* could be defined in certain moments.

Table 1

Numerical value of the symmetrical thermal tension $\sigma_{rr}^{s}, \sigma_{\varphi\varphi}^{s}, \sigma_{zz}^{s}$ ***produced by the symmetrical thermal field***

The experimental rolling with n = 35,7 rot/min;

$$\int_0^1 \bar{t}(\rho,\tau)\rho d\rho = 40{,}1576\,; \qquad \frac{1}{2}(T - T_0) = \frac{78{,}80 - 50}{2} = 14{,}4021$$

Δr [mm]	$\frac{1}{\rho^2}\int_0^\rho \bar{t}(\rho,\tau)\rho d\rho$	$\bar{t}(\rho,\tau)$	$2\int_0^\rho \bar{t}(\rho,\tau)\rho d\rho$	σ_{rr}^{s} [N/mm²]	$\sigma_{\varphi\varphi}^{s}$ [N/mm²]	σ_{zz}^{s} [N/mm²]
0	40,1676	169,52	80,3153	0	- 605,1	- 318,4
1,5	39,7953	172,48	80,3153	1,29	- 614,4	- 329,0
3	38,9434	115,33	80,3153	4,33	- 407,3	- 125,0
6	37,8987	78,80	80,3153	8,06	- 273,2	5,4
9	37,1314	78,86	80,3153	10,80	- 273,2	5,4
620	14,4021	78,84	80,3153	91,94	- 189,5	5,4

The experimental rolling with n = 93 rot/min;

$$\int_0^1 \bar{t}(\rho,\tau)\rho d\rho = 37{,}362\,; \qquad \frac{1}{2}(T - T_0) = \frac{78{,}80 - 50}{2} = 11{,}5781$$

Δr [mm]	$\frac{1}{\rho^2}\int_0^\rho \bar{t}(\rho,\tau)\rho d\rho$	$\bar{t}(\rho,\tau)$	$2\int_0^\rho \bar{t}(\rho,\tau)\rho d\rho$	σ_{rr}^{s} [N/mm²]	$\sigma_{\varphi\varphi}^{s}$ [N/mm²]	σ_{zz}^{s} [N/mm²]
0	37,3628	203,58	74,7256	0	- 593,4	- 460,0
1,5	36,5686	152,01	74,7256	2,83	- 543,4	- 279,4
3	35,7161	93,50	74,7256	5,89	- 327,9	- 67,0
6	34,8239	72,54	74,7256	9,06	- 249,9	7,7
9	33,9215	72,54	74,7256	8,52	- 246,7	7,7
592,5	11,2741	72,54	74,7256	93,13	- 165,8	7,7

The experimental rolling with n = 50 rot/min;

$$\int_0^1 \bar{t}(\rho,\tau)\rho d\rho = 37{,}389\,; \qquad \frac{1}{2}(T - T_0) = \frac{73{,}156 - 50}{2} = 11{,}5781$$

Δr [mm]	$\frac{1}{\rho^2}\int_0^\rho \bar{t}(\rho,\tau)\rho d\rho$	$\bar{t}(\rho,\tau)$	$2\int_0^\rho \bar{t}(\rho,\tau)\rho d\rho$	σ_{rr}^{s} [N/mm²]	$\sigma_{\varphi\varphi}^{s}$ [N/mm²]	σ_{zz}^{s} [N/mm²]
0	37,3892	196,95	74,7785	0	- 703,1	- 436,1
1,5	36,7740	141,64	74,7785	2,2	- 503,4	- 238,7
3	35,9354	97,24	74,7785	5,2	- 341,9	- 80,2
6	35,0534	73,15	74,7785	8,3	- 252,8	5,8
9	34,3619	73,15	74,7785	10,8	- 252,8	5,8
592,5	11,5781	73,15	74,7785	92,1	- 169,0	5,8

The initial temperature for warming up the rolling cylinders is also important to these relations, when time is $\tau = 0$, we know that, according to l'Hospital rule, we have the following calculated value.

- where: T - average temperature to certain levels of the radial section of a rolling cylinder; $T = \bar{t}$; T_0 – initial temperature of warming of the rolling cylinders.

In order to do some calculation, we have considered the initial warming temperature within the rolling cylinders is up to 50^{o} C. In order to determine the thermal tensions produced by symmetrical temperature fields in case of all rolling cylinders calibres, we have performed them for Δr in depth, underneath the surface of the calibres of the radial section of the rolling cylinders. Such depths correspond to some determined levels, which have been measured in case of temperature variations within the gravel filters rolling cylinders. These levels are: Δr = 0; 1,5; 3; 6; 9; R mm. We should also consider that the influence of the symmetrical temperature fields produces only main tensions, such as: σ^{s}_{rr} - radial tensions; $\sigma^{s}_{\varphi\varphi}$ - circumferential tensions; σ^{s}_{zz} - axial tensions; all the other tensions - also tangential tensions - are equal to 0. The results of the calculations based on relations (16), (17), (18), and the main calculation elements are written down in table 1. In order to calculate them, we have used the following constant values: $E = 2{,}1\cdot10^{4}$daN/mm^2; $\alpha = 11{,}9\cdot10^{-6}$1/^{0}K; ν = 0,3; Eα/2(1-ν) = 0,357.

5. Results and conclusions

The analysis of all the results of the calculations comprised in Table no. 1 describes that radial tensions σ^{s}_{rr} on the surface of the cylinder are equal to 0, and the first layers have sub-unit values. These tensions have a positive value and they enable a higher length of the rolling cylinder turns so they get closer to the symmetry axis.

Circumferential tensions $\sigma^{s}_{\varphi\varphi}$ are only negative; they produce some compression and decrease as close as they get to the surface of the cylinder and to the symmetry axis. These tensions are associated with axial tensions produced by asymmetrical tension fields; they cause some longitudinal fissures on the surface of the rolling cylinders. Axial tensions σ^{s}_{zz} reach their highest values on the surface of the cylinders, they have negative values, and decrease as they get closer to the core of the cylinder, and reach their absolute value.

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