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THE SURFACE TEMPERATURE IN SLIDING CONTACTS OF GEAR WORKING FLANKS
PART 1 – ANALYTICAL SOLUTIONS

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Abstract. The paper examines the analytical solution for the surface temperature in sliding contacts, assuming convection of heat. The convection of heat and surface temperature can be analyzed by Laplace transforms and the method of Green's functions. The solution technique is given in general form, i.e. the temperature field can be achieved for arbitrary pressure distributions.

1. INTRODUCTION

Temperature study in the contact zone of rolling and sliding concentrated contacts is frequent in literature. It is seldom related to the thermal flow analysis in the wheel-railway contact.

Knothe şi Liebelt, [3], give a detailed integral form of the heat flow produced during wheel-railway contact. The problem of the temperature of limiting surfaces for rolling and sliding unlubricated contact is revised by Fischer, Werner şi Yan, [1], and Fischer, Werner şi Knote, [2].

The present paper intends a theoretical analysis of thermal exchange and of surface temperature in rolling and sliding contact between flanks of gear teeth. The first part of the work presents the analytical solutions required in solving the problem.

2. CONVECTION OF HEAT

Heat transfer by convection is characterised by the equation with partial derivatives of parabolic type, having the form:

$$\frac{\partial^2\theta}{\partial z^2}-\frac{1}{a_s^2}\frac{\partial\theta}{\partial t}=0; \qquad (1)$$

where z denotes the depth, t is the time of passing through contact, a_s is the diffusion coefficient, $\theta=T-T_S$.

The scheme of sliding contact and the prediction for temperature distribution is shown in Figure 1.

In order to find the solution of the equation with partial derivatives, of parabolic type, the operational calculus is used by applying Laplace transforms and Green functions. The complex variable from Laplace space is denoted s=Laplace(t).

The Laplace transform of temperature with respect to variable t is represented by the following notation:

$$\Theta(z,s)=\text{Laplace}\,\theta(z,t) \qquad (2)$$

Since at contact entry the relative temperature, θ, is zero, the initial condition can be expressed by the relation:

$$\theta(z, t = 0) = 0 \tag{3}$$

and consequently the Laplace transform of the derivative with respect to t is:

$$\text{Laplace}\frac{\partial \theta}{\partial t} = s\,\Theta \tag{4}$$

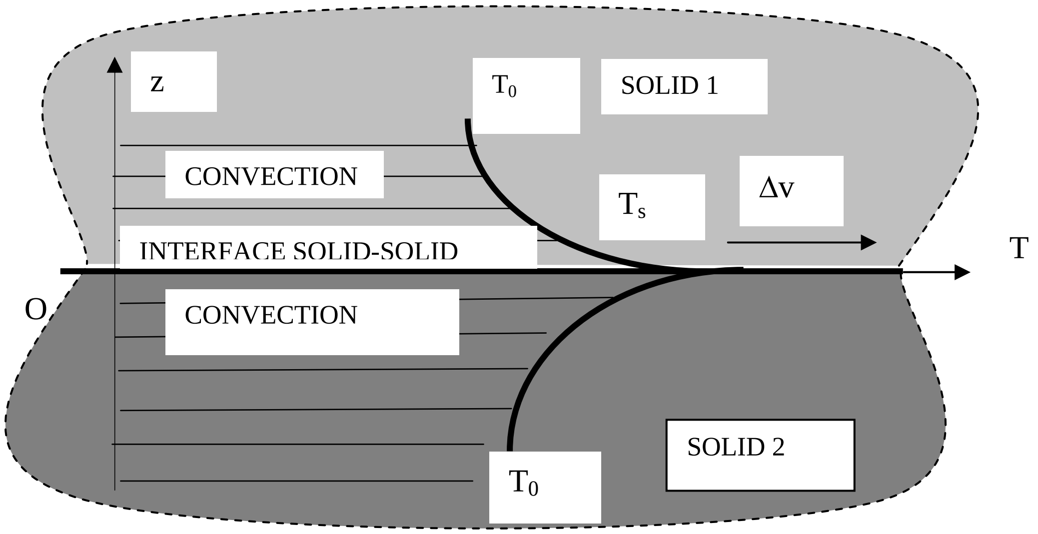

Fig. 1. Scheme of sliding contact

Another aspect considers the rule of differentiation under the Laplace integral with respect to a parameter, and it is obtained:

$$\text{Laplace}\frac{\partial^2 \theta}{\partial z^2} = \frac{\partial^2 \Theta}{\partial z^2} \tag{5}$$

After transforming, the considered equation takes the following form:

$$\frac{\partial^2 \Theta}{\partial z^2} - \frac{s}{a_s^2}\Theta = 0. \tag{6}$$

The solution of the differential equation resulting from Laplace transform is given by the relation:

$$\Theta = C_1 \exp\left(-\frac{z}{a}\sqrt{s}\right) + C_2 \exp\left(\frac{z}{a}\sqrt{s}\right) \tag{7}$$

The constants from integration C_1 and C_2 are found by imposing the limit conditions. The first condition considers that for $z \to \infty$ the temperature must be finite and in this case, the constant C_2 should be zero.

The solution takes a simpler form, as it follows:

$$\Theta = C_1 \exp\left(-\frac{z}{a_s}\sqrt{s}\right). \tag{10}$$

One returns to the first limit condition. In the Laplace space, the heat flow density, Q, is given by the relation:

$$Q = \mathrm{Laplace}(q). \tag{11}$$

In fact, considering the contact between two teeth from the same material, the following relation is valid:

$$Q = \mathrm{Laplace}(q) = \mathrm{Laplace}\left(\frac{\mu\, p\, \Delta v}{2}\right) \tag{12}$$

where:
μ is the friction or traction coefficient;
p is the contact pressure;
Δv is the relative velocity between limiting surfaces of contact.

On the other hand, the heat flow density represents the product between the thermal conductivity coefficient k_f and the temperature gradient on transverse direction that is the temperature derivative with respect to z. The above product must be taken with minus sign because decrease of thermal flow density with temperature increase.

$$q = -k_f \left.\frac{\partial \theta}{\partial z}\right|_{z=0}. \tag{13}$$

Considering that the Laplace transform of the function derivative with respect to a parameter equals the derivative of the function's Laplace transform, by Laplace transform of thermal flow density the image of the original is obtained:

$$Q = -k_f \,\mathrm{Laplace}\frac{\partial \theta}{\partial z} = -k_f \frac{\partial}{\partial z}\left(\mathrm{Laplace}\,\theta\right) = -k_f \frac{\partial \Theta}{\partial z} \tag{14}$$

Calculating the derivative for z=0, after some computation, the constant C_1 is obtaimed:

$$C_1 = \frac{a_f Q}{k_s \sqrt{s}}. \tag{15}$$

The image of temperature in Laplace space after replacing the integration constant, takes the following final form:

$$\Theta = \frac{a_s Q}{k_s \sqrt{s}} \exp\left(-\frac{z}{a_s}\sqrt{s}\right). \tag{16}$$

Using the Mathcad application or a table with usual Laplace transforms and considering the convolution theorem, the original is obtained, or the temperature produced by convection.

$$\theta = \frac{a_s}{k_s\sqrt{\pi}} \int_0^t \frac{q(u)}{\sqrt{t-u}} \exp\left(-\frac{z^2}{h_T^2}\right) du, \tag{17}$$

where the depth of the thermal zone appropriate for convection thermal transfer, h_T is:

$$h_T = 2\sqrt{a_s(t-u)} \tag{18}$$

By substituting the diffusion coefficient a_f with its definition relation, a relation similar to the relations found in the literature is obtained:

$$\theta = \frac{1}{\sqrt{\pi\, k_s c_s \rho_s}} \int_0^t \frac{q(u)}{\sqrt{t-u}} e^{-\frac{z^2}{h_T^2}} du. \tag{19}$$

The thermal flow density with respect to the variable under the integral used in the calculus will be:

$$q(u) = \frac{\mu\, p(u)\, \Delta v}{2}, \tag{20}$$

where μ is the searched for local traction coefficient and p(u) is the contact pressure.

The highest temperatures are on the surface, at z=0 and there are found by the use of relation:

$$\theta_S = \frac{1}{2\sqrt{\pi\, c_s \rho_s k_s}} \int_0^t \frac{\mu\, p(u)\, \Delta v}{\sqrt{t-u}} du. \tag{21}$$

CONCLUSION

The relative temperature from contact due to convective thermal transfer is a punctual function. It decreases by exponential function with respect to depth, z, and increases with time or duration of loading, reaching maximum values towards the contact exit region.

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