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ESTIMATION OF THE INFLUENCE OF THE MICRO-PROFILE IN THE DYNAMICS OF ANTIFRICTION BEARINGS

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Abstract: The paper studies the influence of the micro-profile of an antifriction bearing having a micro-profile that influences its behaviour. It is considered that the micro-profile is a random stationary process, with an index of regularity rapidly decreasing; the spectral composition is obtained by a linear transformation. It is solved an isoperimetric problem that gives the characteristic of transfer of the micro-profile. The behaviour of a shaft-bearing system modelled as three degrees of freedom system is analyzed, in the conditions of the determined micro-profile.

1. INTRODUCTION

Taking into account the micro-profile of the baring represents in fact a valorisation of an important excitation source appearing in the shaft-bearing system, in the most of previously studied situations it was considered the bearing having a sinusoidal shape.

The profile of the bearing surface is build of three distinct components: the macro-profile, the micro-profile and the ruggedness, which influence in a different way the behaviour.

The micro-profile is considered to be shaped by long and gentle dislevelments. This one generates vibrations of the shaft structure, but because of its small wave-lengths, they do not generate major modification on the working regimes.

The use of the micro-profile as an excitation source has some specific features. First of all, the amplitude of the micro-profile is limited. Then, it does not contain variable components. It is a stationary random phenomenon with the index of regularity rapidly decreasing. Thus, two arbitrary portions of the micro-profile situated to a very close distance can be considered independent, meanwhile like a profile it is a random phenomenon with a stationary increasing.

Let us consider the micro-profile $h(l)$ whose spectral composition is obtained by the linear transform H_q, defined by:

$$q(l) = H_q h(l) \tag{1}$$

where:

$$H_q(i\lambda) = \begin{cases} 0 & \text{for} \quad |\lambda| < \lambda_m \\ 1 & \text{for} \quad \lambda_m < |\lambda| < \lambda_M \\ 0 & \text{for} \quad \lambda_M < |\lambda| \end{cases} \quad , \qquad \lambda_m = \frac{2\pi}{L_M}, \ \mathrm{L_M} = \frac{2\pi}{L_m} \tag{2}$$

where L_M and L_m are the maximum and minimum wave-lengths of the micro-profile.

The transform H_q is named the transform of micro-profile [1], and so, the micro-profile is determined by the following elements: the minimum parameters of vibrations, the limiting value of the amplitude of micro-profile and the limiting value of the maximum frequency.

For the given micro-profile *h(l),* it is considered that Δ_h is the deformation of the elastic elements, during its displacement. If it is noted by A the corresponding transform, then, like in (1), it results that Δ_h=Ah. If Δ_q is the deformation of the elastic elements in their motion on the micro-profile q, it results Δ_q=Aq.

For the determination of the transform of micro-profile in the field of low frequencies, it is necessary that:

$$|q| \le q_M, \quad |\Delta_h - \Delta_q| = min \tag{3}$$

The variational problem (2) is studied considering the micro-profile *h(l)* as a normal random generalized stationary process, and the transforms A and H_q, as linear transforms.

The conditions that determine the form of the transform H_q become:

$$D_q \equiv M\left\{|q|^2\right\} \le D_{q\,max} \;\; \text{si} \;\; M\left\{|\Delta_h - \Delta_q|^2\right\} = min \tag{4}$$

where D_{qmax}= (0,05-0,1)q^2_{max} [1] and M is the operator of mathematical expectation [2].

By introducing the Lagrange multiplier, the variational problem consists in the minimization of the functional, defined as follows:

$$f = M\left\{|\Delta_h - \Delta_q|^2 + \lambda |q|^2\right\} = min \tag{5}$$

where λ is the Lagrange multiplier.

2. THE STUDY OF THE ISOPERIMETRIC PROBLEM

In order to solve the problem (5), it is supposed that the aspect of the transform and the spectral density of the excitation proceed from the micro-profile *h(l),* which in the field of low frequencies is determined from the relation:

$$S_h(\omega) = K_n {v_a}^{n-1} \omega^{-n} \tag{6}$$

where ω is the pulsation of the random process *h(t)*, K_n-a coefficient depending on the micro-profile, and v_a-the speed of the vehicle, n∈[2,4].

The limit cases in (6) are:

$$S_h(\omega) = K_2 v_a \omega^{-2} \tag{7}$$

and

$$S_h(\omega) = K_4 {v_a}^3 \omega^{-4} \tag{7'}$$

Because the transform H_q is considered only for the field of low frequencies that are characteristic for the micro-profile of bearings, it is sufficient that the transform A to have the same characteristic and, after calculus, it becomes:

$$A(i\omega) = \frac{(i\omega)^2}{(i\omega)^2 + 2\psi_0 \omega_0 i\omega + \omega_0^2} \tag{8}$$

where y_0 is the coefficient of relative damping of the vibrations, and ω_0 is its minimum proper pulsation.

The variation of the functional:

$$f = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(|A|^2 + |AH_q|^2 - |A|^2 2R_e H_q + \lambda |H_q|^2\right)S_h d\omega \tag{9}$$

in relation to H_q, leads to a functional equation that determines the characteristic of transfer of the transform H_q, under the form:

$$H_q\left(|A|^2 + \lambda\right)S_h - |A|^2 S_h = F \tag{10}$$

where F is an analytic function, defined in the right half-plane [3].

When, for a certain micro-profile, there are not imposed conditions, physically feasible, from (10) it results:

$$H_q\left(|A|^2 + \lambda\right)S_h - |A|^2 S_h = 0 \tag{11}$$

whence $H_q = \dfrac{|A|^2}{|A|^2 + \lambda}$.

For the particular case from (7), taking account of (8) and (10), it results:

$$\begin{aligned} H_q = &\frac{(1+\lambda)(i\omega)^4 + \lambda(2\omega_0^2 - 4\psi_0^2)(i\omega)^2 + \lambda\omega^4}{\left[(i\omega)^2 + 2\psi_0\omega_0 i\omega + \omega_0^2\right]\left[(i\omega)^2 - 2\psi_0\omega_0 i\omega + \omega_0^2\right]}\cdot\frac{K_2}{(i\omega+0)(-i\omega+0)} - \\ &- \frac{(i\omega+0)^2(-i\omega+0)^2}{\left[(i\omega)^2 + 2\psi_0\omega_0 i\omega + \omega_0^2\right]\left[(i\omega)^2 - 2\psi_0\omega_0 i\omega + \omega_0^2\right]}\cdot\frac{K_2}{(i\omega+0)(-i\omega+0)} = F \end{aligned} \tag{12}$$

where, from [3],

$$(i\omega+0)^{-1} = \lim_{\alpha\to 0}(i\omega-\alpha)^{-1} = \frac{1}{i\omega} - \pi\delta(\omega)$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}(i\omega+0)^{-1}e^{i\omega t}d\omega = \begin{cases}1 & \text{pentru} \quad t>0\\ 0 & \text{pentru} \quad t<0\end{cases}$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}(i\omega)^{-1}e^{i\omega t}d\omega = \begin{cases}0{,}5 & \text{pentru} \quad t>0\\ -0{,}5 & \text{pentru} \quad t<0\end{cases}$$

Knowing that the Lagrange multiplier λ<<1, it results:

$$(1+\lambda)(i\omega)^4 + \lambda\left(2\omega_0^2 - 4\psi_0\omega_0^2\right)(i\omega)^2 + \lambda\omega_0^4 \cong \left[(i\omega)^2 + \sqrt{2\sqrt[4]{\lambda}}\,\omega_0 i\omega + \sqrt{\lambda}\,\omega_0^2\right]\left[(i\omega)^2 - \sqrt{2\sqrt[4]{\lambda}}\,\omega_0 i\omega + \sqrt{\lambda}\,\omega_0^2\right]$$

and from (12), it results the simplified form:

$$H_q \cong \frac{(i\omega)^2}{(i\omega)^2 + \sqrt{2\sqrt[4]{\lambda}}\,\omega_0 i\omega + \sqrt{\lambda}\,\omega_0^2} \tag{12'}$$

If instead of minimizing the deformation of the springs, it is minimized the difference of vertical accelerations, it obtains the transform A of form:

$$A(i\omega) = \frac{\left(2\psi_0\omega_0 i\omega + \omega_0^2\right)(i\omega)^2}{(i\omega)^2 + 2\psi_0\omega_0 i\omega + \omega_0^2}$$

i.e. the vibrations are very little different.

The dispersion of q(t) is determined with the relation [3]

$$D_q = \frac{1}{2\pi}\int_{-\infty}^{\infty}|H_q|^2 S_h d\omega \tag{13}$$

which, for the forms (7) and (7'), becomes: $D_q = \dfrac{K_2 v_a}{2\sqrt{2\sqrt[4]{\lambda}}\,\omega_0}$ and $D_q = \dfrac{K_4 v_a}{2\sqrt{2\sqrt[4]{\lambda^3}}\,\omega_0^3}$

In the situation of a characteristic of transfer of form:

$$H_q = \frac{(i\omega)^2}{(i\omega)^2 + 2\psi_1\omega_1 i\omega + \omega_1^2}$$

The expressions for the two particular cases become:

$$D_q = \frac{K_2 v_a}{2}\frac{1}{2\psi_1\omega_1} = \frac{K_2}{2}\frac{1}{2\psi_1\theta_1}$$
$$D_q = \frac{K_4 v_a^3}{2}\frac{1}{2\psi_1\omega_1^3} = \frac{K_4}{2}\frac{1}{2\psi_1\theta_1^3} \quad (14)$$

where $\theta_1 = \frac{\omega_1}{v_a}$ is estimated from the condition of limitation of D_q,

$$\theta_1 = \frac{\omega_1}{v_a} \geq \frac{K_2}{2 \cdot 2\psi_1 D_{q\,max}}$$
$$\theta_1 = \frac{v_1}{v_a} \geq \sqrt[3]{\frac{K_4}{2 \cdot 2\psi_1 D_{q\,max}}} \quad (15)$$

where $\psi_1 \cong \sqrt{0.5}$, and D_{qmax} is the maximum admitted value of the micro-profile.

From the effected analysis, it results that, when the ratio: $\omega_1 / \omega_0 = 0.3 - 0.2$, i.e. the frequency of the section of transform of micro-profile H_q is of 3-5 times smaller than the frequency of proper vibrations, the vibrations of the shaft-bearing system on the profile and micro-profile have important variations.

3. MODEL FOR THE STUDY

One of the cases, more frequently encountered, is the one of a dynamic system, like in the fig.1. It is considered that the frictions are negligible.

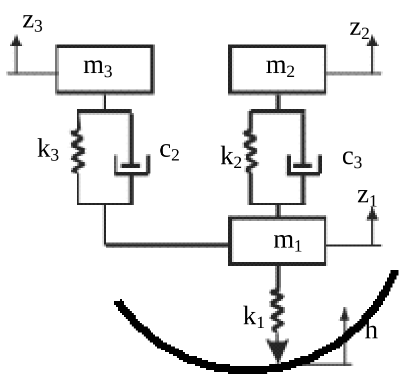

Fig.1

The differential equations of the motion are:

$$m_2\ddot{z}_2 + 2c_2(\dot{z}_2 - \dot{z}_1) + k_2(z_2 - z_1) = 0$$
$$m_1\ddot{z}_1 - 2c_2(\dot{z}_2 - \dot{z}_1) - k_2(z_2 - z_1) + k_1(z_1 - h) = 0$$
$$m_3\ddot{z}_3 + 2c_3(\dot{z}_3 - \dot{z}_1) - k_3(z_3 - z_1) = 0 \quad (16)$$

where m_2 is the mass of the load, z_2, z_1 and z_3-the vertical coordinates, k_2, k_1, k_3-the rigidities of the springs and c_2, c_3-the coefficients of the dampers of the model.

This system of linear differential equations can be transformed in an equivalent system, by introducing the following notations:

$$\Delta = z_2 - z_1;\ \ \Delta_i = z_3 - z_1;\ \ \frac{dz_2}{dt} = -2\Psi_2\omega_2\Delta + \dot{x}_1;\ \ \frac{dx_1}{dt} = -\omega_2^2\Delta;\ \ \frac{dz_0}{dt} = 2\alpha\Psi_2\omega_2\Delta + \dot{x}_2;$$
$$\frac{d\dot{x}_2}{dt} = \omega_1^2(h - z_1) + \alpha\omega_2^2\Delta; \frac{dz_3}{dt} = -2\Psi_3\omega_3\Delta_i + \dot{x}_3; \frac{dx_3}{dt} = -2\omega_3^2\Delta_i \quad (17)$$

The output signal Δ_i determines the characteristic of transfer of the dynamic transformer of micro-profile, which has the form:

$$H_i(p)=\frac{\left(m_r p^2+2c_2 p+k_2\right)c_1}{m_2 m_1 p^4+2c_2(m_2+m_1)p^3+(m_2 k_2+m_1 k_2+m_2 k_1)p^2+2c_2 c_1 p+k_2 k_1}\cdot\frac{m_3 p^2}{m_3+2c_3 p+k_3} \quad (18)$$

or

$$H_i(p)=\frac{\left(p^2+2\psi_2\omega_2 p+\omega_2^2\right)\omega_1^2}{p^4+2\psi_2(\omega_2+\alpha\omega_2)p^3+(\alpha\omega_2^2+\omega_2^2+\omega_1^2)p+2\psi_2\omega_2 p+\omega_2{}^2\omega_1^2}\cdot\frac{p^2}{p^2+2\psi_3\omega_3 p+\omega_3{}^2}$$

This can be transformed in the product of functions of transfer of three vibrating elements, of the form:

$$H_i(p)=\frac{p^2+2\psi_2\omega_2 p+\omega_2^2}{p^2+2\psi_3\omega_2 p+\omega_3^2}\cdot\frac{\omega_1^2}{p^2+2\psi_2\omega_2 p+\omega_2^2}\cdot\frac{p^2}{p^2+2\psi_3\omega_3 p+\omega_3^2} \quad (19)$$

This relation characterizes the field of low frequencies, as well as the one of high frequencies.

So, the problem was again reduced to an isoperimetric problem, like in section 1, i.e.

$$M\{\left|q-q_i\right|^2\}=min$$

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