MULTI-OBJECTIVE OPTIMIZATION OF AN INPUT SHAFT SUB-ASSEMBLY DESIGN

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Abstract: In this paper, we propose a population-based evolutionary multi-objective optimization approach based on the concept of Pareto optimality, in order to design an input shaft sub-assembly. The goals of the optimization were to minimize both the mass of the input shaft sub-assembly (including the mass of the pinion and the mass of the two tapered rolling bearing) and the bending deflection. In the actually optimal design problem solved in this work, three genes and eighteen constraints were taken into consideration. The Pareto optimal set was obtained by running a new genetic algorithm inspired by Non-Dominated Sorting Genetic Algorithm II (NSGA-II) implemented in Cambrian v.3.09 software which belongs to the Optimal Design Centre of Technical University of Cluj-Napoca.

1. INTRODUCTION

Multi-objective optimization (also called multi-criteria optimization, multi-performance or vector optimization) can be defined as the problem of finding [5]: “a vector decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. These functions form a mathematical description of performance criteria which are usually in conflict with each other. Hence, the term optimize means finding such a solution which would give the values of all the objective functions acceptable to the designer.” In these terms, we want to solve multi-objective optimization problems of the form:

Minimize:

$$[f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{1}$$

subject to the *m* constraints:

$$g_i(\vec{x}) \prec 0,\ i = \overline{1, m}\ \ (\prec \text{ means } \le, < \text{ or } =) \tag{2}$$

where: the *k* objective functions are $f_i : \mathrm{R}^n \to \mathrm{R}$. We call $\vec{x} = [x_1, x_2, \ldots, x_n]$ the vector of decision variables. We want to determine from among the set $\Im$ of all sets of numbers which satisfy (2), the particular set $\vec{x}^* = [x_1^*, x_2^*, \ldots, x_n^*]$ which yields the optimum values of all the objective functions. It is rarely the case that there is a single point that simultaneously optimizes all the objective functions. Therefore, we normally look for trade-offs, rather than single solutions when dealing with multi-objective optimization problems. The notion of optimality is therefore, different in this case. The most commonly adopted notion of optimality is called Pareto optimality. We say that a vector of decision variables $\vec{x}^* \in \Im$ is Pareto optimal if there does not exist another $\vec{x} \in \Im$ such that $f_i(\vec{x}) \le f_i(\vec{x}^*)$ for all $i = \overline{1, k}$ and $f_j(\vec{x}) < f_j(\vec{x}^*)$ for at least one j. Unfortunately, the concept of Pareto optimality almost always gives not a single solution, but rather a set of solutions called the Pareto optimal set. The vectors $\vec{x}^*$ corresponding to the solutions included in the Pareto optimal set are called non-dominated. The plot of the objective functions whose non-dominated vectors are in the Pareto optimal set is called the Pareto front. In the following paragraphs we will identify and propose the variables (genes), the objective functions, and the constraints which will be aggregated in the multi-objective optimization program.

2. DESIGN PROBLEM

The aim of our work is to perform a multi-objective optimization in order to obtain an input shaft sub-assembly (fig.1) as lighter as possible and with a minimal bending deflection. Obviously, these two objectives are in conflict and so it is impossible to reach such a result. Therefore we will use the Pareto front in order to deal with these goals. The actual design input data are listed bellow.

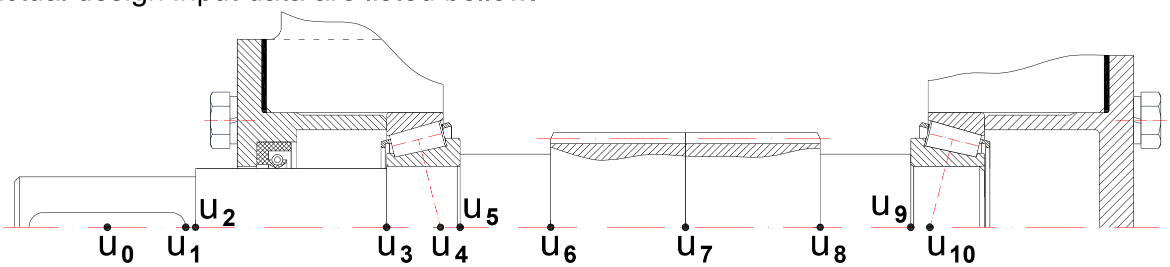

Fig.1. The input shaft sub-assembly sketch

3. DESIGN INPUT DATA

Data regarding shaft material
Shaft material: 41MoCr11 (quenched and tempered): $HB_1 = 3000$ MPa;
Strength tensile, [MPa]: $\sigma_r = 1000$;
Allowable bending stress at static loading: $\sigma_{aI} = 330$ MPa;
Allowable bending stress at pulsating load: $\sigma_{aII} = 150$ MPa;
Allowable bending stress at alternate load: $\sigma_{aIII} = 90$ MPa;
Coefficient of loading: $\alpha = 0.6$;
Fatigue strength at alternate load: $\sigma_{-1} = 500$ MPa;
Fatigue strength at alternate load: $\tau_{-1} = 275$ MPa;
Fatigue strength at pulsating load: $\tau_0 = 495$ MPa;
Young's modulus: $E = 2.1 \cdot 10^5$ MPa;
Shear modulus: $G = 86000$ MPa;
Material density: $\rho_{mat} = 7{,}85 \cdot 10^{-6}$ kg/mm^3;
Allowable fatigue strength coefficient: $c_a = 1{,}5$;
Allowable bending deflection: $\delta_a = 0{,}053$ mm;
Allowable deflection at the supporting point angle: $\varphi_a = 0{,}053$ rad.
Forces and torsion moments:
Tangential force: $F_{t1} = 1689$ N;
Radial force: $F_{r1} = 669$ N;
Axial forces: $F_{a1} = 374$ N;
Torsion moment for the shaft: $T_1 = 26639$ Nmm.
Data regarding gearing geometry:
Centre distance: $a_w = 80$ mm;

Number of pinion's teeth: $z_1 = 34$;
Pinion's width: $b_1 = 55$ mm;
Pinion root diameter: $d_{f1} = 34.326$ mm;
The area of frontal face of teeth: $A_{dp} = 3.62$ mm^2.

In order to perform the optimal design of the sub-assembly is necessary to set up:

- The variables (genes) that uniquely describe the problem (both the objective function and the constraints);
- The objective functions;
- The constraints of the problem.

4. OPTIMAL DESIGN OF THE SUB-ASSEMBLY

4.1. Genes

The first step of the setup of the optimization program consists in the identification of the variables that are able to uniquely describe the problem. These variables should be involved in the calculus of the objective functions and the constraints both. Hereinafter, since the optimization will be performed using genetic algorithms, instead of the term *variable* we will use the term *gene*.

It is worthy to mention here that the notion of *gene* is rather larger that the usual meaning of a variable. A *gene* could be a real or an integer number, as well as an array, a matrix or a list. The objects of the list could be anything one could imagine and that have a numerical coding (representation).

The authors consider that there are three genes that can describe completely the optimization problem. These genes are listed below:

Gene 1: i_1 – index number for the shaft diameter, d_{ca} (values between 0…63);
Gene 2: i_2 – index number for the radial shaft seal of the input shaft, d_{1m} (values between 0…127);
Gene 3: i_3 – index number for the tapered rolling bearing of the input shaft (values between 0…63).

4.2. Objective functions

The objective functions chosen for this application are the mass of the input shaft sub-assemblies (including the mass of the pinion and the mass of the two tapered rolling bearings) and the bending deflection. Note that the mass of the pinion is precisely computed since the area of frontal face of teeth is accurately determined [7].

Obj.1 The mass of the sub-assembly:

$$M_{sub\_assembly} = V_{pinion} \cdot \rho_{mat} + 2 \cdot M_{bearing} \rightarrow \min \quad (3)$$

where:

V_{pinion} – volume of the shaft (witch include also the volume of the pinion), [mm^3];
$M_{bearings}$ – mass of the tapered rolling bearings, [kg].

Obj.2 The bending deflection:

$$\delta(u_7) = \sqrt{\delta_H^{\,2}(u_7) + \delta_V^{\,2}(u_7)} \rightarrow \min \quad (4)$$

where:

$\delta_H^2(x)$ – bending deflection in horizontal plane at the abscissa x, [mm];
$\delta_V^2(x)$ – bending deflection in vertical plane at the abscissa x, [mm].

4.3. Constraints

We attached to the optimization problem a set of forty-one constraints. All values of these constraints have to be negative or zero.

C1. The radial shaft seal diameter should be greater than the input end shaft diameter (fig. 2).

$$g_1 = 1.15 \cdot \frac{d_{ca}}{d_{1m}} - 1 \quad (4)\backslash$$

where:

d_{ca} – input end shaft diameter, [mm];
d_{1m} – radial shaft seal diameter, [mm].

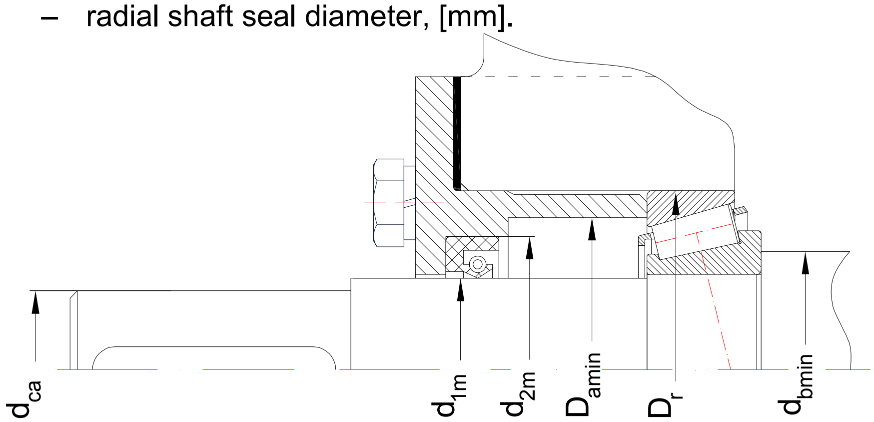

Fig.2. Mounting dimensions for radial shaft seal and the tapered rolling bearing

C2. The output radial shaft seal diameter should be lower than the inner diameter of bearing cap (fig. 2).

$$g_2 = \frac{d_{2m} + 1}{D_{a\min}} - 1 \quad (4)$$

where:

d_{2m} – output radial shaft seal diameter, [mm];
D_{amin} – inner diameter of bearing cap, [mm].

C3. The pinion root diameter should be greater than the input shaft collar diameter.

$$g_3 = \frac{d_{b\min}}{d_{f1}} - 1 \quad (5)$$

where:

d_{bmin} – input shaft collar diameter, [mm];
d_{f1} – pinion root diameter, [mm].

C4. The maximum value of Von Misses equivalent stress in the input shaft should be inferior to the allowable bending stress.

$$g_4 = \frac{\sigma_e(x)}{\sigma_{aiIII}} - 1 \quad (6)$$

C5. The fatigue strength coefficient of the input shaft in section 0 (fig. 3) must be greater or equal to the allowable fatigue strength coefficient for the shaft.

$$g_5 = \frac{c_a}{CSO(u_0)} - 1 \quad (7)$$

where:

CSO(x) – function which returns the value of the fatigue strength coefficient (in section of abscissa *x*).

$$CSO(x) = \frac{C_\sigma(x) \cdot C_\tau(x)}{\sqrt{C_\sigma^2(x) + C_\tau^2(x)}} \tag{8}$$

where:

$C_\sigma(x)$ – fatigue strength coefficient for bending;
$C_\tau(x)$ – fatigue strength coefficient for torsion.

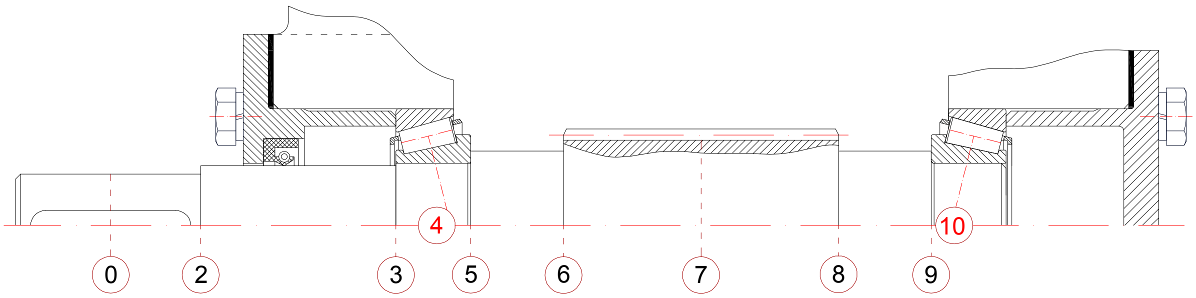

Fig.3. Sections of the shaft where the fatigue strength is checked

C6. The fatigue strength coefficient of the input shaft in section 2 (fig. 3) must be greater or equal to the allowable fatigue strength coefficient for the shaft.

$$g_6 = \frac{c_a}{CSO(u_2)} - 1 \tag{9}$$

C7. The fatigue strength coefficient of the input shaft in section 3 (fig. 3) must be greater or equal to the allowable fatigue strength coefficient for the shaft.

$$g_7 = \frac{c_a}{CSO(u_3)} - 1 \tag{10}$$

C8. The fatigue strength coefficient of the input shaft in section 5 (fig. 3) must be greater or equal to the allowable fatigue strength coefficient for the shaft.

$$g_8 = \frac{c_a}{CSO(u_5)} - 1 \tag{11}$$

C9. The fatigue strength coefficient of the input shaft in section 6 (fig. 3) must be greater or equal to the allowable fatigue strength coefficient for the shaft.

$$g_9 = \frac{c_a}{CSO(u_6)} - 1 \tag{12}$$

C10. The fatigue strength coefficient of the input shaft in section 8 (fig. 3) must be greater or equal to the allowable fatigue strength coefficient for the shaft.

$$g_{10} = \frac{c_a}{CSO(u_8)} - 1 \tag{13}$$

C11. The fatigue strength coefficient of the input shaft in section 9 (fig. 3) must be greater or equal to the allowable fatigue strength coefficient for the shaft.

$$g_{11} = \frac{c_a}{CSO(u_9)} - 1 \tag{14}$$

C12. The bending deflection of the input shaft in section 0 (fig. 3) should be less or equal to the allowable deflection bending for the shaft.

$$g_{12} = \frac{\delta(u_0)}{\delta_a} - 1 \tag{15}$$

where:

$\delta(x)$ – function which return the value of bending deflection at the abscissa *x*, [mm].

C14. The deflection at the supporting point angle of the input shaft in section 4 (fig. 3) should be less or equal to the allowable deflection at the supporting point angle for the shaft.

$$g_{14} = \frac{\varphi(u_4)}{\varphi_a} - 1 \quad (17)$$

where:

$\varphi(x)$ – function which return the value of deflection at the supporting point angle at the abscissa *x* of the supporting point, [rad].

C15. The deflection at the supporting point angle of the input shaft in section 10 (fig. 3) should be less or equal to the allowable deflection at the supporting point angle for the shaft.

$$g_{15} = \frac{\varphi(u_{10})}{\varphi_a} - 1 \quad (18)$$

C16. The torsion angle of the input shaft must be less or equal to the allowable torsion angle of the shaft.

$$g_{16} = \frac{\theta(x)}{\theta_a} - 1 \quad (19)$$

where:

$\theta(x)$ – function which return the value of the torsion angle for the input shaft, [rad].

C17. The basic rating life must be greater to the imposed (accepted) basic rating life of the bearing.

$$g_{17} = \frac{L_{h\_nec}}{L_h} - 1 \quad (20)$$

C18. The bearing stress between key and the key way of the input end shaft should be lower to the allowable bearing stress.

$$g_{18} = \frac{\sigma_s}{\sigma_{sa}} - 1 \quad (21)$$

where:

σ_s – key bearing stress, [MPa];

σ_{sa} – allowable bearing stress, [MPa].

C19. The shear stress of key of the input end shaft should be lower to the allowable shear stress.

$$g_{19} = \frac{\tau_f}{\tau_{fa}} - 1 \quad (22)$$

where:

τ_f – key shear stress, [MPa];

τ_{fa} – allowable shear stress, [MPa].

4.4. Results

The optimal Pareto set was obtained using Cambrian v.3.1 software belonging to the Optimal Design Centre of the Technical University of Cluj-Napoca. The resulted Pareto front is presented in fig. 4.

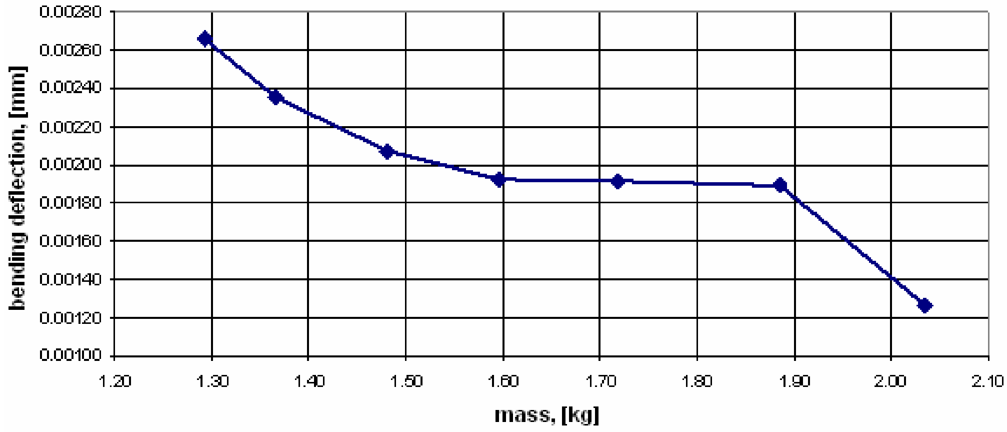

Fig.4. Pareto front (bending deflection vs. sub-assembly mass)

4.5. Conclusions

Obviously, it is up to the designer to choose, from the optimal Pareto set (fig. 4), the appropriate design solution. Analyzing the Pareto front we can observe that the solution of the shaft sub-assembly with the lower mass (1.292 kg) has the highest bending deflection ($2.656 \cdot 10^{-3}$ mm). The solution with the greater stiffness is two time heaviest than the lightest solution. If there are no special stiffness conditions, obviously the lightest solution will be selected.

In the table bellow is presented a comparison between the lightest solution and the solution with the greater stiffness.

Table 1 Comparison between the lightest solution and the solution with the greater stiffness

	The lightest design solution	The solution with the greater stiffness	Variation
Mass [kg]	$M_{sub\_assembly} = 1.292$	$M_{sub\_assembly} = 2.032$	57,4 %
Bending deflection [mm]	$\delta(u_7) = 2{,}656 \cdot 10^{-3}$	$\delta(u_7) = 1{,}265 \cdot 10^{-3}$	110,0 %

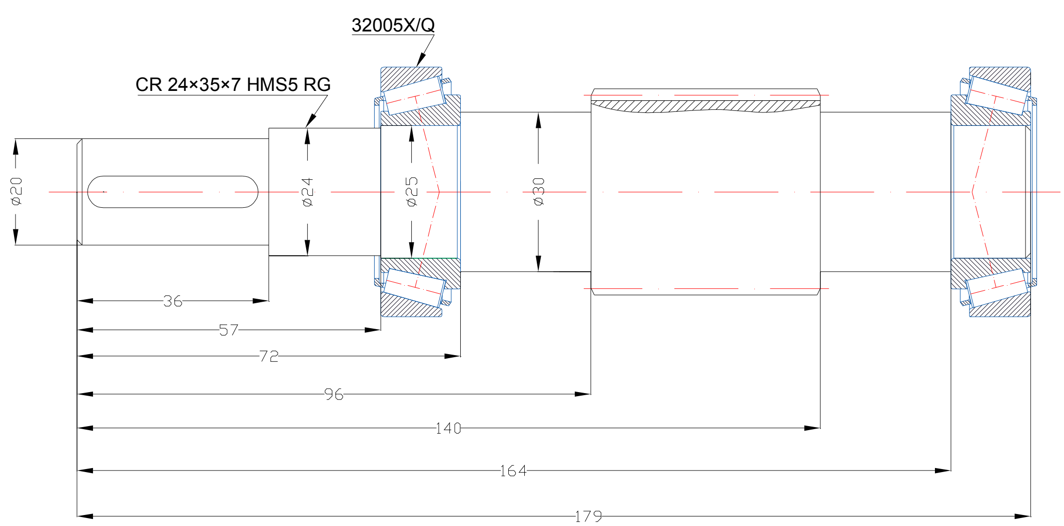

Fig.5 Optimal input shaft sub-assembly design for minimal mass solution

For the lightest input shaft sub-assembly solution the values of the genes are presented bellow:

End shaft:

End shaft length: $l_{ca} = 36$ mm

End shaft diameter: $d_{ca} = 20$ mm

Radial shaft seal (CR 24x35x7 HMS5 RG):

Radial shaft seal diameter: $d_{1m} = 24$ mm

Exterior radial shaft seal diameter: $d_{2m} = 35$ mm

Radial shaft seal width: $b_m = 7$ mm

Tapered rolling bearing (32005 X/Q):

Interior bearing diameter: $d_r = 25$ mm

Exterior bearing diameter: $D_r = 47$ mm

Bearing width: $T_r = 15$ mm

Sub-assembly mass: $M_{sub\_assembly} = 1.292$ kg

Maxim bending deflection: $\delta(u_7) = 2{,}656 \cdot 10^{-3}$ mm

REFERENCES

[1] Arora, J., (1989) *Introduction to Optimum Design*, McGraw-Hill, New-York.
[2] Coello Coello, C. A., Lamont, B. L., Van Veldhuizen, D. A., (2007) *Evolutionary Algorithms for Solving Multi-Objective Problems*, Second Edition, Springer, New York.
[3] Deb, K., (2004) Multi-Objective Optimization using Evolutionary Algorithms, Wiley, Chichester.
[4] Deb, K., TIWARI, S., (2003), *Multi-speed gearbox design using multi-objective evolutionary algorithms*, ASME Transactions on Mechanical Design, pp. 609-619.19.
[5] Osyczka, A., (1985) *Multi-criteria Optimization for Engineering Design*, in J.S. Gero, Ed., “Design Optimization”, Academic Press, pp. 193-227.
[6] Tudose, L., Pop, D., Haragâş, S., Nistor, G., Jucan, D., Pustan, M., (2006) *Optimal Design of Complex Systems, Mediamira*, Cluj-Napoca, (in Romanian).
[7] Tudose, L., Pop, D., Buiga, O., Codre, C., (2007), *Minimal mass approach of helical gear design*, The First International Conference “Advanced Engineering in Mechanical Systems”, Cluj-Napoca, Romania, pp 359-366.