ANNALS of the ORADEA UNIVERSITY.

Fascicle of Management and Technological Engineering, Volume VII (XVII), 2008

MATHEMATIC MODEL TO DETERMINE THE PREVENTIVE MAINTENANCE OF BALL MILLS

Teodor VASIU, Adina BUDIUL BERGHIAN

Engineering Faculty of Hunedoara, Department of Engineering and Management
Revolutiei str., no.4, 331128 Hunedoara
e-mail teodor.vasiu@fih.upt.ro, adina.budiul@fih.upt.ro

Keywords: mathematical model, ball mills, maintenance

Abstract : In this study is presented a mathematical model of preventive maintenance which takes into account several stochastic factors that influence the failure rate and working life of an entity. It is assumed that preventive maintenance is done through imperfections: it is not reduced only the proper operation but the failure probability as well as much as the number of maintenance works is increased. The objective of this study is to determine the optimal diagram for planned maintenance works so that the failure rate to not exceed a certain prescribed value.

1. INTRODUCTION

A condition for preventive maintenance success is to determine the moment of its execution. One of the most used concepts is periodical preventive maintenance, which specifies interventions done at equal times of continuous operation (for example in the case of ball mills this is of 670 hours [8]). Another concept is the so-called sequential preventive maintenance, which means that planned interventions to be done at unequal times of continuous operation. The first concept is more comfortable, but the sequential preventive maintenance is more realistic because it takes into account that an entity should be often recovered as much as its working life increases. The bond between these two concepts is done by corrective maintenance, which is applied when the entity fails. Corrective maintenance solves the cause of that failure only and not all the status problems of the entity. By other words, corrective maintenance doesn't change the failure rate and doesn't increase the working life either.

The most used two methods to determine the periods of performing the preventive maintenance are based on minimization of maintenance costs and keeping the failure rate under a prescribed value, but this study approaches the second method only, the first method being approached in [9].

In [5] are adopted some correction factors for failure rates $h(t)$ and for the proper operation time elements t within the preventive maintenance i.e.:

1. Failure rate during the next operational time element is $ah(t)$, where $h(t)$ refers to the previous time element, $a \geq 1$ is a correction factor and $t \geq 0$ represents the time refluxed from the previous intervention.
2. Time element t for proper operation of the entity before preventive maintenance is reduced to bt after intervention, where $b \leq 1$ is a proper operation reduction factor.

According to the proposal of survey [5], for a failure rate $h(t)$, $t \in (0, t_1)$, the preventive maintenance started at the moment t_1 determines a new failure rate $g(t)$, $t \in (t_1, t_2)$, which depends on the previous failure rate and maintenance work. In the survey [5] is proposed a form for *g(t)*, using both the concept of increasing the failure rate and reducing the proper operation according to the entity exploitation:

$$g(t_1 + x) = ah(bt_1 + x) \tag{1}$$

where $a \geq 1$, $0 \leq b \leq 1$ and $x \in (0, t_2 - t_1)$. For $a = 1$, the proposed model is resized to the one of working life reduction, and b = 0 is the equivalent of increasing the failure rate.

Using the proposed model is developed an optimal policy of preventive maintenance, with a major implication i.e. renunciation to the classic, uneconomical approach of constant periods for preventive maintenance execution.

2. DESCRIPTION OF THE MODEL AND OPTIMAL SOLUTIONS

At this moment is necessary to specify the notations that follow to be used:

h(t) – failure rate;
H(t) – cumulated failure rate;
λ - maximum value accepted for the failure rate;
x_k – moments of preventive maintenance execution, $k = 1, 2,..., N$;
$t_k = x_1 + x_2 + ... + x_k$, $k = 1, 2,..., N$;
y_k – continuous operation time immediately after the number „*k*” of preventive maintenance, $k = 1, 2,..., N$;
N – number of proper operation time elements;
a_k – failure rate correction factor after the number „*k*” of preventive maintenance;
$1 = a_0 \leq a_1 \leq a_2 \leq ... \leq a_{N-1}$;
$A_k = \prod_{i=0}^{k-1} a_i$, $k = 1, 2,..., N$;
b_k – correction factor of proper operation time;
$0 = b_0 \leq b_1 \leq b_2 \leq ... \leq b_{N-1} < 1$;
c_m – corrective maintenance cost;
c_p – preventive maintenance cost;
c_r – overhaul cost (entity replacement);
C – average cost of entity;

It is considered the situation in which an entity is subjected to the preventive maintenance at the moments t_1, t_2, ..., t_{N-1} and overhauled or replaced at the moment t_N. Corrective maintenance is executed as a consequence of failures appeared between the works of preventive maintenance. The overhaul from the moment t_N makes the entity to be like a new one. The entity has the failure rate $A_k h(t)$ between the numbers „ *k*-1” and „*k*” of preventive maintenances, i.e. in the range (t_{k-1}, t_k). The proper operation time is $b_{k-1}y_{k-1}$, immediately after the number „ *k*-1” of preventive maintenance, becoming $y_k = x_k + b_{k-1}x_{k-1} + ... + b_{k-1}b_{k-2}...b_2b_1x_1$ after the number „*k*” of preventive maintenance, meaning that the proper operation time is changed from $b_{k-1}y_{k-1}$ la y_k in the range (t_{k-1}, t_k). Obviously $y_k = x_k + b_{k-1}y_{k-1}$ or $x_k = y_k - b_{k-1}y_{k-1}$.

From [6], the entity average cost is:

$$C = \frac{c_r + c_p(N-1) + c_m \sum_{k=1}^{N} A_k\left[H(y_k) - H(b_{k-1}y_{k-1})\right]}{\left[\sum_{k=1}^{N-1}(1-b_k)y_k + y_N\right]} \qquad (2)$$

The next target is to determine the periods of proper operation in order to limit failure rate.

By this model, the preventive maintenance is done any time the failure rate of the entity riches the prescribed value λ, i.e. at the moment t_i (i = 1, 2, ..., N) [4] :

$$\lambda = A_k h(y_k),\ k = 1, 2, ..., N \qquad (3)$$

Solving the equation (3) are obtained y_k (k = 1, 2, ..., N) as functions of λ. Replacing these mathematic expressions in (2) and differentiating C related to λ, from $\frac{\partial C}{\partial \lambda} = 0$ is obtained :

$$\frac{\sum_{k=1}^{N-1} \frac{h(y_k) - a_k b_k h(b_k y_k)}{h'(y_k)} + \frac{h(y_N)}{h'(y_N)}}{\sum_{k=1}^{N-1} \frac{1-b_k}{A_k h'(y_k)} + \frac{1}{A_N h'(y_N)}} = \frac{C}{c_m} \qquad (4)$$

From equation (4) can be determined λ depending on N; then can be find N which minimizes

$$\frac{\sum_{k=1}^{N-1} \frac{h(y_k) - a_k b_k h(b_k y_k)}{h'(y_k)} + \frac{h(y_N)}{h'(y_N)}}{\sum_{k=1}^{N-1} \frac{1-b_k}{A_k h'(y_k)} + \frac{1}{A_N h'(y_N)}} \qquad (5)$$

Consequently the algorithm to determine the periods when preventive maintenance should be done is:

(i) Solving the equation (3) and getting the y_k depending on λ;
(ii) Replacing the y_k in equation (4) and getting the value of λ;
(iii) It is choosen N in order to minimize the function in the mathematical expression (5) where y_k (k = 1, 2, ..., N) is obtained from the steps **(i)** and **(ii)**;
(iv) Calculation of y_k (k = 1, 2, ..., N) using the mathematic expressions of steps **(i)** and **(ii)**, for value of N from the step **(iii)**;
(v) Calculation of $x_k = y_k - b_{k-1}y_{k-1}$, k = 1, 2, ..., N .

3. NUMERICAL EXAMPLE

It is considered the case of a ball mill for which the failure rate is in accordance with the Weibull distribution law:

$$h(t) = \beta t^{\alpha-1}$$

where $\beta = 6{,}148\times10^{-9}$ and $\alpha = 2{,}462$. Values of β and α have been obtained from operation/failure in plant regime of a ball mill of ∅2700×3000.

Solving the equation (3) are obtained the values :

$$y_k = \left(\frac{\lambda}{A_k\beta}\right)^{\frac{1}{\alpha-1}}, \; k = 1, 2, ..., N \tag{6}$$

Replacing (6) in (4) is obtained the value λ depending on N:

$$\lambda = \beta^{\frac{1}{\alpha}}\left[\frac{c_r + c_p(N-1)}{\left(1-\frac{1}{\alpha}\right)c_m E(N)}\right]^{\frac{\alpha-1}{\alpha}} \tag{7}$$

where

$$E(N) = \sum_{k=1}^{N-1}\left(1 - a_k b_k^{\alpha}\right)A_k^{-\frac{1}{\alpha-1}} + A_N^{-\frac{1}{\alpha-1}}$$

Thus, the mathematic expression (5) becomes :

$$\beta^{\frac{1}{\alpha}}\left[\frac{c_r + c_p(N-1)}{\left(1-\frac{1}{\alpha}\right)c_m}\right]^{\frac{\alpha-1}{\alpha}}\frac{[E(N)]^{\frac{1}{\alpha}}}{F(N)} \tag{8}$$

where :

$$F(N) = \sum_{k=1}^{N-1}\left(1 - b_k\right)A_k^{-\frac{1}{\alpha-1}} + A_N^{-\frac{1}{\alpha-1}}$$

Minimization of the function given by the mathematic expression (8) is equivalent with minimization of :

$$Q(N)=\frac{\left[c_r+c_p(N-1)\right]^{\frac{\alpha-1}{\alpha}}\left[E(N)\right]^{\frac{1}{\alpha}}}{F(N)}$$

In order to determine the optimal value of N^* there should be meet the following inequalities $Q(N+1)\geq Q(N)$ şi $Q(N)\prec Q(N-1)$, which involves :

$$W(N)\geq\frac{c_r}{c_p} \text{ şi } W(N-1)\prec\frac{c_r}{c_p} \tag{9}$$

where $W(N)=\dfrac{\left[E(N+1)\right]^{\frac{1}{\alpha-1}}\left[F(N)\right]^{\frac{\alpha}{\alpha-1}}}{\left[E(N)\right]^{\frac{1}{\alpha-1}}\left[F(N+1)\right]^{\frac{\alpha}{\alpha-1}}-\left[E(N+1)\right]^{\frac{1}{\alpha-1}}\left[F(N)\right]^{\frac{\alpha}{\alpha-1}}}-(N-1)$

Values x_k are of the form $x_k=y_k-b_{k-1}y_{k-1}$, k = 1, 2, ..., N, where y_k are given by (6) and (7).

In order to calculate accurately the values x_k, k = 1, 2, ..., N is necessary to be known the economical parameters c_m, c_p şi c_r, parameters α and β of the Weibull distribution law and coefficients a_k and b_k. For the economic parameters is enough to know the ratios $\frac{c_r}{c_p}$ and $\frac{c_r}{c_m}$. Coefficients a_k and b_k are recommended in [4]. In this numerical example $\frac{c_r}{c_p}$ = 2, 5, 10, 20, 50, $\frac{c_r}{c_m}$ = 4, and $a_k=\dfrac{6k+1}{5k+1}$; $b_k=\dfrac{k}{2k+1}$, k = 0, 1, 2,...

Solving the prescribed algorithm has been done using the MathCAD [7] software, and the results of the simulation are given in the table 1.

Table 1. Cycles of the proposed preventive maintenance for ball mill

c_r/c_m	2	5	10	20	50
N	1	2	3	4	5
Continuous operation time elements between two planned interventions [hours]					
x_1	1864.53	2235.32	2554.08	3012.29	3891.66
x_2	1056.44	1266.53	1447.13	1706.75	2205.00
x_3		989.77	1130.91	1333.80	1723.17
x_4		826.38	944.23	1113.62	1438.72
x_5			807.81	952.73	1230.85
x_6			699.14	824.57	1065.28
x_7			608.95	718.20	927.86
x_8				627.97	811.29
x_9				550.44	711.13
x_{10}				483.31	624.40

x_{11}					548.90
x_{12}					482.96
x_{13}					425.22
x_{14}					374.58
x_{15}					330.09

4. CONCLUSION

This survey presents an execution method for the preventive maintenance starting from the observation that the repair works don't reduce only the life time of an entity but in the same time change the failure rate. As relevant example has been chosen a ball mill with the failure rate according to Weibull distribution law, because the algorithm described in this survey can not be solved for a general case.

From table 1 has been ascertained that operation time elements prescribed between two successive repair works are thus reduced. This thing shows that the studied ball mill suffers failures whose solve doesn't have as effect „an entity as a new one" after rehabilitation. The reasons should be analyzed each time, no matter if their causes are of human nature or not.

REFERENCES

[1] Brown M., Proschan F., *Imperfect repair*, Journal of Applied Probability 20 (1983), pp. 851-859.
[2] Fontenot R. A., Proschan F., *Some imperfect maintenance models*, Reliability Theory and Models (Academic Press, 1984), pp. 83-101.
[3] Lie C. H., Chun Y. H., *An algorithm for preventive maintenance policy*, IEEE Transactions on Reliability R-35 (1986), pp. 71-75.
[4] Lin D., Zuo M. J., Yam R. C. M. Y., *General Sequential Imperfect Preventive Maintenance Models,* International Journal of Reliability, Quality and Safety Engineering, Vol. 7, No. 3 (2000), pp. 253-266.
[5] Nakagawa T., *Optimum policies when preventive maintenance is imperfect*, IEEE, Transactions on Reliability, R-28 (1979), pp. 331-332.
[6] Sheu S.-H., Liou C.-T., *A generalized sequential preventive maintenance policy for repairable systems with general random minimal repair costs*, International Journal of Systems Science 26 (1995), pp. 681-690.
[7] Scheiber E., Lixăndroiu D., *MathCAD. Presentation and solved problems,* Technical Publishing House, Bucharest (1994).
[8] Vasiu T., *Increasing the reliability of mineral processing equipment*, PhD Thesis, Petroşani (1996).
[9] Vasiu T., Budiul-Berghian A., *Reliability of ball mills*, "Infomin" Publishing House, Deva (2007).