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A BIOMECANICAL MODEL OF THE HUMAN ARM

lect. univ. dr. Mircea BĂRBUCEANU,
University of Piteşti, Faculty of Sciences, ROMANIA,
e-mail: mircea_barbuceanu@yahoo.com

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Abstract
One of the important problems of biomechanics is the assessment of human joint torque. By means of new calculation systems the specific methods of theory of mechanism was adapted: multibody dynamics, inverse mechanics etc. All these require a valid mathematical model of the real human system which can be made only by human factor. In this paper we present a mathematical model for human arm.

1. INTRODUCTION

Multybody analyses are applied extensively in biodynamic modeling related to experimental and theoretical analyze methods like kinematics analyze by videocomputing (3D video-based human motion analysis systems), inverse mechanics, tensiometrical interferometer etc. Today, their application into sport is expanding and becoming a standard also among sport technicians.

All these methods require a valid mathematical model of the real human system which can be made only by human factor. There is at this moment a great need for biomechanical models of different parts of human body. In this way, the motivation of the study is to develop a mathematical model for human arm during voluntary free movement in 3D coordinates which allows the assessment of joint torque by inverse mechanics method.

2. THE MECANO-GEOMETRICAL MODEL OF HUMAN ARM

In figure 1 it is schematized experimental conditions for by videocomputing analyze of the free voluntary moving of the human arm.

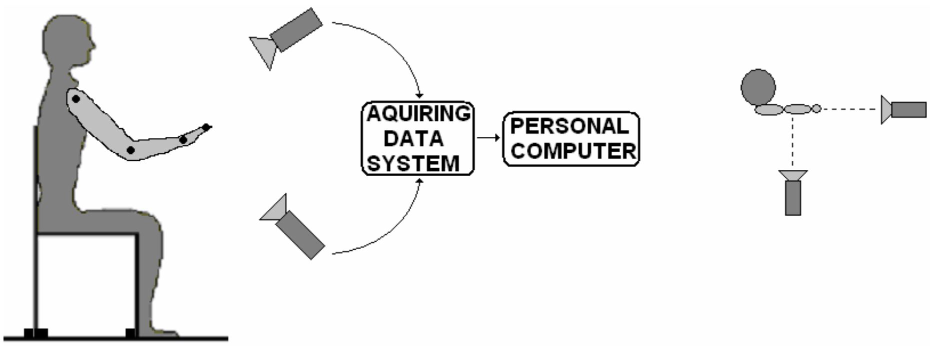

Fig. 1. Biomechanical experiment conditions

The motion of the arms has been tracked by means of a video-computerized system for human motion analysis. The system used during this study was mainly composed of:

- two analog video cameras Panasonic with capturing frequency of 36 Hz (enough for qualitative data acquiring);
- a PC video card for grabbling the images recorded with the analog video cameras;

- software for camera calibration, digitization and analysis of the markers position, velocity and acceleration (*World in motion*).

The system described is a common tool in biomechanics. The video camera image provides a 2D projection of the 3D space. The aim is to compute the spatial position of a marker from the knowledge of its position into the camera image space (the solution of the problem in 3D space requires at least two different projections; a well-known mapping procedure is the direct linear transformation).

After experiment, the human arm has been modeled by means of three rigid body segments: arm, forearm and hand (Figure 2). The shoulder has been considered fixed and it is assumed that the inertia properties of the hand segment are not influenced by the motion of the fingers.

The first body (the arm) is connected to the frame (upper torso) through a spherical joint. Arm and forearm are joined through a kinematics pair composed of two revolute pairs whose intersecting axes form a valgus angle $\alpha \approx 95^{O}$. The relative motions of the forearm versus the arm are flexion-extension and pronation-supination.

The kinematics pair joining hand and forearm is composed of two revolute pairs with intersecting axes forming an angle of 90^{O} (a cardanic joint).

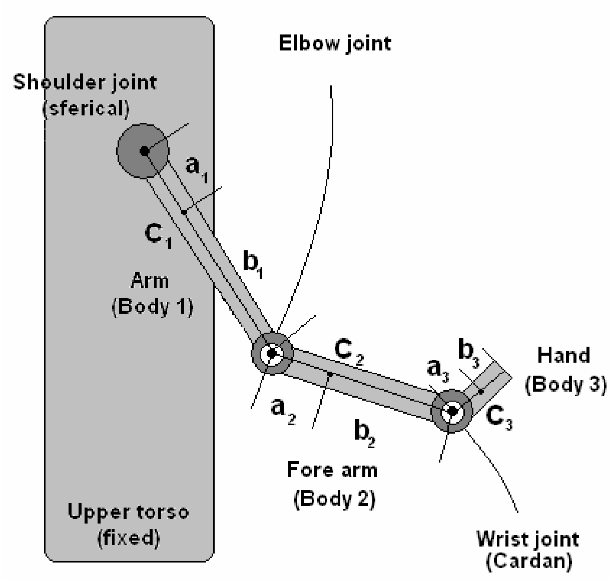

Fig. 2. The mechanical model of the human arm

3. MATEMATICAL MODEL OF THE HUMAN ARM

The geometrical considerations and the notations are showed in figure 3.

The number of freedom degrees of bodies and the number of freedom degrees left out of each joint (scalar constraints) are presented in Table 1.

Table 1. Degrees of freedom and of constraint

Kinematics elements	**Number of degrees**
3 rigid bodies	+18
Shoulder (spherical kinematics pair)	-3
Elbow (two kinematics pairs of rotation)	-4
Wrist (cardanic kinematics pair)	-4
Total system degrees	**7**

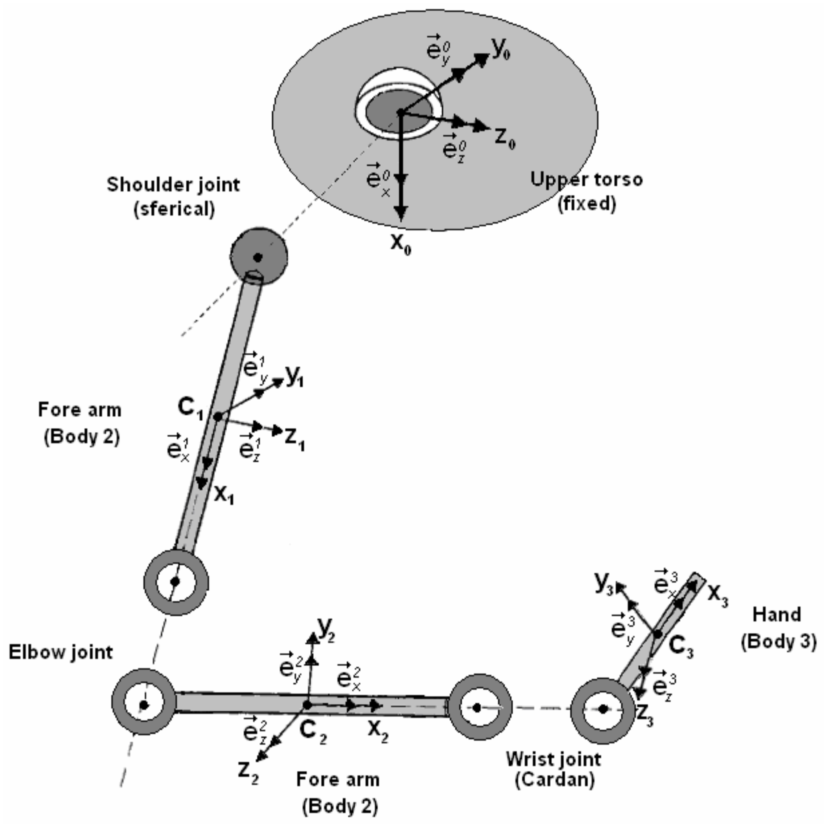

Fig. 3. The geometrical considerations and the notations

Thus the model has seven degrees of freedom. Taking into account the three sets of Euler parameters, we can conclude that the vector of position $\{q\}$ has 7 components:

$$\{q\} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \theta_7 \end{bmatrix}^T \tag{1}$$

and the Jacobian matrix associated with constrains $[\Psi_q] \equiv [\Psi]$ has 14 components.

For the deduction of the kinematics and dynamics equations it has started to the general form of these:

$$[M(q)]\{\ddot{q}\} + [g^c(q,\dot{q},t)] = [g^e(q,\dot{q},t)] \tag{2}$$

where:

$$[M(q)] = \sum_{i=1}^{n} \left(m_i R_{T_i}^T R_{T_i} + J_{S_i}^z R_{R_i}^T R_{R_i} \right) \text{ - mass matrix} \tag{3}$$

$$\vec{g}^{\,c}(q,\dot{q},t) = \sum_{i=1}^{n} \left(m_i R_{T_i}^T \vec{a}_{S_i} + J_{S_1}^z R_{R_i}^T \vec{\varepsilon}_i \right) \text{ - resultant of generalized inertia forces} \tag{4}$$

$$\vec{g}^{\,e}(q,\dot{q},t) = \sum_{i=1}^{n} (R_{T_i}^T \vec{f}_i^{\,e} + R_{R_i}^T \vec{M}_{S_i}^{e,z}) \text{ - resultant of generalized applied forces} \tag{5}$$

Notations have the follow signification:

n - number of rigid bodies of system

m_i - mass of the i-th body

$J_{C_i}^z$ - moment of inertia of the i-th body versus its center of mass C_i

R_{T_i} - matrix of translation of the Cartesian frame attached to the i-th body, C_i - $x_i y_i z_i$ (versus fixed Cartesian frame C_o - $x_o y_o z_o$, attached to the upper torso)

$R_{R_i}, R_{R_i}^T$ - matrix of rotation of the Cartesian frame attached to the i-th body, C_i - $x_i y_i z_i$ (versus fixed Cartesian frame C_o - $x_o y_o z_o$, attached to the upper torso)

$\vec{a}_{C_i}$ - relative (local) acceleration of the i-th body

$\vec{\varepsilon}_i$ - angular relative (local) acceleration of the i-th body

$\vec{f}_i^{\,e}$ - external applied forces

$\vec{M}_{C_i}^{e,z}$ - external applied moments reduced relative to center of mass.

In our study, for a free movement of the arm, it is convenient to split the vector of generalized forces into three parts:

$$\vec{g}^{\,e} = \{\vec{R}\} + \{\vec{F}_w\} + \{\vec{F}_c\} \tag{6}$$

and to write the vector of generalized inertia forces $\vec{g}^{\,c}(q,\dot{q},t)$ into Lagrange's form:

$$\vec{g}^{\,c} = [\Psi_q]^T \vec{\lambda} \tag{7}$$

($[\Psi_q]$ is the Jacobian matrix associated with constraint vector).

The first vector $\{\vec{R}\}$ depends on the unknown torque components at the joints, whereas $\{\vec{F}_w\}$ contains the generalized forces due to the weight. Thus, separating the unknowns (torque components and internal constraint forces) from the known forces (inertia, centrifugal and weight forces), one has:

$$[M][\ddot{q}] + [\Psi_q]^T \vec{\lambda} = \{\vec{R}\} - \{\vec{F}_w\} - \{\vec{F}_c\} \tag{8}$$

or explicitly:

$$[R\Psi_q^T]_{21x21} \begin{Bmatrix} \{\vec{\tau}\} \\ \{-\vec{\lambda}\} \end{Bmatrix} = [M][\ddot{q}] - \{\vec{F}_w\} - \{\vec{F}_c\} \tag{9}$$

In the last equation form $\{\vec{\tau}\}$ is the vector of torque components into the joints and $\{\vec{\lambda}\}$ is the vector of Lagrange's multipliers.

In the following the analytical expressions of the terms of the equation (9) will be expressed:

- mass matrix:

$$[M] = \begin{bmatrix} M_i & 0 & 0 \\ 0 & M_i & 0 \\ 0 & 0 & M_i \end{bmatrix}, \qquad [M_i] = \begin{bmatrix} \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix} & 0_{3x4} \\ 0_{4x3} & \left[4[G_i]^T [J^{(i)}][G_i]\right] \end{bmatrix} \tag{10}$$

where $[A_i] = [E_i][G_i]^T$ is well-know transformation matrix of the i-th body with Euler parameters.

- generalized forces:

The external forces considered are due to the weight $\{\vec{F}_w\}$, to the quadratic velocity terms $\{\vec{F}_c\}$ (centrifugal forces), and to applied joint torques $\{\vec{R}\}$ (the torques due to damping at the joints have been ignored).

- centrifugal forces:

$$\{\vec{F}_c\} = \begin{Bmatrix} \{F_{c_1}\} \\ \{F_{c_2}\} \\ \{F_{c_3}\} \end{Bmatrix} \qquad \{F_{c_i}\} = \begin{Bmatrix} 0_{3x1} \\ 8[\dot{G}_i]^T [J^{(i)}][\dot{G}_i]\vec{e}_i \end{Bmatrix} \qquad [\dot{G}_i] = \frac{1}{2}\left([G_i][\overline{\omega}^{(i)}] - \{q_i\}\{\vec{\omega}^{(i)}\}^T\right) \tag{11}$$

- generalized forces due to weight forces:

$$\left\{\vec{F}_w\right\} = \begin{Bmatrix} \{F_{w_1}\} \\ \{F_{w_2}\} \\ \{F_{w_3}\} \end{Bmatrix} \qquad \left\{F_{w_i}\right\} = \begin{Bmatrix} m_i g \\ 0_{6x1} \end{Bmatrix} \tag{12}$$

- generalized forces due to joint torques on the i-th body:

$$\left\{\vec{R}_j\right\} = \begin{bmatrix} 0_{3x1} \\ 2[G_i]^T\left\{\vec{T}^{(i)}\right\} \end{bmatrix} = T_{i_k}\begin{bmatrix} 0_{3x1} \\ 2[G_i]^T\left\{\vec{e}^{\,i}_k\right\} \end{bmatrix} \tag{13}$$

- if the components of applied torque $\{\vec{\tau}\}$ are given in the i-th body reference frame ($k = x, y, z$)

Consequently:

- o generalized forces due to the motor torque exerted on the upper-arm (body 1) through the spherical join, $\left\{\vec{T}^{(11)}\right\} = \left\lfloor T_{11_x} \quad T_{11_y} \quad T_{11_z} \right\rfloor$

$$\left\{\vec{R}_1\right\} = T_{11_x}\begin{bmatrix} 0_{3x1} \\ 2[G_1]^T\left\{\vec{e}^{\,1}_x\right\} \end{bmatrix} \qquad \left\{\vec{R}_2\right\} = T_{11_y}\begin{bmatrix} 0_{3x1} \\ 2[G_1]^T\left\{\vec{e}^{\,1}_y\right\} \end{bmatrix} \qquad \left\{\vec{R}_3\right\} = T_{11_z}\begin{bmatrix} 0_{3x1} \\ 2[G_1]^T\left\{\vec{e}^{\,1}_z\right\} \end{bmatrix} \tag{14}$$

- o generalized forces due to the motor torque exerted on the upper-arm (body 1) through the elbow joint, $\left\{\vec{T}^{(12)}\right\} = \left\lfloor T_{12_x} \quad T_{12_y} \right\rfloor$

$$\left\{\vec{R}_4\right\} = T_{12_y}\begin{bmatrix} 0_{3x1} \\ 2[G_1]^T\left\{\vec{e}^{\,2}_y\right\} \end{bmatrix} \qquad \left\{\vec{R}_7\right\} = -T_{12_x}\begin{bmatrix} 0_{3x1} \\ 2[G_1]^T[A_1]^T[A_2]\left\{\vec{e}^{\,2}_x\right\} \end{bmatrix} \tag{15}$$

- o generalized forces due to the motor torque exerted on the arm (body 2) through the elbow joint, $\left\{\vec{T}^{(22)}\right\} = \left\lfloor T_{22_x} \quad T_{22_y} \right\rfloor$

$$\left\{\vec{R}_5\right\} = -T_{22_y}\begin{bmatrix} 0_{3x1} \\ 2[G_2]^T[A_2]^T[A_1]\left\{\vec{e}^{\,2}_y\right\} \end{bmatrix} \qquad \left\{\vec{R}_6\right\} = T_{22_x}\begin{bmatrix} 0_{3x1} \\ 2[G_2]^T\left\{\vec{e}^{\,2}_x\right\} \end{bmatrix} \tag{16}$$

- o generalized forces due to the motor torque exerted on the arm (body 2) through the wrist joint, $\left\{\vec{T}^{(23)}\right\} = \left\lfloor T_{23_y} \quad T_{23_z} \right\rfloor$

$$\left\{\vec{R}_8\right\} = -T_{23_y}\begin{bmatrix} 0_{3x1} \\ 2[G_2]^T\left\{\vec{e}^{\,2}_y\right\} \end{bmatrix} \qquad \left\{\vec{R}_{10}\right\} = T_{23_z}\begin{bmatrix} 0_{3x1} \\ 2[G_2]^T\left\{\vec{e}^{\,2}_z\right\} \end{bmatrix} \tag{17}$$

- o generalized forces due to the motor torque exerted on the hand (body 3) through the wrist joint, $\left\{\vec{T}^{(33)}\right\} = \left\lfloor T_{33_y} \quad T_{33_z} \right\rfloor$

$$\left\{\vec{R}_9\right\} = -T_{23_y}\begin{bmatrix} 0_{3x1} \\ 2[G_3]^T[A_3]^T[A_2]\left\{\vec{e}^{\,3}_y\right\} \end{bmatrix} \qquad \left\{\vec{R}_{11}\right\} = T_{23_z}\begin{bmatrix} 0_{3x1} \\ 2[G_3]^T[A_3]^T[A_2]\left\{\vec{e}^{\,3}_z\right\} \end{bmatrix} \tag{18}$$

Thus, a vector $\{R\}$ can be defined as

$$\{R\} = [R]_{21x7}\,\vec{\tau} \tag{19}$$

where

$$[R]_{21x7} = \begin{bmatrix} R_1 & R_2 & R_3 & R_4 & -R_7 & 0 & 0 \\ 0 & 0 & 0 & .R_5 & R_6 & R_8 & R_{10} \\ 0 & 0 & 0 & 0 & 0 & -R_9 & -R_{11} \end{bmatrix} \tag{20}$$

and

$$\vec{\tau} = \begin{bmatrix} T_{11_x} \\ T_{11_y} \\ T_{11_z} \\ T_{12_x} \\ T_{12_y} \\ T_{33_y} \\ T_{33_z} \end{bmatrix} \qquad (21)$$

is the vector of unknown torque components.

4. CONCLUSIONS

The equation (9) can be readily solved versus $\{\vec{\tau}\}$ and $\{\vec{\lambda}\}$, once all the terms of that are explicated, inertia properties are known and the kinematics experimentally measured. If needed, the constraint forces can be computed from the Lagrange multipliers (*inverse mechanics method*).

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