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COMPARATIVE ANALYSIS ON THE MAIN PARAMETERS INFLUENCE FOR THE PINION TEETH ADDENDUM THICKNESS IN EXTERNAL AND INTERNAL GEARS

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Abstract: The present paper is presenting by comparison the influence of the principal parameters on the on the tooth addendum thickness of the pinion in internal and external cylindrical spur gears. The parameters compared are: teeth number of pinion, ratio gear, real gearing angle.

1. INTRODUCTION

By analyzing tooth geometry of external spur gears, it is known that a positive profile modification would result in a decrease in tooth addendum thickness and an increase in tooth dedendum thickness, respectively. This modification occurs in the pinion that presents external toothing of gears and the wheel with internal toothing of gears. Specialist literature [2, 3, 4, 5] does not present a detailed analysis of the influence of geometric parameters on the tooth addendum thickness with internal gears; in the current paper, the authors will undertake by comparison this particular analysis, that is necessary mostly for pinions that are simultaneously in internal and external gearing, as the automotive planetary gearing as well as in the speed-increasing planetary gears used in wind turbines.

2. THEORETICAL BASIS

For internal and external gearing the pinion has external toothing. We shall use the following relation [4, 5] so as to determine tooth thickness on a circle of diameter d_y

$$s_{y1} = \left[s_1 - mz_1\left(\mathrm{inv}\,\alpha_y - \mathrm{inv}\,\alpha\right)\right]\frac{\cos\alpha}{\cos\alpha_y}. \tag{1}$$

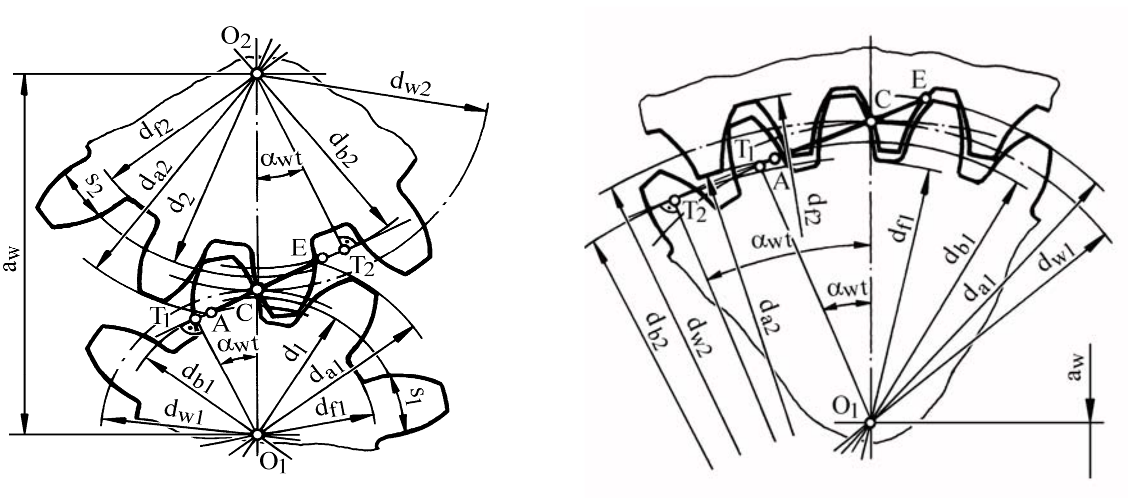

Figure1. External and internal gearing

And the relation below, respectively, in orders to determine tooth thickness on the pitch circle

$$s_1 = m\left(\frac{\pi}{2} + 2x_1 \tan\alpha\right). \tag{2}$$

To determine the pinion tooth thickness, the following relation will be used

$$s_{a1} = \frac{\cos\alpha}{\cos\alpha_{a1}} \cdot \left[s_1 + (\mathrm{inv}\,\alpha_{a1} - \mathrm{inv}\,\alpha) m z_1\right]. \tag{3}$$

After realizing the substitutions the relation become

$$s_{a1} = m \cdot \left[\left(\frac{\pi}{2} + 2x_1 \tan\alpha\right) + (\mathrm{inv}\,\alpha_{a1} - \mathrm{inv}\,\alpha) z_1\right] \frac{\cos\alpha}{\cos\alpha_{a1}}. \tag{4}$$

In connection with the gear's modulus *m* there can be established the relations used in analysing the influence of geometric parameters on the pinion tooth addendum thickness

$$\frac{s_{a1}}{m} = \left[\left(\frac{\pi}{2} + 2x_1 \tan\alpha\right) + (\mathrm{inv}\,\alpha_{a1} - \mathrm{inv}\,\alpha) z_1\right] \frac{\cos\alpha}{\cos\alpha_{a1}} \le (s_a/m)_{adm}, \tag{5}$$

A correct manufacturing of external and internal gears has to comply with condition [2] $(s_a/m)_{adm}$= 0.25 – with gears made of normalized and deep hardened steel, and $(s_a/m)_{adm}$= 0.4 – with gears made of hardened, blistered or nitro-carburized steel. [1, 2, 4].

The real gearing angle by the relation

$$\mathrm{inv}\,\alpha_w = \mathrm{inv}\,\alpha + 2\frac{x_d}{z_2 - z_1}\tan\alpha \tag{6}$$

for the external gear, respectively

$$\mathrm{inv}\,\alpha_w = \mathrm{inv}\,\alpha + 2\frac{x_d}{z_2 + z_1}\tan\alpha \tag{7}$$

for internal gear, where $x_d = x_2 - x_1$.

In calculating the angles β_{a1} and α_{a1} it is consider the tooth addendum modification of the pinion thus the standard clearance at the addendum tooth is maintained. The tip diameter of external gear is determined by the relation

$$d_{a1} = m\left[z_2 - 2\left(h_a^* - x_2 - k_{\mathrm{ext}}\right)\right], \tag{8}$$

for the internal gear by the relation

$$d_{a1} = m\left[z_2 + 2\left(h_a^* + x_2 + k_{\mathrm{int}}\right)\right], \tag{9}$$

The pressure angle on the tip diameter d_a is determined by the following relation

$$\alpha_{a1} = \arccos\left(\frac{d_1}{d_{a1}}\cos\alpha\right). \tag{10}$$

Considering relations (8), (9), (10) it can be established that

$$\alpha_{a1} = \arccos\left(\frac{z_1}{z_2 - 2\left(h_a^* - x_1 - k_{ext}\right)}\cos\alpha\right) \tag{11}$$

for external gear, respectively

$$\alpha_{a1} = \arccos\left(\frac{z_1}{z_2 + 2\left(h_a^* + x_1 - k_{ext}\right)}\cos\alpha\right) \tag{12}$$

for internal gear.

In the relations below there war introduced shortening coefficients of the tooth addendum that can be determinate by the relation (13) for external gear and by relation (14) for internal gear:

$$k_{ext} = \left(x_1 + x_2\right) - \left(\frac{z_1 + z_2}{2}\right)\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right), \tag{13}$$

$$k_{int} = \left(x_2 - x_1\right) - \left(\frac{z_2 - z_1}{2}\right)\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right) \tag{14}$$

3. SOFTWARE

To analyse the influence of the main parameters on the addendum thickness for the pinion, a software was developed with the following features: easy entering of input data; the possibility of choosing the variable on the x-axis, the variable parameter which limits the diagram curves, as well as the constant parameters; the tabular distribution of the results for easy representation of the curves.

The ***Main Menu*** of the program Figure 2 includes toolbars as: *Input data, Calculus, Output data* and *Exit.*

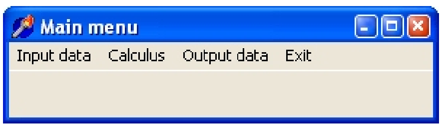

Figure 2 Main Menu

The Input Data window ***Figure 3*** offers the possibility of choosing the variable on the *x*-axis, the variable parameter which limits the diagram curves, as well as the constant values of the other parameters. The constant parameters can be changed according to the previously selected parameters, the other *Combo Boxes* remaining inactive.

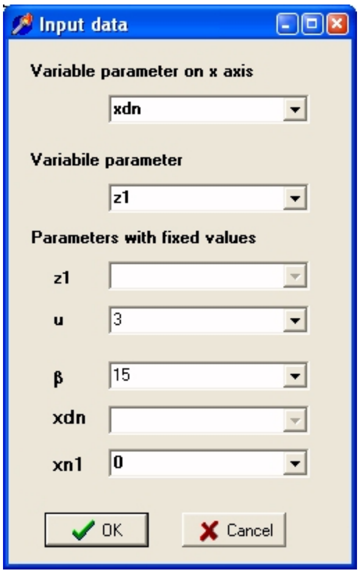

Figure 3 Input data window

A piece of the *Output Data* window is shown in Figure 4 the results are presented in a tabular format.

Results - ratio san/mn

xn1=	-0.5	-0.4	-0.3	-0.2	-0.1
		z1=20	u=3	xdn=0.5	
beta=0	0.000	0.000	1.827	1.755	1.667
beta=5	0.000	0.000	1.832	1.761	1.667
beta=10	0.000	1.920	1.849	1.778	1.667
beta=15	0.000	1.949	1.878	1.790	1.665
beta=20	2.063	1.993	1.912	1.789	1.663
beta=30	2.153	2.031	1.907	1.781	1.652

Figure 4 Results - Window

4. COMPARATIVE ANALISIS

From the above information, it follows that the addendum thickness pinion depends on both the pinion profile modification coefficient and the number of teeth, as well as on the gear ratio, the profile modification coefficient and the helix angle.

The relations previously presented are valid for both external and internal gear. The following diagrams were plotted for a standard, reference rack with: α_n=20º, $h^*_{an} = h_{an}/m_n = 1$; $c^*_n = c_n/m_n = 0.25$.

In ***Figure 5***, the variation of ratio s_a/m is presented, for both external and internal spur gear (β=0º), with respect to the pinion profile modification coefficient x_1 and ratio gear u for tooth pinion number z_1=20 and the profile modification coefficient x_d=0.5.

By analyzing the diagram, several conclusions can be drawn:

- it can be remarked a diminution on the tooth addendum thickness, as expected, on both exterior and interior gear pinion;
- the curves that describe the tooth addendum thickness of the pinion term to ratio gear are practically parallels for both types of gears;
- the maximum values of the ratio s_a/m are major for the internal gear (appreciative-1.2) unlike the external gear where the maximum value dose not exceeds 0.8, for major ratio gear the ratio s_a/m is increased.

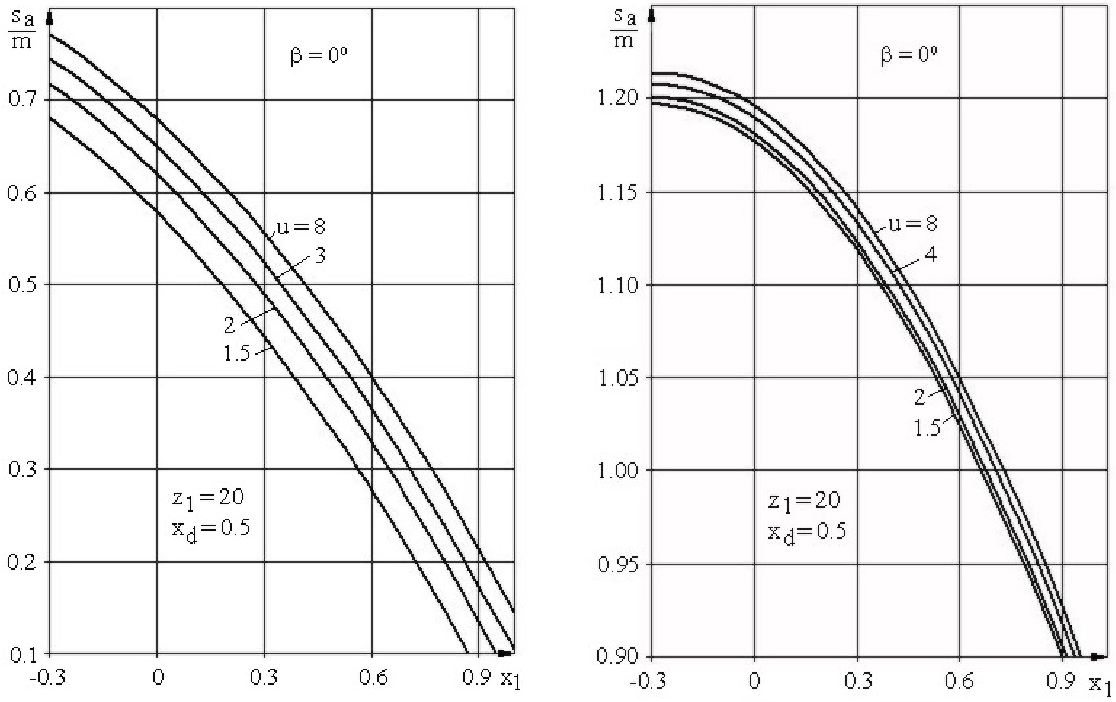

Figure 5 Ratio s_a/m variation by u

In ***Figure 6***, the variation of ratio s_a/m is presented, for both external and internal spur gear (β=0º), with respect to the profile modification coefficient x_d and the pinion profile modification coefficient x_1 for ratio gear u=3 and pinion tooth number z_1=20.

By analyzing the diagram, several conclusions can be drawn:

- for pinion profile modification coefficient x_1 the ratio s_a/m value is increased for the pinion of the external gear as well as for the internal gear pinion;
- the curves that describe the tooth addendum thickness of the pinion term to the pinion profile modification coefficient x_1 are practically parallels between them for the external gear as for the internal gear, and the addendum tooth thickness is increasing until reaching the 0 value of the profile modification coefficient x_d decreasing afterwards;
- for the internal gear pinion is notable an increasing of the tooth addendum thickness as well as an convergence of the tooth thickness value after a certain value of the pinion profile modification coefficient;
- while the maximum value reached by the ratio s_a/m of the pinion on external cylindrical gear is sub-unit approaching 0.8 for the pinion of an internal cylindrical gear the ratio s_a/m can reach a value near 1.5.

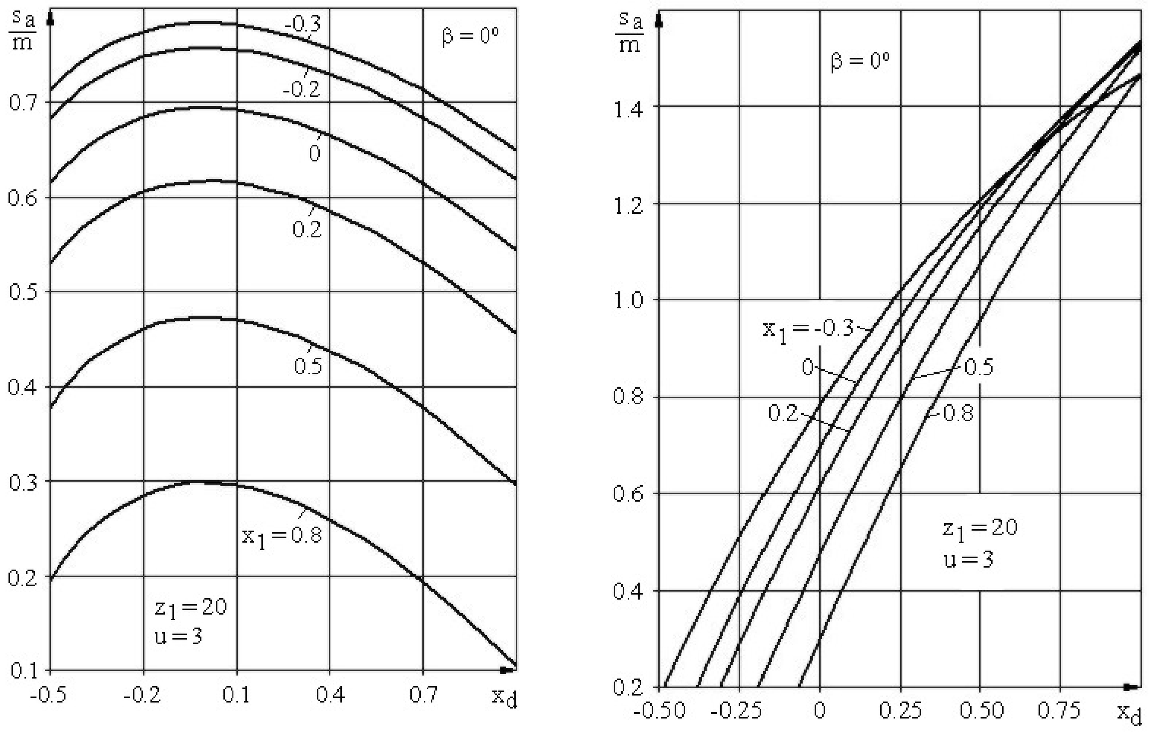

Figure 6 Ratio s_a/m variation by x_1

In ***Figure 7*** the variation of ratio s_a/m is presented, for both external and internal spur gear (β=0º), with respect to pinion profile modification coefficient x_1 and the pinion tooth number z_1, for ratio gear u=3 and profile modification coefficient x_d=0,5.

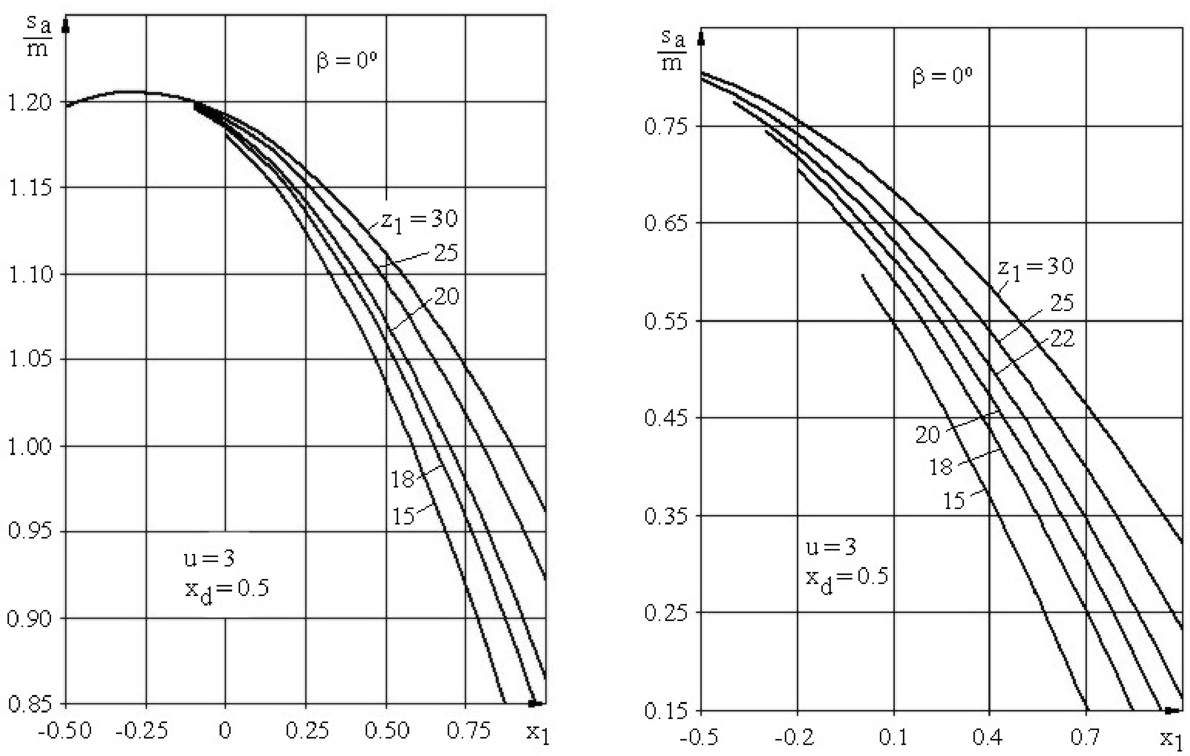

Figure 7 Ratio s_a/m variation by z_1

By analyzing the diagram, several conclusions can be drawn:

- the pinion tooth number influences the ratio s_a/m, (which represents the tooth addendum thickness) by increasing it's value with the increasing of the pinion number of teeth for both external and internal cylindrical gears;
- the decrease is more pronounced for a minor number of pinion number of teeth for both external and internal cylindrical gears;
- the curves that describes the variation on addendum tooth thickness term to pinion number of teeth z_1 have the same shape for both external and internal cylindrical gears;
- for external cylindrical gear is notable a growth of the addendum tooth thickness reaching a value x_1= -0.25 afterwards the value of the ratio s_a/m decreasing.

5. CONCLUSIONS

Based on the previous diagrams following conclusions can be drawn:

- the value of the ratio of the tooth addendum growth s_a/m enlargement is decreasing in the same time with the decreasing of the pinion profile modification coefficient x_1, the decrease is more pronounced for the case when the number of teeth of the pinion is smaller, it's value reaching for x_1=1, a medium value of 0.9 for the external gear pinion and 0,15 for internal gear pinion;
- the curves that describes the variation on addendum tooth thickness s_a/m have similar shape for both internal and external gears with helix (β=0º) for gearing ratio *u* values smaller then 3 as well as for values between 3 till 8, the medium value reached for x_1=1 is 0.1 for external gears pinion and bigger, about 0.9 for internal gear pinion;
- the value of the ratio of the tooth addendum growth s_a/m decreases with the enlargement of the pinion profile modification coefficient x_1 for both types of spur cylindrical gears and it grows term to the enlargement of the profile modification coefficient x_d for the pinion of the internal gear, comparing with the diminution of it value for the pinion of the external gear. The value of the ratio of the tooth addendum growth s_a/m reaches a medium value of 1.5 for a cylindrical internal gear.

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