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LOGISTIC EQUATION – ORDER AND CHAOS

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Abstract: The morphological theories offer the necessary instrument for describing, in its specificity, the forms' world – a world in continuous motion, so complex, that it can not be subjected to the ordinary quantitative analysis. This instrument allows us to progress beyond the barrier established on the idea that mathematics stops where the nature begins, and which the founders of the modern mathematics succeeded to exceed it just by the denying of nature itself. The morphological theories make us to understand the reality and not only to predict it.

1. INTRODUCTION

The universe of the morphological theories if that of the world close to us, so close, that we often forget it: the universe of the objects around us. These theories study the world rationality where we live, the world subjected to occurrence, that the classic physics has chose to ignore it. The growth of a tree, the branches forming, the burgeoning, the covering with leaves, etc., all of these reveal an own rationality, that escapes to the ration that persists to calculate. The rationality of the world close to us is discovered and considered thoroughly by the morphological theories, which replace the infinitesimal calculation with qualitative disciplines as topology and geometry.

The fractal theory, proposes to study o certain family of forms, namely those "fractals". This qualifier suggests about what kind of forms they are. Fractal means fragmented, fractionated, irregular, interrupted. It deals with forms that are characterised by an intrinsic complexity, by a fundamental irregularity that manifests itself at any observation scale.

A fractal theory contains two indissoluble connected aspects: in the first stage, it is mathematically defined o fractal set, in the second stage, it is explored, as possible systematically, the sets that correspond to the chosen definition.

The fractal sets are abstract geometrical structures. Their construction uses the mathematical theory of the complex function iteration. The geometrical structures with these properties (or at least the most of them) are probably presented in very diverse forms.

The preferred area of the chaos theory is that of the irregular or "chaotic" forms. The chaotic term is reserved to the systems whose behaviour defies any description. The chaos theory appeals, in a great measure, to the mathematical apparatus of the **dynamic systems science**. This theory opens the way to understanding of a phenomena category apparently disordered and chaotic.

The irregular forms and the chaotic processes abound in nature. There are not necessary long observations to discover them, so long as the simplest experiments, day after day, bring them to our attention. A river flowing has not ever a constant velocity, but it is whirling because of the obstacles it meets. A leaf that takes off does not fall in a straightforward, but following a sinuous and complex trajectory. A vehicle that passes through an environment (a boat on the water or a plane in flight), leaves behind a turbulent trail. All these irregular phenomena are familiar examples of chaotic processes and forms, and the order leaves its place to the chaos.

The chaos theory treats the irregular or chaotic structures.

The fractals theory takes into consideration, as the previous one, the irregular forms, but it distinguishes by the absence - almost complete - of the considerations of dynamic order. It does not propose to take into consideration the form genesis, but it elaborates a new reading grid of the existent forms.

The universe of the classic science is not only uniform, but also simple (or rather simplified). This simplicity was obtained by the diversity homogenisation.

In the spirit of science, to understand a phenomenon means to analyse it, i.e. (in an etymological sense) to break it up into simple elements. In this respect, it is outlined a confrontation between the two areas of experience: on the one hand, that of the simple and fundamental reality, and on the other hand, that of the complex and accidental reality. Thus, it is considered that: a complex behaviour can be resulted only from complex causes (or from a complex of causes) and, inversely, a simple system can have only a simple behaviour. In a certain way, the whole structure of the classic science is based on this double axiom. According to the morphological theories, the complexity of behaviour (for instance, the turbulence) results from the action of more independent factors that perturb, in an uncontrollable manner, the system evolution.

2. LOGISTIC EQUATION

The chaos theory reveals that, far to be antagonist, the simplicity and complexity can coexist. The systems with a small number of degrees of freedom can manifest – in contradiction with that was believed until now – very complicated behaviours. The discover of the complexity of simple systems represents, without doubt, an essential result and one of the most remarkable features of the modern chaos theory.

The probability that the stable structural systems to execute complicated motions, exponential unstable, represents one of the most important discovery, made by the differential equations theory. Not too long ago, it was supposed that in the systems of generic differential equations there could be only stable simple regimes: equilibrium positions and cycles. If the system was more complicated (for instance, a conservative system), it is admitted that, under the effect of a small equations changing, the complicated motions are “decomposed” into simple motions.

An example of simple system with complex behaviour is the equation with differences. This equation allows the determination, step by step, of the successive values of a variable. An equation with differences can be written with the following general formula:

$$X_{t+1} = F(X_t) \tag{1}$$

where *F* represents a mathematical function. Knowing the initial value X_0, it can be calculated:

$$X_1 = F(X_0),\ X_2 = F(X_1)\ldots \tag{2}$$

Analysing the behaviour of a simple equation with differences, it can be find that the simple mathematical models can contain very complex dynamics.

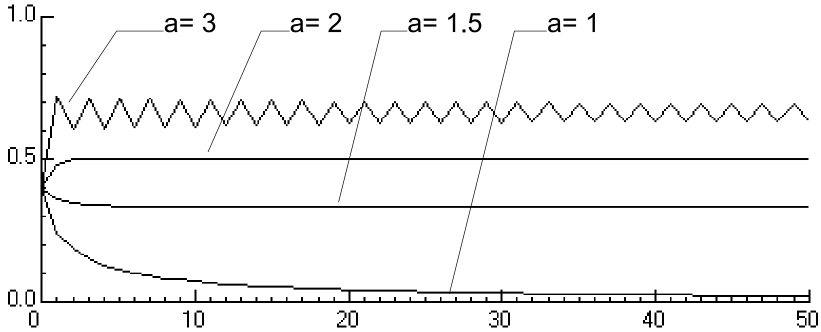

Fig. 1. Tendency to a fixed point

In the next step of the presentation, it is considered the equation, sometimes named logistic equation:

$$X_{t+1} = a\,X_t\left(1 - X_t\right) \qquad (3)$$

where *a* represents a parameter that can be construed as increasing factor.

For the study of the logistic equation, the series X_i will be analysed for different values of *a* factor and the seed $X_0 = 0.4$.

- If the parameter is sub-unitary, all the equation iterations tend to origin.
- If the parameter is greater than *1*, the behaviour is complicated.
  - For an *a*-value between *1* and *3*, the regime is stationary; indifferently of the initial condition value, the iterations tend to an unique fixed point that depends on *a*, Figure 1.
  - If the *a*-value is greater than 3, the regime is bifurcated; it is not anymore stationary, it becomes cyclic. The cycles are for the first of 2 period that is the equation takes alternately two values.
  - If the parameter continues to increase, the cycles become of *4* period, and the *8* period and so on, Figure 2. If the parameter *a* exceeds the accumulation point (*3.5699*), the system becomes chaotic, aperiodic, but however the trajectories remain between certain limits (to a finite distance from origin) [1]. The equation iterations create apparently a random series, depending significantly of X_0, the initial condition value, Figure 3.

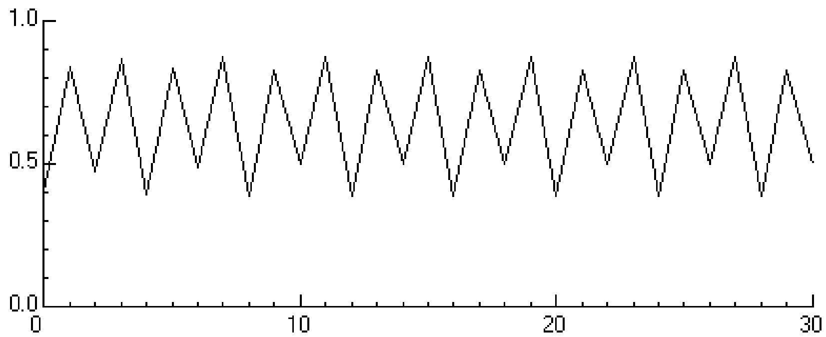

Fig. 2. Behaviour close to 4-cycle, a=3.5

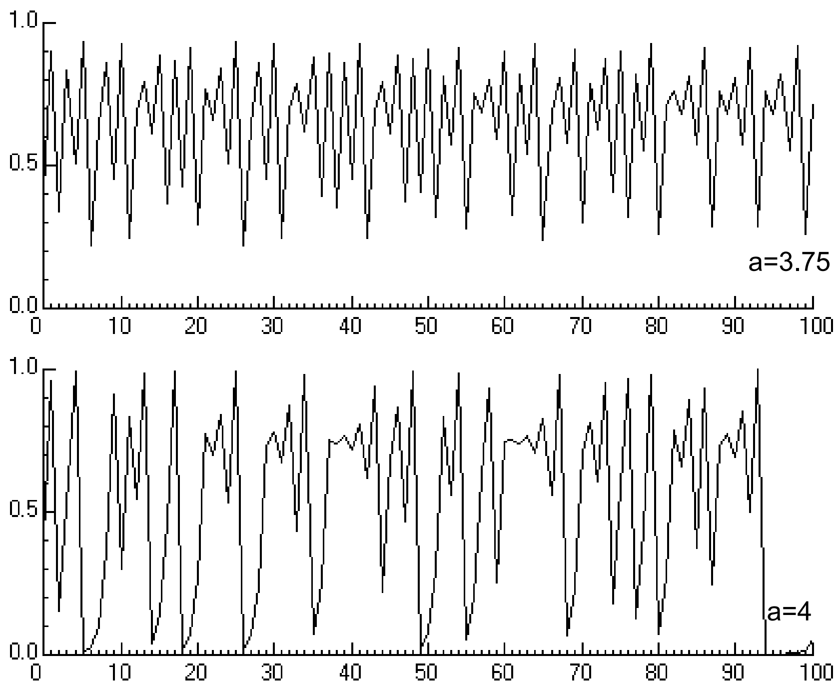

Fig. 3. The system becomes chaotic (the trajectories still remain between certain limits)

3. LOGISTIC EQUATION – THE MECHANISM OF PERIOD DOUBLING (THE BIFURCATION DIAGRAM)

For a better understanding of the tendency of X_i iterations series, it is proposed the graphic representation of this depending on the increasing factor *a*, Figure 4. By means of this diagram, it can be established the value of the parameter *a*, for what the trajectories tend to a desired value or estimate a cyclic behaviour by a certain period [2]. These diagrams also allow the determination of the accumulation point of cycles (close by the value of *3.68*, Figure 4).

Figures 5, 6 and 7 present the iterations of logistic equation for values of the parameter *a*, that are close by some characteristic points, such as:

- the pass from the cycle of *2* period to the cycle of *4* period, Figure 5;
- the pass from the cycle of *4* period to that of *8* period, Figure 6;
- the pass of the accumulation point of cycles, respectively the pass to the zone corresponding the chaos, Figure 7.

For an illustration more suggestive of the manner of X_i iterations variation, in Figure 8 are represented 4 successive iterations X_i.

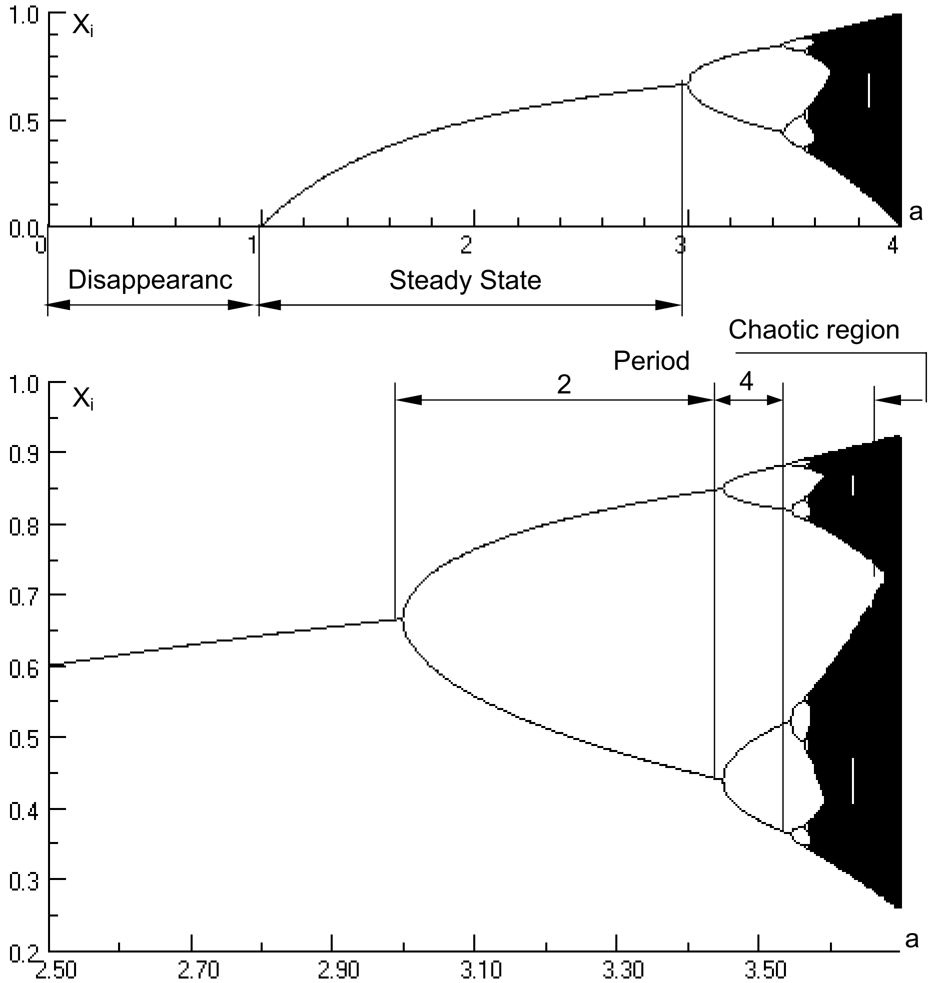

Fig. 4. The logistic equation and the mechanism of period doubling

4. CONCLUSIONS

Morphological theories try to describe, and as it is possible, to explain the creation, formation, evolution and the disappearance of forms, try the genesis understanding and their stability, in a multitude of domains.

The nature of the opposition between the physics and the morphological branches of science can be elucidated, getting thoroughly into the problems continuity / discontinuity. The opposition continuity / discontinuity has inverse roles in these cases: if the morphological branches of science start from continuity for division it, the physics starts from the discrete particles and brings together, extends and aggregates them in a continuum. The lack of interest of the modern physics for the natural phenomena forms can be explained to begin by the inappropriate mathematical instrument in the discontinuity study. The form is always emphasised on a background, expressing the presence of a discontinuity of the environment properties.

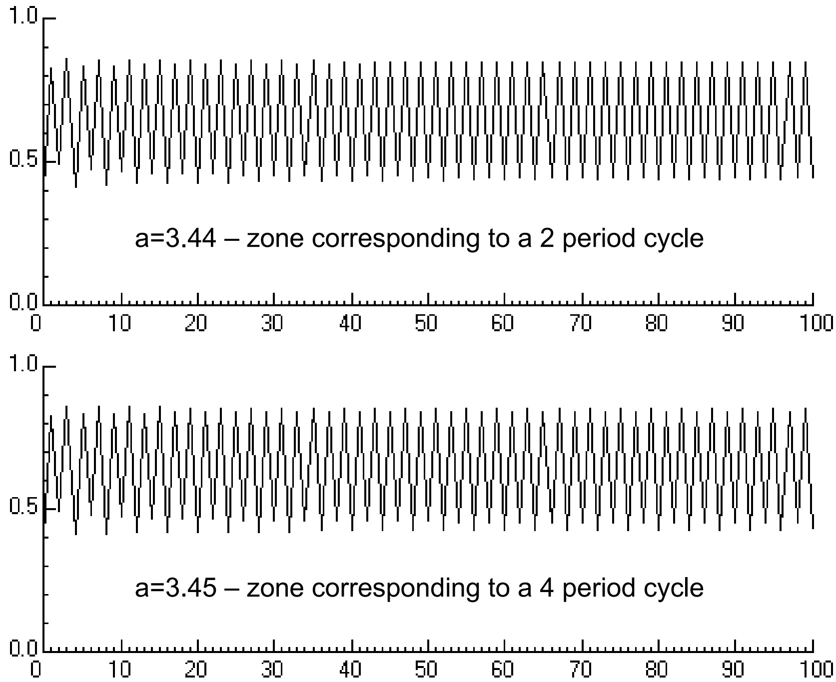

Fig. 5. The pass from the zone corresponding to the 2 period cycle to that of 4 period

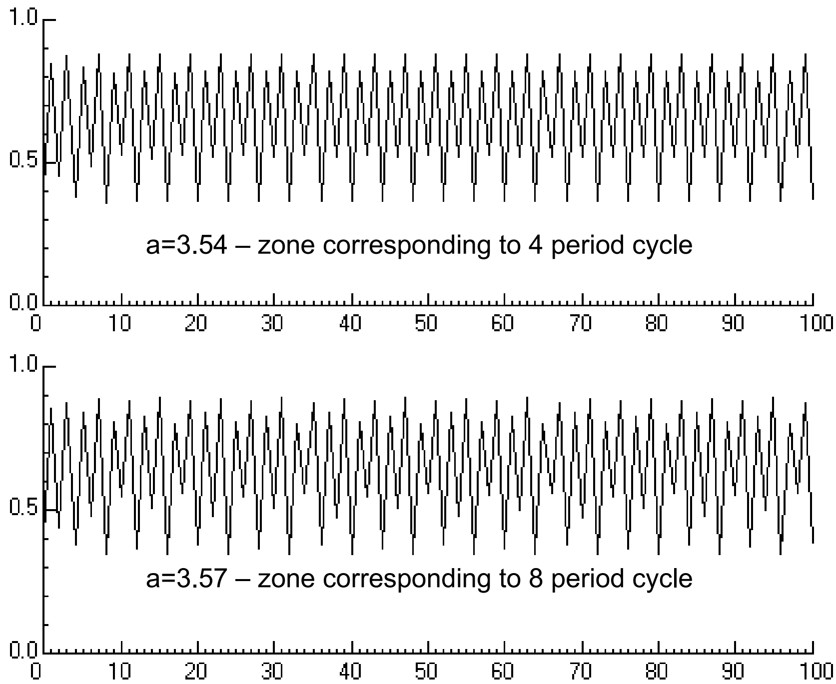

Fig. 6. The pass from the zone corresponding to a 4 period cycle to that of 8 period

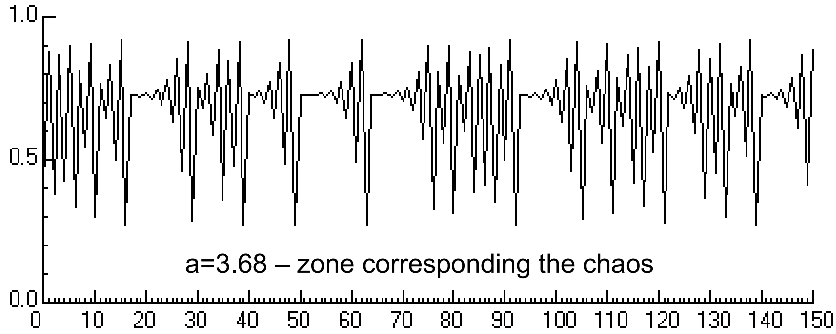

Fig. 7. Characteristic points on the bifurcation diagram of logistic equation

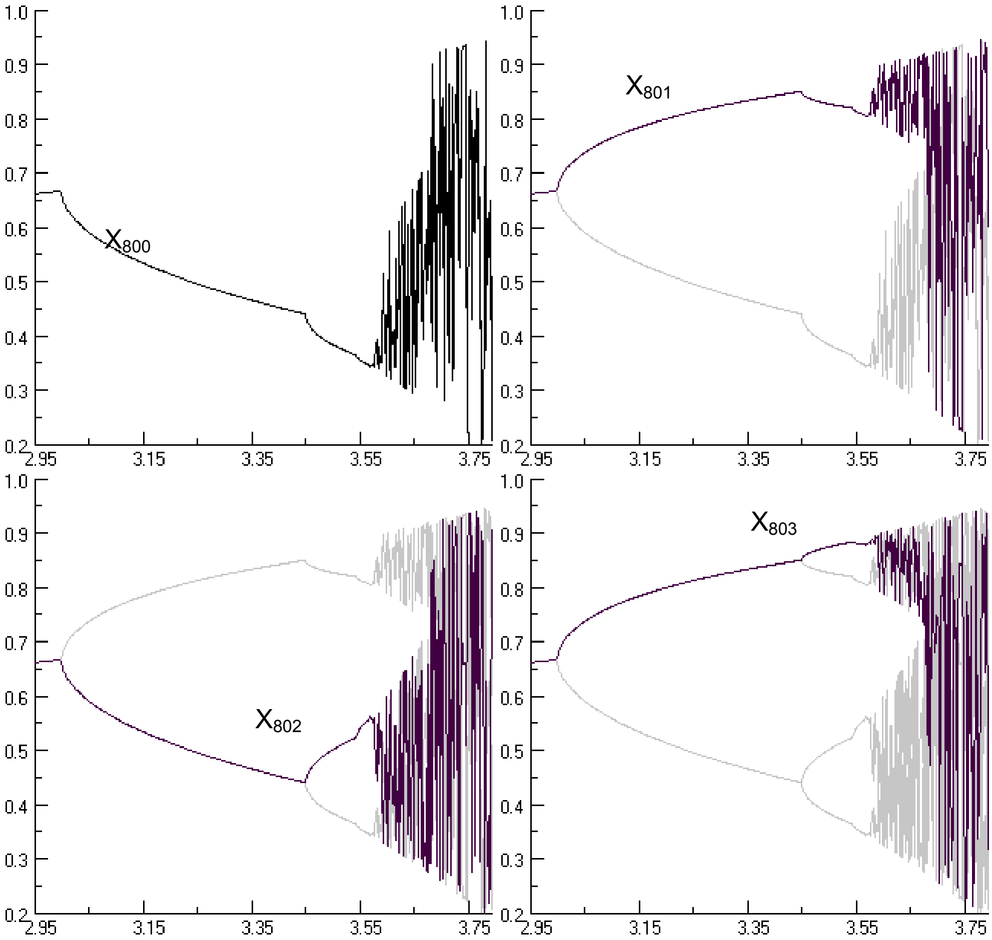

Fig. 8. The representation of 4 successive iterations of the series X_i

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