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MORPHOLOGICAL THEORIES – REGULARITY AND IRREGULARITY

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Abstract: In general, the physical models are proved incapable to describe in a formal manner, empirical discontinuities, and this for a very simple reason: they use regular functions that are by their nature continuous. This continuity primate is emphasised by the necessity of accurate prevention of the phenomena in their evolution, of deduction of any state starting from the initial state. The applied method in this area is always the same: on experimental ways, it is found out how evolves a phenomenon at local scale and on an infinitesimal time (between t and t+dt), then the local evolution assembly is integrated in a global evolution.

1. INTRODUCTION

According to the morphological theories, the complexity of behaviour (for instance, the turbulence) results from the action of more independent factors that perturb, in an uncontrollable manner, the system evolution.

For example, in mechanics, the irregularity of trajectories is put on the account of the diversity of a multitude of elementary forces that actuate on the mobile.

This traditional approach of the complexity is based on at least two suppositions [1]:

- a complex behaviour can appear only in systems with a high number of degrees of freedom (or infinite). In the simple systems, that has only a reduced number of degrees of freedom, it can not be manifested a turbulent behaviour;
- the behaviour complexity is not intrinsic to the system, but extrinsic (or accidental). It results from the exterior elements action that affects the system evolution. To put it differently, a system adopts an irregular and chaotic behaviour when its "regular" evolution (unperturbed) is compromised by the action of the exterior environment and becomes uncontrollable.

The morphological theories (especially the chaos theory and the fractal theory) bring back into discussion these suppositions, offering, in the same time, another image about the complexity.

The chaos theory is not the only one among the morphological disciplines that put under question, in such a radical manner, the traditional representation and analysis of the complexity. The fractal theory proposes to make similar changes. The Mandelbrot set is generated by iteration. Iteration means to repeat a process over and over again. In mathematics this process is most often the application of a mathematical function. For the Mandelbrot set, the function involved is the simplest non-linear function imaginable, namely X^2+c, where *c* is a constant.

To iterate X^2+c, we begin with a *seed* for the iteration. This is a (real or complex) number that we denote by X_0. Applying the function X^2+c to X_0 yields the new number

$$X_1 = (X_0)^2 + c \tag{1}$$

Now, we iterate, using the result of the previous computation as the input for the next. That is

$$X_2 = (X_1)^2 + c,\ X_3 = (X_2)^2 + c,\ X_4 = (X_3)^2 + c,\ \ldots \tag{2}$$

and so forth. The list of numbers $X_0, X_1, X_2,\ldots$ generated by this iteration is called the *orbit* of X_0 under iteration of X^2+c. One of the principal questions in this area of mathematics is: What is the fate of typical orbits? Do they converge or diverge? Do they cycle or behave erratically? (the Mandelbrot set is a geometric version of the answer to this question).

The time series is the plot of orbit values in order. That is, it is the graph of the points (0, X_0), (1, X_1), (2, X_2), ... When many points are plotted, the ordering can be emphasised by drawing lines connecting successive points. This is one of the most common ways to visualise temporal patterns in data.

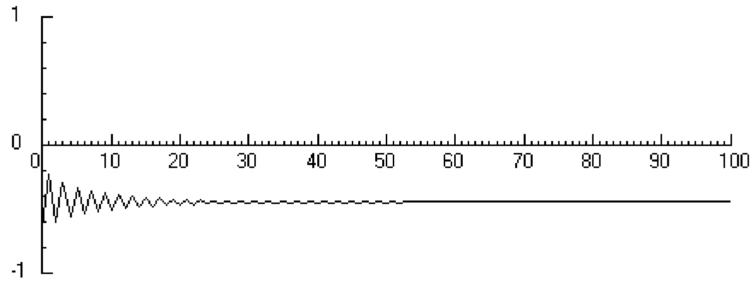
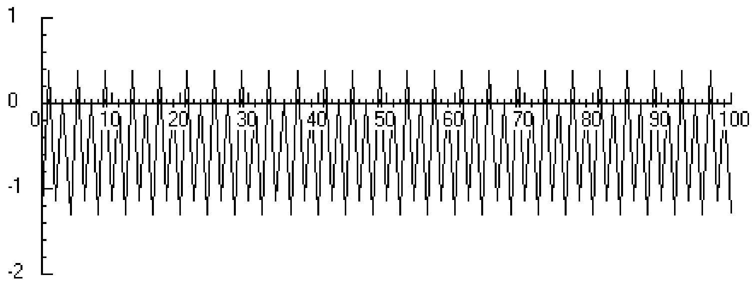
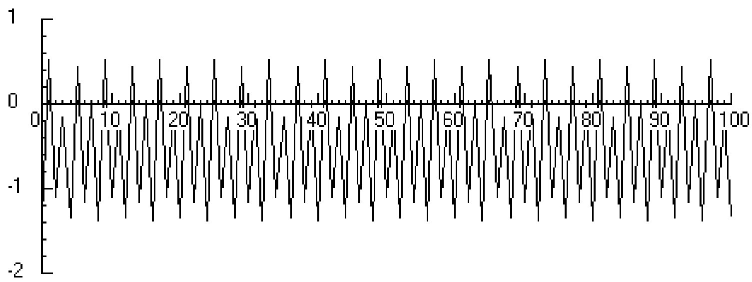


Fig. 1. Time series for X^2+c, tendency to a fixed point, c= -0.65



a. c=-1.3, 4-cyle



b. c= -1.38, 8-cycle

Fig. 2. Behaviour close to n-cycle

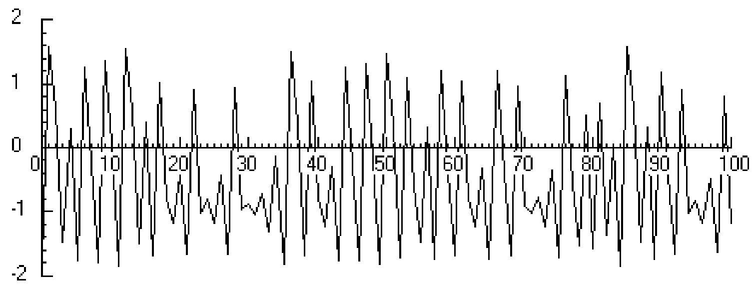


Fig. 3. Chaotic behaviour - c=-1.85

In the next step of the presentation we will analyse a few examples. Suppose we start with the constant c= 1. Then, if we choose the seed 0, the orbit is

$$X_0 = 0, X_1 = 1 = 0^2 + 1, X_2 = 2, X_3 = 5, X_4 = 26, X_5 = \text{BIG}, X_6 = \text{BIGGER}, \ldots \quad (3)$$

and we see that this orbit tends to infinity.

As another example, for c= 0, the orbit of the seed 0 is quite different: this orbit remains *fixed* for all iterations

$$X_0 = 0,\ X_1 = 0,\ X_2 = 0, \ldots \quad (4)$$

If we now choose *c*= *-1*, something else happens. For the seed *0*, the orbit is

$$X_0 = 0,\ X_1 = -1,\ X_2 = 0,\ X_3 = -1,\ \ldots \qquad (5)$$

Here we see that the orbit bounces back and forth between *0* and *-1*, a *cycle of period 2*.

To understand the fate of orbits, it is most often easiest to proceed geometrically. Accordingly, a time series plot of the orbit often gives more information about the fate of the orbits. In the plots from Figure 1, Figure 2 and Figure 3, there were displayed the time series for X^2+c for different values of c. In each case we have computed the orbit of *0*. Note that the fate of the orbit changes with *c*.

- For *c*= *-0.65*, the orbit tends to a fixed point, Figure 1.
- For *c*= *-1.1*, the orbit approaches a 2-cycle. For *c*= *-1.3*, the orbit tends to a 4-cycle, Figure 2,a. For *c*= *-1.38*, we see an 8-cycle, Figure 2,b.
- When *c*= *-1.6*, *-1.85* or *-1.9*, there is no apparent pattern for the orbit; mathematicians use the word *chaos* for this phenomenon, Figure 3.

2. TIME SERIES – THE MECHANISM OF PERIOD DOUBLING (THE BIFURCATION DIAGRAM)

For a better understanding of the tendency of X_i iterations series, it is proposed the graphic representation of this series limits depending on the constant values *c*. In this respect, a diagram similar to that of the period doubling mechanism (bifurcation diagram) is obtained, for the logistic equation. By means of this diagram, there can be established the values of the constant *c*, for a certain behaviour of the X_i iterations. It can be also determined the value of the accumulation point of cycles.

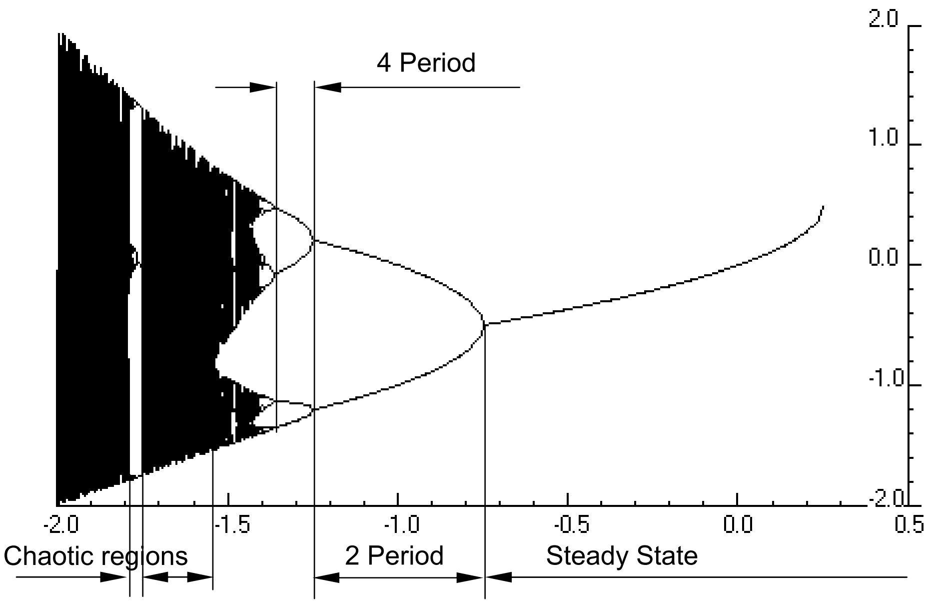


Fig. 4. The mechanism of period doubling for the time series

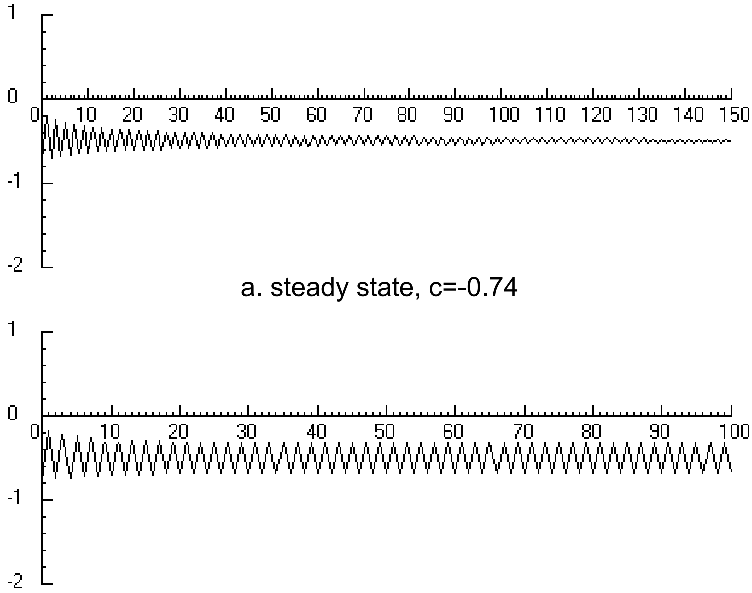
The bifurcation diagram was plotted for values of the constant *c*, included between *–2* and *0.25* [2]. The behaviour analysis of iterations series will be made on the basis of this diagram and on the basis of the iteration series representation around the characteristic points (transition points from a regime to another). In this respect there can be worded the following conclusions:

- For values of the constant *c*>*-0.74*, it is obtained a behaviour specific to a steady state, the iterations tending to a single fixed point, Figure 5,a;

- To a value of parameter *c≅-0.74*, the regime if bifurcated, becoming cyclic:
  - the cycles are, for the first, of period *2*, *c∈(-1.24, -0.75)*, Figure 5,b and Figure 6,a;
  - for *c∈(-1.35, -1.25)*, the cycles are of *4* period, Figure 6,b and Figure 7,a;
  - for *c∈(-1.42, -1.36)*, the cycles are of *8* period, Figure 7,b and Figure 8;
  - for *c∈(-1.43, -1.54)*, the period of cycles increases until the value of *c* exceeds the accumulation point, Figure 9;
  - if *c∈ (-1.74, -1.55)*, the regime becomes chaotic, Figure 10 and Figure 11,a;
  - if *c∈ (-1.75 –1.79)*, the regime is periodical, Figure 11,b and Figure 12,a;
  - if *c∈ (-1.8, -2)*, the regime becomes chaotic, Figure 12,b.

Remark

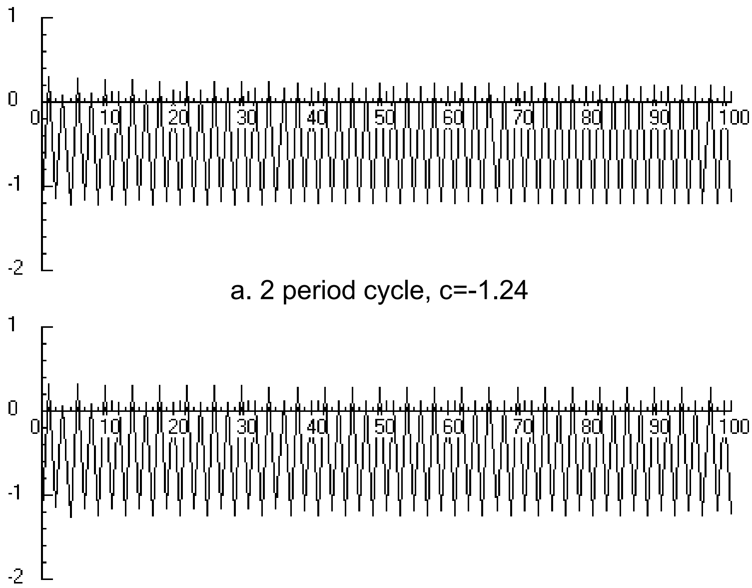
It can be notice, inside the chaotic behaviour zone, a variation region of *c* parameter (between *–1.75* and *–1.79*), where the behaviour becomes periodical.



a. steady state, c=-0.74

b. periodical regime, c=-0.78

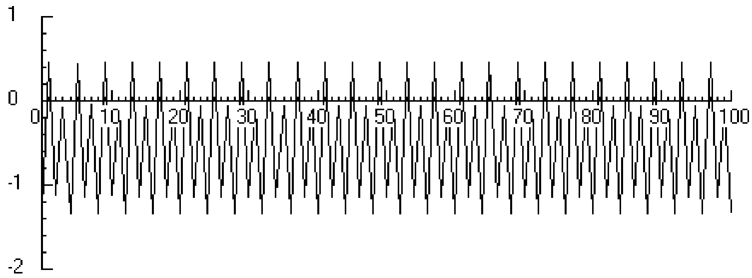
Fig. 5. The pass from a steady state to a cyclic regime of 2 period



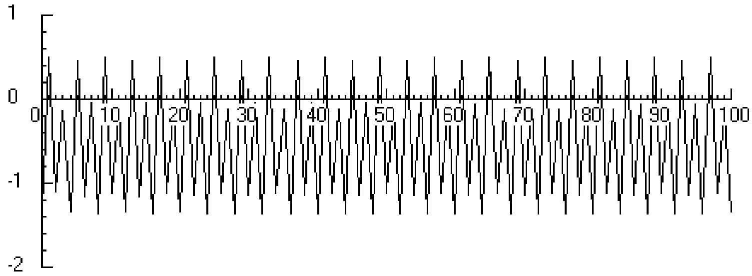
a. 2 period cycle, c=-1.24

b. 4 period cycle, c=-1.26

Fig. 6. The pass from the 2 period cycle to that of 4 period



a. 4 period cycle, c=-1.35



b. 8 period cycle, c=-1.37

Fig. 7. The pass from the 4 period cycle to that of 8 period

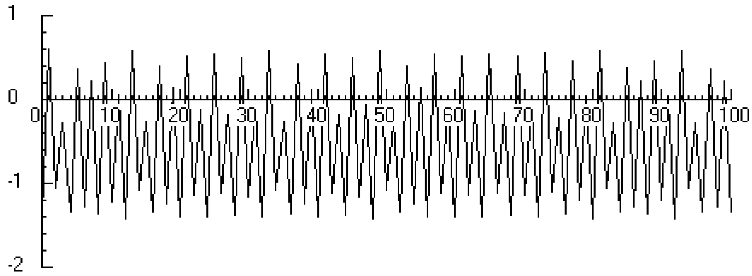


Fig. 8. 8 period cycle, c=-1.42

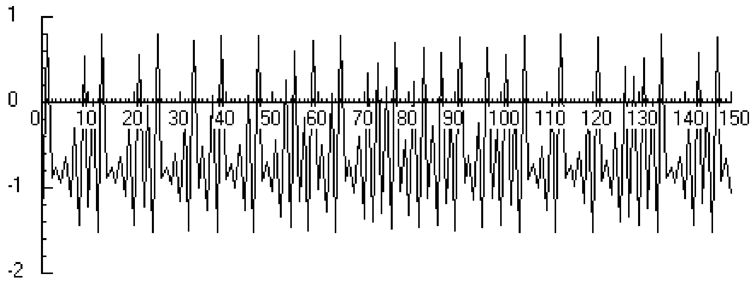


Fig. 9. Close to the accumulation point of cycles - c=-1.53

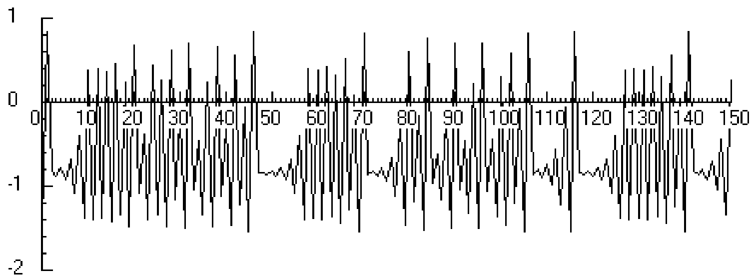
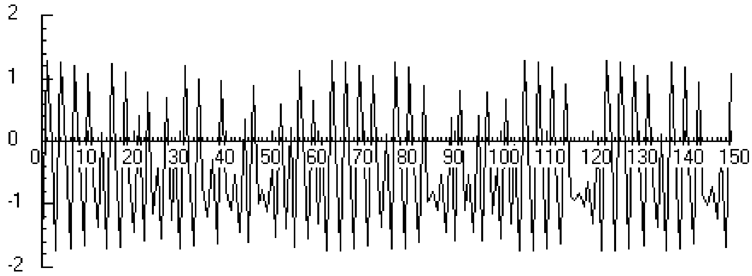


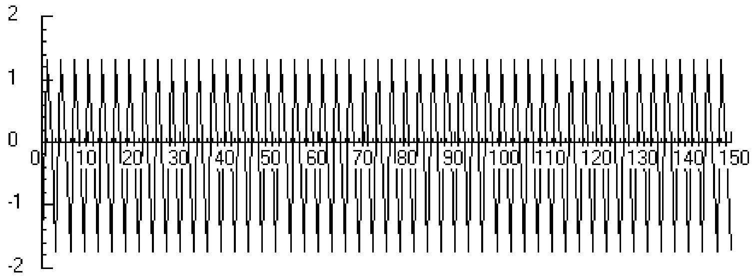
Fig. 10. Chaotic regime, c=-1.55

3. CONCLUSIONS

The complexity of the iteration series presented is not explained by the perturbational action of the unknown elements of the system (like it is considered by the traditional approach), but it is inherent to this. They are included in the form, the equations structure and it resulted from the fact that X_{t+1} does not depend linearly on the X_t (it is not proportional to X_t). There is a characteristic feature of the chaotic systems: they are non-linear systems. But the classic science is linear; it ignores the non-linearity or tries to estimate them by linear equations. In its position of non-linear science, the chaos theory emphasises the fictitious, and consequently the artificial character of the classical simplicity.

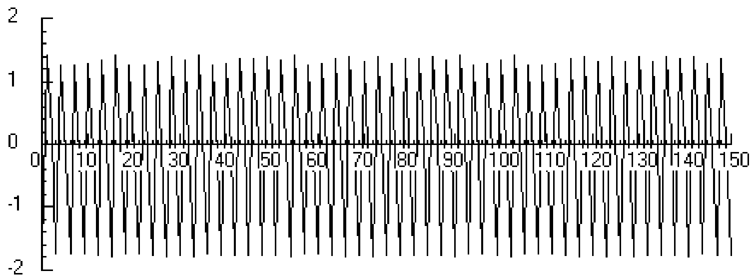


a. chaotic regime, c=-1.74

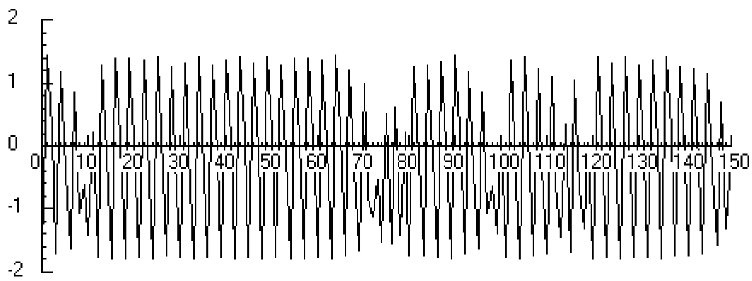


b. periodical regime, c=-1.75

Fig. 11. The pass from chaotic regime to periodical regime



a. periodical regime, c=-1.79



b. chaotic regime, c=-1.8

Fig. 12. The pass from the periodical regime to that chaotic

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