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THE INFLUENCE OF GEOMETRICAL PARAMETERS ON THE Z_B AND Z_D GEARING FACTORS FOR EXTERNAL CYLNDRICAL GEARS

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Abstract: The contact stress calculus of external cylindrical gears is very important because, many times, this calculus imposes the gear overall dimension. The ISO contact stress calculus method considers three situations of teeth contact in which maximum contact stress could appear. Regarding the contact points ISO 6336-2 stipulates: for the pinion, generally, the σ_H actual contact stress is determined considering the contact of the teeth to happen in the highest point of single tooth contact point (*B* point) and, *in special situations*, in the *C* gearing pole; for the driven wheel, generally, the σ_H actual contact stress is determined considering the considering the contact of the teeth to happen in the *C* gearing pole and, *in special situations* (small gearing ratios) in the highest point of single tooth contact (*D* point). In this paper, the authors present the influence of certain gearing parameters on the Z_B and Z_D factors.

1. THEORETICAL BASIS

Pitting of the external cylindrical gear active flanks may be avoided by means of correct contact stress calculus, restricting the effective stress to be smaller than the endurance limit for contact stress. The calculus model of a spur external cylindrical gear, presented in fig. 1, is based of the Hertzian model for the contact (on the generatrix) of two cylinders pushed together with an F_n normal force. Accepting the hypothesizes presented in [6], the σ_H contact stress equation established by Hertz is

$$\sigma_{H} = Z_{E} \sqrt{\frac{F_{n}}{I_{k}} \frac{1}{\rho}}$$
(1)

where: Z_E is the elasticity factor of the two toothed wheels; $1/\rho = 1/\rho_1 + 1/\rho_2$ – the reduced curvature of the teeth profiles, ρ_1 , ρ_2 being the curvature radiuses of the teeth in the considered contact point; I_k – the teeth contact. The calculus model considers that there is only a pair of teeth in contact (ε_α =1.0) and that the teeth make contact in the Y point [5, 6].

For an external cylindrical gear the (1) equation has to be corrected with factors which take account of the differences between the adopted calculus model and the real gear, differences presented in [1, 5, 6]. Therefore, the (1) equation becomes

$$\sigma_{H} = Z_{E} \sqrt{\frac{F_{nc}}{I_{k}} \frac{1}{\rho}} = Z_{E} \sqrt{\frac{F_{nc}}{I_{k}} \frac{\rho_{Y1} + \rho_{Y2}}{\rho_{Y1} \rho_{Y2}}}.$$
 (2)

Since $\rho_{Y_1}+\rho_{Y_2}=T_1T_2=$ const., the maximum contact stress appears where the curvature





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radiuses product is minimal. This corresponds to the *A* point for the pinion and to the *E* point for the driven wheel (fig. 2). Still, on the *AB* and *DE* segments two teeth pairs are in contact and the load is distributed among them. Therefore the maximum contact stress appears in the highest points of single tooth contact *B* and *D* (fig. 3 [1]).



Fig. 2. Radii curvature



Fig. 3. Contact stress diagram

Comparing to the σ_{HC} contact stress in the gearing pole, also called σ_{H0} base stress, the stress in the Y point is

$$\frac{\sigma_{HY}}{\sigma_{H0}} = \frac{\sqrt{1/\rho_{Y}}}{\sqrt{1/\rho_{C}}} = \sqrt{\frac{\rho_{C1}\rho_{C2}}{\rho_{Y1}\rho_{Y2}}} = Z_{Y}.$$
(3)

For the highest points of single tooth contact, *B* and *D*, the Z_B and Z_D gearing factors are determined with these equations [4, 7, 8] (see also fig. 2):

$$Z_{B} = \sqrt{\frac{\rho_{c_{1}}\rho_{c_{2}}}{\rho_{B_{1}}\rho_{B_{2}}}} = \frac{tg\alpha_{w}}{\sqrt{\left[\sqrt{\frac{d_{a_{1}}^{2}}{d_{b_{1}}^{2}} - 1} - \frac{2\pi}{z_{1}}\right]\left[\sqrt{\frac{d_{a_{2}}^{2}}{d_{b_{2}}^{2}} - 1} - (\varepsilon_{\alpha} - 1)\frac{2\pi}{z_{2}}\right]};$$
(4)

$$Z_{D} = \sqrt{\frac{\rho_{C1}\rho_{C2}}{\rho_{D1}\rho_{D2}}} = \frac{tg\alpha_{w}}{\sqrt{\left[\sqrt{\frac{d_{a2}^{2}}{d_{b2}^{2}} - 1} - \frac{2\pi}{z_{2}}\right]} \left[\sqrt{\frac{d_{a1}^{2}}{d_{b1}^{2}} - 1} - (\varepsilon_{\alpha} - 1)\frac{2\pi}{z_{1}}\right]}.$$
(5)

Referring to the spur cylindrical gear calculus for contact stress, the ISO/DIN calculus method [7, 8] recommends the following:

• for the pinion, generally, the σ_H actual contact stress is determined considering the contact of the teeth to happen in the highest point of single tooth contact point (*B* point) and, **in special situations**, in the *C* gearing pole;

• for the driven wheel, generally, the σ_H actual contact stress is determined considering the contact of the teeth to happen in the *C* gearing pole and, **in special situations** (small

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gearing ratios) in the highest point of single tooth contact (*D* point).

In this paper, the authors present a detailed analysis of the Z_B and Z_D highest point of single tooth contact (HPSTC) gearing factors with mentioning the conditions in which the maximum stress appears in the *C* gearing pole or in the *B*, respectively *D* points on the gearing line.

2. SOFTWARE

In order to generate data and to be able to perform a comprehensive analysis of the Z_B and Z_D factors dependency, a software application was developed with easy usage, portability and expandability in mind.

The application reads input data from XML files which are easy to modify and benefits of the optimization methods presented in [Deaky 2006 - 1] to increase processing speed and dynamic behavior.

Results are displayed (fig. 1) in spreadsheet format and the special charting/diagram windows may be used to graphically study (including zooming) the dependency of the Z_B and Z_D factors of different parameters. The left options panel of the charting windows allows the user to choose the parameter on *x* axis, the variable chart element and the values for fixed parameters. The variable parameter multiple curves are drawn using a gradient of colors (black to red) to suggest the evolution.

21	B, ZD)									🛃 ZB Graphs		
	Make ZB Graphs Make ZD Graphs							Make ZD	Graphs		ZB dependecy (xn1, z1, Beta = 0, u = 5, xsn = 1)		
Beta	I≊Z1	u	Xn1	Xsn	k	EpsAlpha	da1perdb1	da2perdb2	ZB	ZD 🔥	Values on X axis 1.311-x	z1 = 15	
0	20	2	-0.2	1	0.094	1.389	1.139	1.176	1.249	1.016	xn1 💌	z1 = 20.	
0	20	2	-0.1	-0.5	0.039	1.771	1.156	1.094	1.037	0.976	1.274	z1 = 25. z1 = 30.	
0	20	2	-0.1	0	0	1.644	1.16	1.123	1.088	0.986	Variable element	z1 = 40.	
0	20	2	-0.1	0.5	0.027	1.521	1.157	1.148	1.142	0.995 📃	1.237-	z1 = 50. z1 = 60.	
0	20	2	-0.1	1	0.094	1.394	1.15	1.171	1.206	1.003			
0	20	2	0	-0.5	0.039	1.753	1.166	1.089	1.015	0.962	1.201		
0	20	2	0	0	0	1.635	1.171	1.117	1.062	0.974	Fixed element		
0	20	2	0	0.5	0.027	1.518	1.168	1.143	1.112	0.984			
0	20	2	0	1	0.094	1.396	1.161	1.166	1.17	0.992			
0	20	2	0.1	-0.5	0.039	1.731	1.177	1.083	0.997	0.952	u 5 -		
0	20	2	0.1	0	0	1.623	1.181	1.112	1.041	0.964			
0	20	2	0.1	0.5	0.027	1.513	1.178	1.137	1.087	0.973	xsn 🛛 🔽		
0	20	2	0.1	1	0.094	1.396	1.171	1.16	1.139	0.982	1.054		
0	20	2	0.2	-0.5	0.039	1.705	1.188	1.078	0.983	0.946			
0	20	2	0.2	0	0	1.608	1.192	1.107	1.023	0.955	1.017		
0	20	2	0.2	0.5	0.027	1.504	1.189	1.132	1.065	0.965			
0	20	2	0.2	1	0.094	1.393	1.182	1.155	1.113	0.973	0.98		
0	20	2	0.3	-0.5	0.039	1.675	1.198	1.073	0.971	0.943			
0	20	2	0.3	0	0	1.589	1.203	1.101	1.007	0.949	0.943 0.25 0.2 0.05 0.1 0.25 0.4 0.55 0.7 0.95 1		
0	20	2	0.3	0.5	0.027	1.493	1.2	1.127	1.046	0.958	-0.0 -0.05 -0.2 -0.05 0.1 0.25 0.4 0.05 0.7 0.85 1		
	-												

The charts may be saved as bitmap or jpeg files.

Fig. 4. Results display (spreadsheet and configurable diagrams)

3. THE INFLUENCE OF GEOMETRICAL PARAMETERS Z_B AND Z_D FACTORS

By running the developed software several diagrams were made for the Z_B and Z_D factors, studying the influence of the x_1 pinion addendum modification coefficient, the *u* gearing ratio, the x_s sum of addendum modifications coefficient, considering that the gears are manufactured using standard rack ($\alpha = 20^\circ$, $h_a^* = 1.0$, $c^* = 0.25$). The $d_{a1,2}$ gear tip diameters and the ε_{α} transverse contact ratio were determined taking account of the teeth shortening to maintain the clearance. The shortening coefficient is determined with [3]:

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$$\boldsymbol{k} = \left(\boldsymbol{x}_{n1} + \boldsymbol{x}_{n2}\right) - \left(\frac{\boldsymbol{z}_1 + \boldsymbol{z}_2}{2}\right) \left(\frac{\cos \alpha_t}{\cos \alpha_{wt}} - 1\right).$$
(6)

Fig. 5 presents the variation of the Z_B factor while fig. 6 shows the variation of the Z_D factor for a spur external cylindrical gear, depending of the pinion addendum modification coefficient x_1 and the pinion teeth number z_1 . The gear has a u=5 gearing ratio and a



Fig. 5. $Z_B = f(x_1, z_1, \beta = 0^\circ, u = 5, x_s = 1)$

Fig. 6. $Z_D = f(x_1, z_1, \beta = 0^\circ, u = 5, x_s = 1)$

 x_s =1 sum of addendum modification coefficient. The analysis of these diagrams leads to the following conclusions:

- Both highest point of single tooth contact gearing factors rapidly decrease with the increase of the x₁ addendum modification coefficient value;
- The value of the Z_B factor increases with the increase of the z_{n1} pinion teeth number (see fig. 5), while the value of the Z_D factor decreases with the increase of the same pinion teeth number (see fig. 6).
- The values x_1 of the addendum modification coefficient for which $Z_B \ge 1$ are smaller then, for $z_1=60$ and smaller then 0.8, for $z_1=15$, while when $Z_D\ge 1$ the same values are smaller then 0.1, for $z_1=60$ and smaller then zero for $z_1=20$; therefore, there are zones where both $Z_B > 1$ and $Z_D > 1$, where $Z_B > Z_D > 1$, respectively where $Z_B < 1$ and $Z_D < 1$, when the maximum stress appears in the *C* gearing pole.

Fig. 7 shows the variation of the Z_B factor while fig. 8 shows the variation of the Z_D factor for a spur external cylindrical gear, depending of the x_1 pinion addendum modification coefficient and the *u* gearing ratio, when the gear has $z_1=25$ and $x_s=1$. The diagram analysis leads to these conclusions:





Fig. 8. $Z_D = f(x_1, u, \beta = 0^\circ, z_1 = 25, x_s = 1)$

- The values of the Z_B and Z_D factors decrease with the increase of the of the x₁ pinion addendum modification coefficient, except for the Z_D factor when the gearing ratio is u=1;
- The Z_B gearing factor decreases with the increase of the *u* gearing ratio, while the Z_D factor reaches the same value, no matter of the gearing ratio value, when $x_1=0$ (all curves pass trough the same point);
- Except when we have u=1, where the values of both factors fulfill $Z_B \ge 1$ and $Z_D \ge 1$, in the other case one can see the previously presented situations (referring to the diagrams from fig. 5 and fig. 6).

The influence of the sum of addendum medications coefficient of the external cylindrical gear on the Z_B and Z_D factors is shown in fig. 9 and fig. 10, for a spur gear with z_1 =25 and u=5. The analysis of these diagrams allows pointing out that:

- Compared to the previously presented cases, where we considered $x_s = 1$, the Z_B and Z_D factors decrease with the decrease of the sum of addendum medications coefficient and the x_1 pinion addendum modification coefficient;
- The Z_B HPSTC gearing factor reaches values >1 even for relatively big values of the x_1 pinion addendum modification coefficient, while the Z_D factor reaches values >1 only for smaller values of x_1 ;
- If the x_s coefficient has a relatively reduced value and the x_1 coefficient has a value near to its maximum, both Z_B and Z_D coefficients have values <1, meaning that the maximum stress appears in the *C* gearing pole.





Fig. 10. $Z_D = f(x_1, x_s, \beta = 0^\circ, z_1 = 25, u = 5)$

When the gearing ratio u=1, the Z_B and Z_D factors variation curves change. They are presented in fig. 11 and fig. 12 and lead us to these conclusions:







• The curves representing the variation of the two factors have the shape of a parabola, their values being described by the $Z_B \ge 1$ and $Z_D \ge 1$ inequalities;

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- The curves representing the variation of the two HPSTC factors reach their minimum for more increased values of the x₁ with the increase of the u gearing ratio. This x₁ values increase is more pronounced for the Z_B factor and slower Z_D;
- At high values of the pinion addendum modification coefficient and of the pinion teeth number, the maximum stress appears in the *D* HPSTC of the driven wheel (*Z_D* > *Z_B* >1).

The influence of the helix angle on the Z_B and Z_D factors is shown in fig. 13 and fig. 14, for an external cylindrical gear characterized by $z_1=25$ pinion teeth number, u=5 gearing ratio and $x_{sn}=1.0$ sum of addendum modifications coefficient. Below are some of the conclusions obtained by means of the analysis of these diagrams.





Fig. 14. $Z_D = f(x_{n1}, \beta, z_1 = 25, u = 5, x_{sn} = 1)$

- The values of the Z_B and Z_D decrease with the increase of the x_{n1} addendum modification coefficient in two different stages; in the x_{n1}∈(-0.5...-0.15) interval for Z_B, respectively x_{n1}∈(-0.5...0.6) for Z_D, the values of the factors are smaller for greater β helix angle values, while on the remaining interval the values of Z_B and Z_D factors are greater for smaller β angles;
- The value of the Z_B factor is ($Z_B < 1$) only for big values of the x_{n1} pinion addendum modification coefficient and for $\beta \le 25^\circ$ helix angle values, while the value of the Z_D factor is ($Z_D < 1$) for $\beta > 25^\circ$ no matter of the x_{n1} coefficient value.

4. CONCLUZII

The analysis regarding the influence of some of the geometrical parameters of an external cylindrical gear on the values of the Z_B and Z_D HPSTC gearing factors allows the

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choice of those geometrical parameters in order to obtain gears with higher load capacity, therefore obtaining reduced overall dimensions for the transmissions using cylindrical gears. The main conclusions drawn from the effected analysis may be synthesized as follows:

- One can affirm for certain that the Z_B and Z_D HPSTC gearing factor values decrease with the increase of the x_1 (x_{n1}) pinion addendum modification coefficient, except the case when the gearing ratio u=1, when the values of this factors have an interval of decrease followed by an increase (see fig. 13 and fig. 14);
- Regarding the increase of the z_1 pinion teeth number, the values of the Z_B and Z_D factors behave differently. While the Z_B value decreases, The Z_D value increases;
- If the gearing ratio u>1, both single tooth contact factors, Z_B and Z_D , decrease with the increase of the gearing ratio; the Z_D factor reaches a constant value $Z_D = 1$ for $x_1=0$, no matter of the *u* gearing factor value;
- With the decrease of the x_s (x_{sn}) sum of addendum modifications coefficient, the values of the Z_B and Z_D factors decrease when the gearing ratio u>1; for the external cylindrical gear where u=1, the Z_B and Z_D factors have a parabolic variation with their values described by the $Z_B \ge 1$ and $Z_D \ge 1$ expressions (see fig. 13 and fig. 14);
- The β helix angle influences the values of Z_B and Z_D in the same directions, but on different intervals of the x_{n1} pinion addendum modification coefficient;

From the effected analysis one can see that the maximum contact stress may appear in the highest points of single tooth contact as well as in the gearing pole; the parameters with the biggest influence are the sum of addendum modifications coefficient, and the pinion addendum modification coefficient.

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