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GENERALIZED EQUATION OF THE HELICAL WORM

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Abstract: The worm drives uses different helical worms (or screws), named Archimedes's worm (or ZA, [5]), involute worm (or ZE, [5]) or another types (ZN, ZK). Every helical worm type has an own equation for his helical surface, large described in [1]. In this work I demonstrate that the mostly worms types, excepting ZK type, can have the same equation for their helical surface, named generalized equation of the helical worm. A concrete equation can be obtained there out if two generalized parameters adopt concrete values.

1. INTRODUCTION

The helical worm is generated as a result of a displacement of a curve *(Γ)* along of another helical curve, named directrix *(Δ),* fig. 1 [1]. In the worm transmission as much as at hobbing cutter [2, 3] are used different helical worms (or screws), exempligratia Archimedes's worm, involute worm, ZN or ZK worms [4]. Any worm has its own equation, deduced which is based on the generation mode, see fig. 1. The equation of the helical surface contain an array of the helical curve *(Δ)* with the equation [1,3]

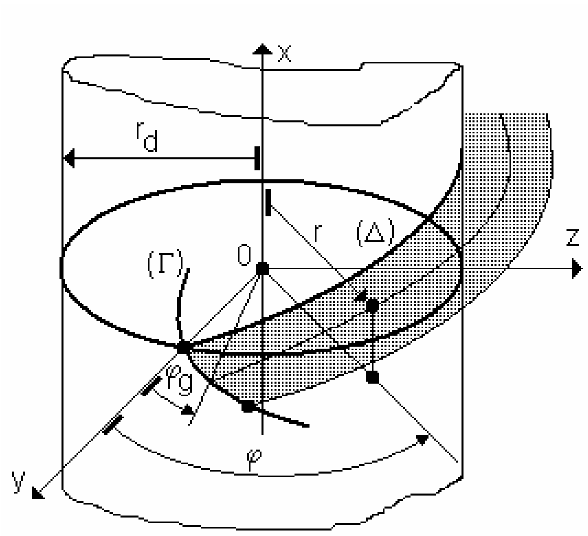

Fig. 1. Generation of the helical worm

$$\begin{cases} x = p_d\ \varphi \\ y = r_d \cos(\varphi) \\ z = r_d \sin(\varphi) \end{cases} \tag{1}$$

where

$$p_d = \frac{P_d}{2\pi} \tag{2}$$

P_d = axial pitch of the screw. All curves *(Δ)* must verify the equation of the curve *(Γ)* given from

$$\varphi_g = \varphi_g(r) \tag{3}$$

and the helical surface can be written therefore

$$\begin{cases} x = p_d(\varphi - \varphi_g) \\ y = r\,cos(\varphi) \\ z = r\,sin(\varphi) \end{cases} \quad (4)$$

An Archimedes's helical worm surface (or Archimedes's helical screw) appears in cross section a pure Archimedes's spiral, in axial section a pure straight line. An involute worm appears in cross section a pure involute spiral, in a contact plane on the ground cylinder from radius r_g a pure straight line only on a single flank, see fig. 2.

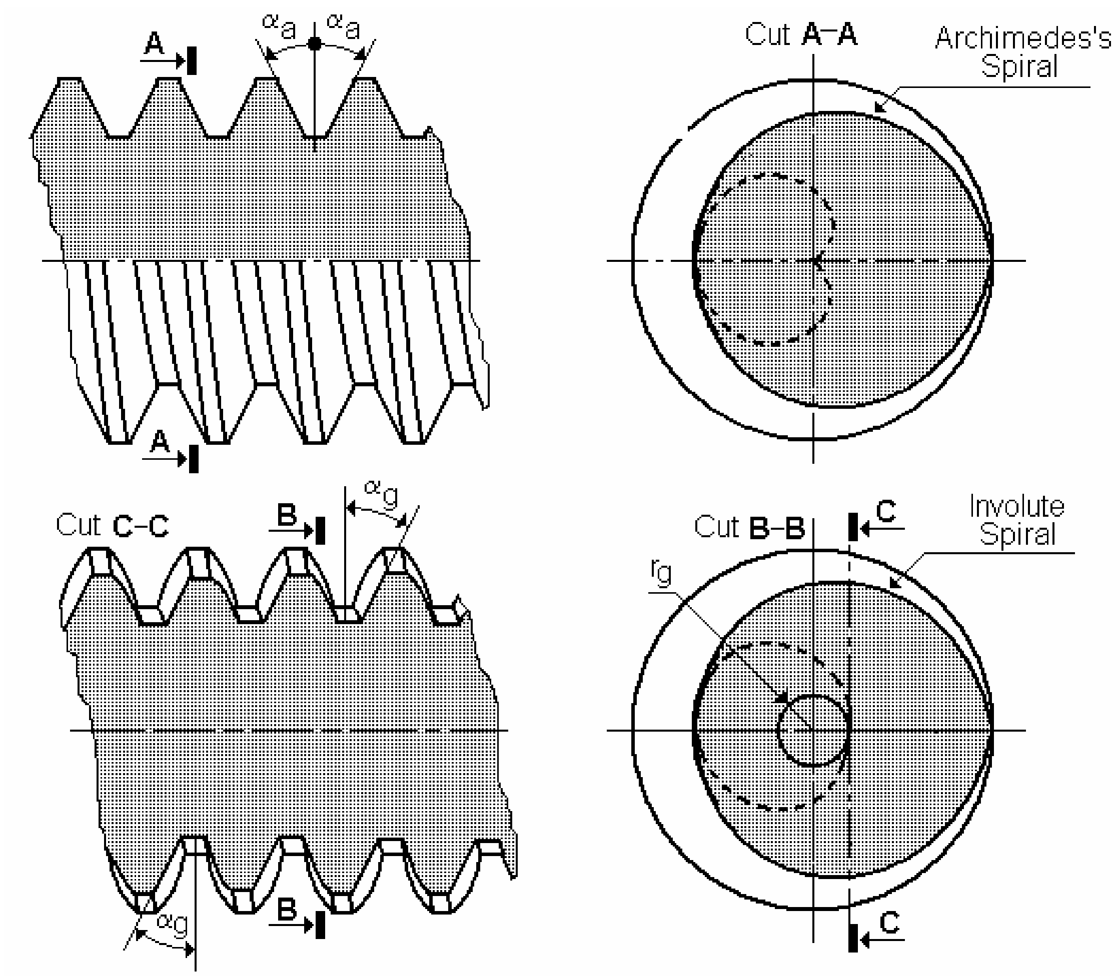

Fig. 2. Archimedes's worm (up) and involute worm (down)

2. THE GENERALIZED EQUATION OF THE HELICAL WORM

The generalized equation of the helical worm arises from a generalized equation of the curve (*Γ*), see fig. 3. A bee line (*D*) circulates slippingless on a ground circle r_g . In the point *A* appears a perpendicular *AB* with the length "*a*". The segment *AB* is considered positive ($a > 0$) if the point *B* belongs under the circle and negative ($a < 0$) if the point *B* belongs out of circle. If *a=0* belongs the point *B* exactly on the line (*D*) in same position on the point *A*.

The equation of the curve (Γ) can be easier parametric written. Because the line (D) circulates slippingless on the circle r_g I can write

$$arc(TA) = \overline{TA'} \Rightarrow r_g(\nu + \alpha) = r_g tg(\alpha) \Rightarrow \nu = tg(\alpha) - \alpha \tag{5}$$

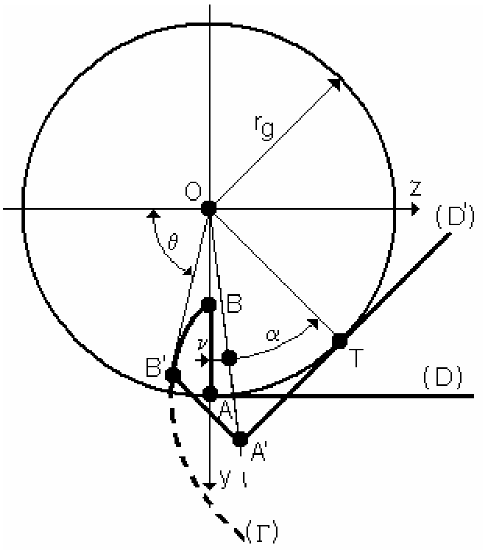

Abb. 3. The generalized curve of (Γ)

and the coordinate of the point B' gives the equation of the curve (Γ)

$$\begin{cases} y = R\cos(\nu) - a\cos(tg(\alpha)) \\ z = R\sin(\nu) - a\sin(tg(\alpha)) \end{cases} \tag{6}$$

where $R = \overline{OA'} = \dfrac{r_g}{\cos(\alpha)}$. If $a=0$ the equation describes a pure involute. If $a = r_g$ we have

$$\begin{cases} y = \dfrac{r_g}{\cos(\alpha)}\cos(tg(\alpha) - \alpha) - a\cos(tg\{\alpha\}) \\ z = \dfrac{r_g}{\cos(\alpha)}\sin(tg(\alpha) - \alpha) - a\sin(tg\{\alpha\}) \end{cases}$$

$$\begin{cases} y = \dfrac{r_g}{\cos(\alpha)}[\cos(tg(\alpha))\cos(\alpha) + \sin(tg(\alpha))\sin(\alpha)] - r_g\cos(tg\{\alpha\}) = r_g\sin(tg(\alpha))tg(\alpha) \\ z = \dfrac{r_g}{\cos(\alpha)}[\sin(tg(\alpha))\cos(\alpha) - \cos(tg(\alpha))\sin(\alpha)] - r_g\sin(tg\{\alpha\}) = -r_g\cos(tg(\alpha))tg(\alpha) \end{cases}$$

$$r = \sqrt{y^2 + z^2} \Rightarrow r = r_g tg(\alpha) \tag{7}$$

Because $a = r_g$, namely $\overline{A'B'} = \overline{OT}$, the polygon $OTA'B'$ is a rectangle. The angle θ pas into

$$\theta = \alpha + \nu = \alpha + tg(\alpha) - \alpha = tg(\alpha) \tag{8}$$

and the equation (7) can be written

$$r = r_g\theta \tag{9}$$

The equation (9) is therefore a pure Archimedes's spiral with the radial pitch

$$P_{sp} = 2\pi\, r_g \tag{10}$$

A certain value for a leads to an elongated *(a>0)* or short-involute *(a<0)* spiral and to adequacy helical surface. The angle φ_g from (3) is to calculate with

$$\varphi_g = arctg\frac{z}{y} = arctg\frac{R\,sin(\nu) - a\,sin(tg(\alpha))}{R\,cos(\nu) - a\,cos(tg(\alpha))} \tag{11}$$

In order to have a parametrical equation like (4) of the helical worm, as function only from current radius r and angle φ, the values for ν and α from (11) must be calculated as functions from r and φ. From fig. 3 we have

$$R = \sqrt{r^2 - a^2 + 2ar_g} \tag{12}$$

and

$$\alpha = arccos\frac{r_g}{R}; \quad \nu = tg(\alpha) - \alpha \tag{13}$$

3. EXAMPLES

A helical worm surface has following specific basic parameters:
- normal module, m_n
- divided circle diameter, D_d
- number of the screw thread, i
- normal angle of pressure on the divided circle, α_{nd}

Another parameters are to calculate as
- the helical angle on the divided circle

$$\omega_d = \arcsin\frac{i\,m_n}{D_d} \tag{14}$$

- axial pitch of the helical line (Δ) fig. 1

$$P_d = \frac{m_n\pi}{cos(\omega_d)} \tag{15}$$

3.1. ARCHIMEDES'S WORM (ZA HELICAL WORM)

The flank angle α_a, fig. 2, calculated with $tg(\alpha_a) = \frac{tg(\alpha_{nd})}{cos(\omega_d)}$ [1]. The Archimedes's worm has the property $tg(\alpha_a) = \frac{P_{sp}}{P_d}$ [1] and the radial pitch of the spiral comes from $P_{sp} = P_d tg(\alpha_a) = P_d \frac{tg(\alpha_{nd})}{cos(\omega_d)}$. The ground cylinder have the radius

$r_g = \frac{P_{sp}}{2\pi} = \frac{P_d \, tg(\alpha_{nd})}{2\pi \cos(\omega_d)}$ and $a = r_g$. If the flank angle α_a must have another value, the calculus continues with this new value. The ground cylinder is not truth at Archimedes's worm, he is an imaginary cylinder.

3.2. INVOLUTE WORM (ZE HELICAL WORM)

The ground cylinder is truth, his radius is given from $r_g = \frac{i m_n \cos(\alpha_{nd})}{2\sqrt{1-\cos^2(\alpha_{nd})\cos^2(\omega_d)}}$ and *a=0*. The equations of the helical surface result direct and easy.

3.3. ZN AND ZK HELICAL WORM

The ZK helical worm hasn't a rectilinear flank and the curve (Γ) is a spatial curved line [5] and therefore can't have the generalized equation for the helical surfaces. The ZN helical worm have a bee line in a plane perpendicular to helical line (Δ) fig. 1, inclined with angle ω_d, [3], [5]. A continuation of this bee line tangents a cylinder named directory cylinder and its diameter is to calculate from [3]. In a cross section appear an elongated involute *(a>0)* and the value of *a* can be determined only on numerical way with the condition of a rectilinear cut of the equation (4) with a plane inclined with angle ω_d, perpendicular to helical line (Δ). The ground cylinder is again imaginary, the radius r_g is to calculate similar to Archimedes's helical worm.

4. CONCLUSION

The mostly helical worms from different type can be analytically defined with a single generalized equation. This fact permits an easier modeling of the worms and, concurrently, a united technology of the fabrication. The generalized equation of the worms can be used alike for the modeling of the worm hobbing cutter [3], [4] and for the gear-cutting by generation.

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