ANNALS of the ORADEA UNIVERSITY.

Fascicle of Management and Technological Engineering, Volume VII (XVII), 2008

KINEMATICAL ANALYSIS OF THE MECHANISM IN CASE OF HYDRAULIC MACHINES WITH AXIAL PISTONS AND WITH ROTATING MOTION TRANSMITTING BY MEANS OF CONNECTING RODS

Simion Parepa
Petroleum-Gas University of Ploieşti, Department of Oilfield and Petrochemical Equipment
e-mail: sparepa@upg-ploiesti.ro

Keywords: hydraulic machines with axial pistons, mechanism, kinematical analysis

Abstract: In the work a kinematical analysis of the mechanism in case of hydraulic machines with axial pistons and inclined block without double cardan joint, taking into account its peculiarity, namely the cylinder block driving in rotating motion by means of connecting rods, is presented. Kinematical schemes equivalent to the constructive scheme by the useless motion liberty identification are realized, and the mobility degree is calculated, the mechanism desmodromy being demonstrated. The variation laws of the piston displacement, of the connecting rod inclination angle in comparison with the cylinder axis, of the angle of rotation and of the cylinder block angular speed, making evident the non-uniformity of the cylinder block rotating motion, are presented.

1. INTRODUCTION

Although the hydraulic machines with axial pistons and with rotating motion transmitting by connecting rods (HMMTCR) are built up and studied for a long time (from the 50 years of the last century − see for example [1]), however a structural and kinematical analysis of their mechanism, taking into account the specific way of transmitting of the rotating motion, has not been carried out.

Naturally, there are more or less accurate approaches by the adopted hypotheses of these mechanism kinematics (in the frame of the three theories: elementary, approximate and accurate, though they are not so called always), as they in the papers [5] and [6] are presented.

In the work [2] the authors enter upon a mechanism kinematical analysis in case of a pump with axial pistons, inclined block, with and without double cardan couple between the driving shaft and the cylinder block, but without making a difference between the two types of mechanisms regarding the rotating motion transmitting to the cylinder block. These determine the piston displacement by using a geometrical way, taking into account that the pump driving shaft and the cylinder block will be synchronously rotated, but this is true only in case of hydraulic machines having double cardan joint between the shaft and the cylinder block.

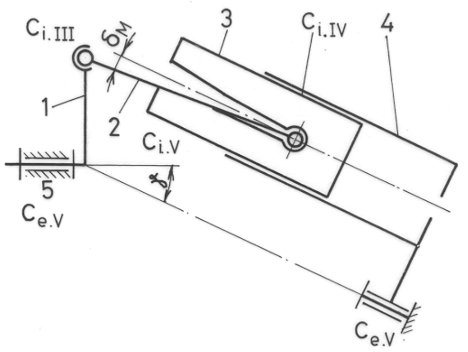
In the work [7] a structural analysis of these machine mechanism are carried out, making evident the functional role of the component elements and defining its kinematical structure.

2. KINEMATICAL ANALYSIS OF THE HYDRAULIC MACHINE MECHANISM WITH MOTION TRANSMITTING BY MEANS OF CONNECTING RODS

By using the considerations presented in the paper [7], in fig. 1.a the structural scheme of the HMMTCR mechanism with a piston, in case where the connecting rod comes in contact with the piston cup, and in fig. 1.b the structural scheme of the same mechanism in case where the connecting rod does not come in contact with the piston cup are shown. Therefore, the mechanism has 5 elements, from which one of them (the carcass noted with 5) is fixed, bounded between them by 5 kinematical couples, two from them being outside. All the kinematical elements of the mechanism are simple, having the rank 2 (because they have two joints with neighbor elements). According with [2], the mechanism is spatially, of family zero ($f = 0$). In this way, the mobility degree is determined by using V. V. Dobrovolski's formula or Somov-Malîşev's formula ([3], [4]):

$$M = 6 \cdot (n-1) - \sum_{k=1}^{5} k \cdot (n_{C.e.k} + n_{C.i.k}), \quad (1)$$

where n is the total number of the kinematical elements in the mechanism structure (inclusively the fixed element); k – the class of the kinematical couple; $n_{C.e.k}$ – the number of the outside kinematical couples of class k ($C_{e.k}$); $n_{C.i.k}$ – the number of the inside kinematical couples of class k ($C_{i.k}$).



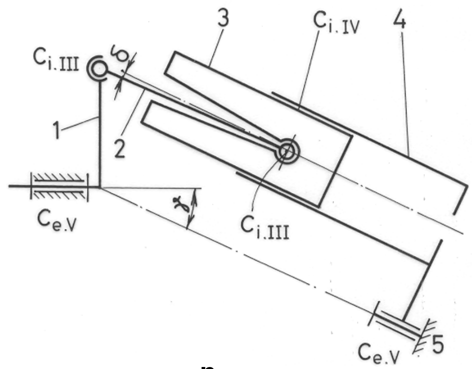


Fig. 1. Structural scheme of the HMMTCR mechanism with a piston: a) in case where the connecting rod comes in contact with the piston cup; b) in case where the connecting rod does not come in contact with piston cup; δ – inclination angle between the connecting rod axis and the piston axis; δ_M – the maximal inclination angle, $\delta < \delta_M$.

Table 1. The mobility degree of the HMMTCR mechanism, having z pistons, taking in consideration the structural schemes in fig. 1.a and 1.b

Crt. nr.	z	Structural schemes for a connecting rod and (z-1) connecting rods	n	$n_{Ce.k}+n_{Ci.k}=n_{C.k}$			M
				k=5	k=4	k=3	
1	1	1(fig. 1.a)	5	2+1=3	0+1=1	0+1=1	2
2	2	1 (fig. 1.a); 1 (fig. 1.b)	7	2+1=3	0+2=2	0+3=3	4
3	3	1 (fig. 1.a); 2 (fig. 1.b)	9	2+1=3	0+3=3	0+5=5	6
4	5	1 (fig. 1.a); 4 (fig. 1.b)	13	2+1=3	0+5=5	0+9=9	10
5	7	1 (fig. 1.a); 6 (fig. 1.b)	17	2+1=3	0+7=7	0+13=13	14
6	9	1 (fig. 1.a); 8 (fig. 1.b)	21	2+1=3	0+9=9	0+17=17	18
7	z	1 (fig. 1.a); z-1 (fig. 1.b)	$2\cdot z+3$	2+1=3	$0+z=z$	$0+(2\cdot z-1)=2\cdot z-1$	$2\cdot z$

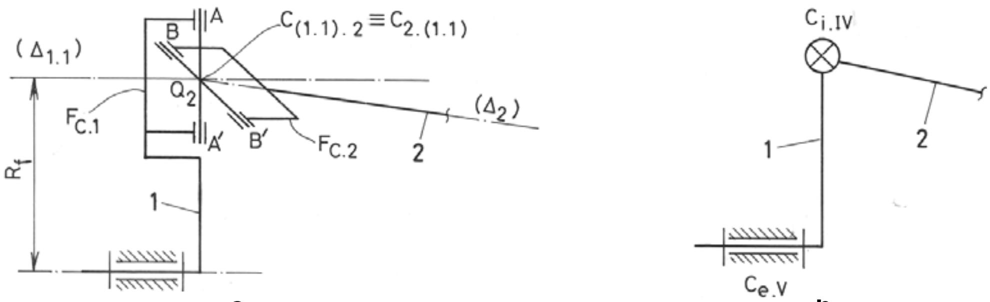
Considering that HMMTCR has z pistons, $z \in \{1, 2, 3, 5, 7, 9\}$, and only a connecting rod from these z is leaning on the piston cup to which is coupled (according to fig. 1.a) and the other z-1 connecting rods make an angle δ with the corresponding piston axes, so that $\delta < \delta_M$ (according to fig. 1.b), the degree of mobility is calculated by means of formula (1), the results being concentrated in table 1.

It is generally ascertained that for a mechanism having z pistons, the mobility degree is $2\cdot z$, the $2\cdot z$ independent motions being: the rotation motion of the driving shaft/flange (1) − of the driving element for the hydrostatic machine of hydrostatic generator (HG) type − involving also the rotating motion of the cylinder block, by means of the driving connecting rod, and $2\cdot z$-1 independent motions of the connecting rods and pistons, from which z rotating motions round the own axes of the pistons coupled with the corresponding non-driving connecting rods. The $2 \cdot z$-1 independent motions are useless motion liberties which have no influence on the desmodromy of the cylinder block.

In order to prove the validity of what have been stated regarding the $2 \cdot z$-1 useless motion liberties, their removal is tried. Firstly, with this end in view, the spherical joint between the connecting rod and the driving flange with a cardan joint or a universal joint is replaced [3]. Therefore, it is considered that between the driving flange (1) and the connecting rod (2) there is a passive joint like the type of cardan joint (see fig. 2), where the driving fork ($F_{c.1}$) is fastened by the driving flange, in the way that the arm axis AA' of this fork to be radially oriented against the flange. The symmetry axis of the fork $F_{c.1}$, noted with ($\Delta_{1.1}$), ($\Delta_{1.1}$)$\perp$AA', is situated at the same distance (R_f) in comparison with the driving shaft/flange axis, in the same way as the center Q_2 of the spherical joint which it replaces. The connecting rod is fastened by the driven fork ($F_{c.2}$). The arm of this fork is noted BB'. It is known that (cf [3]) the arms AA' and BB' are perpendicularly. The connecting rod axis, which is also the symmetry axis of the driven fork ($F_{c.2}$), is noted with (Δ_2); (Δ_2)$\perp$BB'. The axes ($\Delta_{1.1}$) and (Δ_2) are concurrent in a point, $C_{(1.1).2} \equiv C_{2.(1.1)}$, which is even the center of the cardan joint (cross) (C_c), situated in the point Q_2, that is

$$(\Delta_{1.1}) \cap (\Delta_2) \equiv AA' \cap BB' = C_{(1.1).2} \equiv C_{2.(1.1)} \equiv Q_2.$$

The theoretical point of coupling between the connecting rod and the driving flange is considered the cardan cross (Q_2).



a b

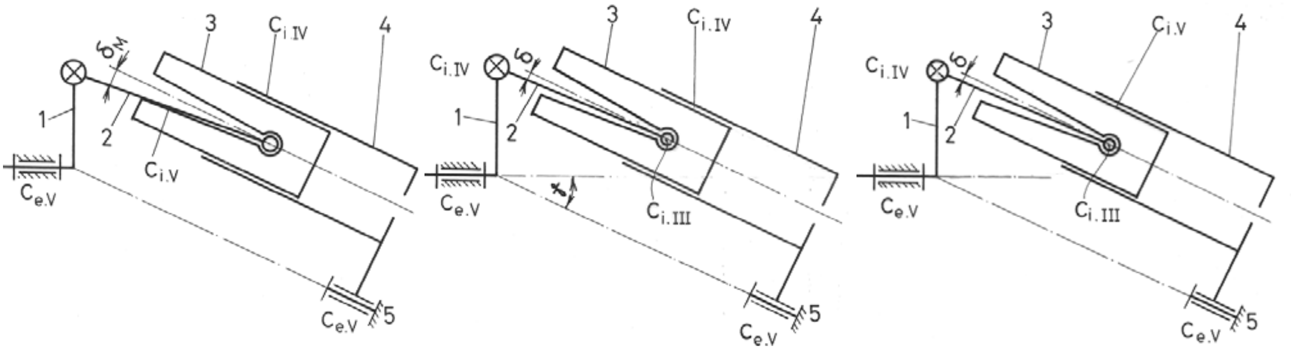
Fig. 2. Schematic (a) and symbolic (b) representation of the cardan joint used as a joint between the connecting rod and the driving flange: $F_{c.1}$ – driving fork; AA' – driving fork arm axis; ($\Delta_{1.1}$) – $F_{c.1}$ fork symmetry axis; $F_{c.2}$ – driven fork; BB' – arm axis of the driven fork; (Δ_2) – $F_{c.2}$ fork symmetry axis; $C_{(1.1).2} \equiv C_{2.(1.1)}$ – point of intersection of the axes ($\Delta_{1.1}$) and (Δ_2); Q_2 – cardan crosshead center (C_c); $Q_2 \equiv C_{(1.1).2} \equiv C_{2.(1.1)}$; R_f – circle radius of disposing of the cardan crosshead centers of the cardan joints between the connecting rods and the driving flange.

The cardan joint is included in the frame of the kinematical couples of IV class (cf [3]), allowing to the connecting rod only two rotating motions (round the arms AA' and BB'), by fastening of the driving fork ($F_{c.1}$) by the flange, in this way suppressing the liberty of motion round the own axis, which was allowed by the spherical joint.

Therefore, replacing the spherical joints between the connecting rods and the driving flange by cardan joints, the figures 3.a (in case where the connecting rod is leading on the piston cup to which it is coupled) and 3.b (in case where the connecting rod does not come in contact with the corresponding piston cup) are obtained.

Calculating the mobility degree by using the relationship (1) in case of a mechanism having z connecting rods, $z \in \{1, 2, 3, 5, 7, 9\}$, coupled by cardan joints with the driving flange, accepting that only one connecting rod is driving from the z ones (according to fig. 3.a) and the others have an inclination angle δ (in comparison with the corresponding piston axes) lower than δ_M (according to fig. 3.b), the results presented in table 2 are obtained.

It is really ascertained that the mobility degree has been reduced to a half by comparison with the previous situation, where the connecting rods have been coupled to the driving flange by means of spherical joints. Therefore, there are now *z* independent motions, where a (rotating) motion belongs to the driving element (to the driving shaft/flange), and the others (*z*-1) represent the rotating motions of the pistons, which are not coupled to the driving connecting rods, and represent useless motion liberties, which have no influence on the desmodromy of the cylinder block.



a **b** **c**

Fig. 3. Structural scheme of the HMMTCR mechanism with a piston: a) in case where the connecting rod is coupled to the driving flange by means of a cardan joint and comes in contact with the piston cup; b) in case where the connecting rod does not come in contact with the piston cup ($\delta < \delta_M$) and the joint piston-cylinder is a cylinder couple; c) in case where the connecting rod does not come in contact with the piston cup and the joint piston-cylinder is a translation couple.

Table 2. The mobility degree of the HMMTCR mechanism with z pistons, taking into consideration the structural schemes in figures 3.a and 3.b

Crt. nr.	*z*	Structural schemes for a connecting rod and (*z*-1) connecting rods	*n*	$n_{Ce.k}+n_{Ci.k}=n_{C.k}$			*M*
				k=5	*k*=4	*k*=3	
1	1	1(fig. 3.a)	5	2+1=3	0+2=2	0	1
2	2	1 (fig. 3.a); 1 (fig. 3.b)	7	2+1=3	0+4=4	0+1=1	2
3	3	1 (fig. 3.a); 2 (fig. 3.b)	9	2+1=3	0+6=6	0+2=2	3
4	5	1 (fig. 3.a); 4 (fig. 3.b)	13	2+1=3	0+10=10	0+4=4	5
5	7	1 (fig. 3.a); 6 (fig. 3.b)	17	2+1=3	0+14=14	0+6=6	7
6	9	1 (fig. 3.a); 8 (fig. 3.b)	21	2+1=3	0+18=18	0+8=8	9
7	*z*	1 (fig. 3.a); *z*-1 (fig. 3.b)	$2\cdot z+3$	2+1=3	$0+2\cdot z=2\cdot z$	0+(*z*-1)=*z*-1	*z*

In order to show that these *z*-1 useless motion liberties are really the rotating motions of the accurately stated pistons, it is tried their removal by replacing the cylindrical couples piston-cylinder, in case of those pistons on which there is no leaning the respective connecting rods, by translation couples – couples belonging to the V class, and to the type 2 ([4], [8]) (see fig. 3.c) –, maintaining the cylindrical couple piston-cylinder for the piston which is coupled to the driving connecting rod. In this situation the figures 3.a and 3.c are valid, and the results regarding the mobility degree calculation are presented in table 3.

The mobility degree being 1, without taking into consideration the number of the pistons (z), it is ascertained that the *z*-1 couples piston-cylinder, for the pistons coupled with the connecting rods, which are not driving connecting rods, run really as some translation couples. Only the couple piston-cylinder, whose piston is coupled with the driving connecting rod, needs also a rotating motion round its own axis, excepting the main motion, that of translation. Therefore, taking into consideration the joints between the kinematical elements of type mentioned in the structural schemes presented in fig. 3.a (for an ensemble made up from connecting rod-piston-cylinder, where the connecting rod is a driving one) and fig. 3.c (for the others, that is *z*-1 of the ensembles connecting rod-piston-cylinder, where the connecting rod is not a driven one), the HMMTCR mechanism with z pistons is desmodrome, the single motion being the rotating of the driving shaft/flange, which gives the rotating motion of the cylinder block by means of the driving connecting rod.

Table 3. The mobility degree of the HMMTCR mechanism with z pistons, considering the structural schemes in figures 3.a and 3.c

Crt. nr.	z	Structural schemes for a connecting rod and (z-1) connecting rods	n	$n_{Ce.k}+n_{Ci.k}=n_{C.k}$			M
				k=5	k=4	k=3	
1	1	1 (fig. 3.a)	5	2+1=3	0+2=2	0	1
2	2	1 (fig. 3.a); 1 (fig. 3.c)	7	2+2=4	0+3=3	0+1=1	1
3	3	1 (fig. 3.a); 2 (fig. 3.c)	9	2+3=5	0+4=4	0+2=2	1
4	5	1 (fig. 3.a); 4 (fig. 3.c)	13	2+5=7	0+6=6	0+4=4	1
5	7	1 (fig. 3.a); 6 (fig. 3.c)	17	2+7=9	0+8=8	0+6=6	1
6	9	1 (fig. 3.a); 8 (fig. 3.c)	21	2+9=11	0+10=10	0+8=8	1
7	z	1 (fig. 3.a); z-1 (fig. 3.c)	$2\cdot z+3$	$2+z$	$0+(z+1)=z+1$	$0+(z-1)=z-1$	1

Of course, in case when at least two connecting rods came in contact with the corresponding piston cups, then the mechanism would be blocked up. Therefore, cannot be than a single driving connecting rod, by its contact with the inside wall of the piston to which it is coupled.

On the basis of the equivalent kinematical schemes in the figures 3.a and 3.c, the geometry of the HMMTCR mechanism may be studied, and then the variation laws of the kinematical sizes of the piston and cylinder block in the frame of an accurate theory of kinematics (cf [6]) may be determined.

In this way, the piston displacement expression depending on the angle of inclination of the connecting rod in comparison with the cylinder axis (δ) (according to [6]) is obtained as it follows:

$$s = R_f \cdot \sin\gamma \cdot \left(\cos\varphi_{1.l} - \cos\varphi_1\right) + l \cdot \left(\cos\delta - \cos\delta'_l\right), \qquad (2)$$

where R_f is the circle radius of disposing of the connecting rod joints with the driving flange; l – the connecting rod length; γ – the angle of inclination of the cylinder block in comparison with the driving shaft; φ_1 – the angle of rotation of the driving shaft/flange; $\varphi_{1.l}$ – the angle of rotation of the driving shaft/flange when the piston is in the lower dead point (LDP); δ'_l – the angle of inclination of the connecting rod in comparison with the cylinder axis when the piston is at LDP, and

$$\cos\delta = \frac{1}{l} \cdot \sqrt{l^2 - \left\{\left[R_c \cdot \cos(\varphi_1 - \Delta\varphi) - R_f \cdot \cos\gamma \cdot \cos\varphi_1\right]^2 + \left[R_c \cdot \sin(\varphi_1 - \Delta\varphi) - R_f \cdot \sin\varphi_1\right]^2\right\}}, \qquad (3)$$

where R_c is the the circle radius of disposing of the cylinder centers being in the cylinder block; $\Delta\varphi$ – the lagging angle between the driving flange and the cylinder block.

The angle of rotation of the cylinder block (φ_4) is expressed (cf [6]) by the following relationship:

$$\varphi_4 = \varphi_1 - \Delta\varphi + \left(\Delta\varphi'_l - \varphi_{1.l}\right), \qquad (4)$$

where $\Delta\varphi'_l$ is the initial lagging angle between the driving flange and the cylinder block.

The angular velocity of the cylinder block (ω_4) is obtained (cf [6]) in dependence with the angular velocity of the driving shaft/flange in the shape of the expression:

$$\omega_4 = \omega_1 \cdot \left(1 - \frac{d\Delta\varphi}{d\varphi_1}\right), \qquad (5)$$

which makes evidently the non-uniformity of the cylinder block rotation motion. In [6], the variation law $\Delta\varphi = f(\varphi_1)$ depending on the tipple angle of the cylinder block γ is presented.

3. CONCLUSIONS

In the work a kinematical analysis of the HMMTCR mechanism taking into account its peculiarity, namely the driving of the cylinder block in rotating motion by means of the connecting rods, is presented.

The HMMTCR mechanism is a spatial complex mechanism having *z* pentajoints, *z* being the number of axial pistons (cf [7]).

On the basis of the structural analysis made in the work [7], kinematical schemes, equivalent to the constructive scheme, by identification of the useless motion liberties, are realized, and the mobility degree is calculated, showing the mechanism desmodromy.

It is shown that, at a time, cannot be than a single driving connecting rod, by its contact with the inside wall of the piston to which it is coupled.

The kinematical analysis carried out in this work allows the study of the HMMTCR mechanism geometry, and then determination of the variation laws in case of kinematical sizes of the piston and cylinder block in the frame of an accurate theory of kinematics (cf [6]). In this way, the variation laws of the piston displacement, of the angle of inclination of the connecting rod in comparison with the cylinder axis, of the rotation angle, and of the angular velocity in case of cylinder block are presented, making evident the rotating motion non-uniformity of the cylinder block.

In the frame of a next work, the geometrical and kinematical features of the HMMTCR mechanism will be developed, taking into account its peculiarity regarding the way of transmitting of the rotating motion from the driving flange to the cylinder block by means of connecting rods, making evident the (kinematical, hydraulic and dynamic) running effects, and also the effect on the distributing process development, which are specifically to this way.

BIBLIOGRAPHY

1. Bloch, *Theoretische und experimentale Untersuchungen an einem Flüssigkeitsgetriebe.* Dissertation ETH Zürig, 1953.
2. Handra-Luca, V., Stoica, I. A., *Introducere în teoria mecanismelor. Vol. I.* Editura Dacia, Cluj-Napoca, 1982.
3. Mangeron, D., Irimiciuc, N., *Mecanica rigidelor cu aplicații în inginerie. Vol. II. Mecanica sistemelor de rigide.* Editura tehnică, Bucureşti, 1980.
4. Manolescu, N., Kovacs, Fr., Orănescu, A., *Teoria mecanismelor şi a maşinilor.* Editura didactică şi pedagogică, Bucureşti, 1972.
5. Parepa, S., *About the Kinematics of the Axial Piston Hydraulic Machines with Motion Transmitting by Connecting Rods*. Petroleum-Gas University of Ploieşti BULLETIN, Technical Series, Vol. LIX, No. 2/2007, p. 43-50.
6. Parepa, S., *Study related to Rotation System of Large Hole Drilling Rigs.* Doctorate thesis. University of Petroşani, 2007.
7. Parepa, S., *Structural Analysis of the Mechanism in Case of Hydraulic Machines with Axial Pistons and with Rotating Motion Transmitting by Connecting Rods.* ANNALS of the ORADEA UNIVERSITY. FASCICLE of MANAGEMENT and TECHNOLOGICAL ENGINEERING. Editura Universității din Oradea. CD-ROM EDITION, Volume VII (XVII), 2008.
8. ⁂*Manualul inginerului mecanic. Vol. III.* Editura tehnică, Bucureşti, 1976.