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THE STUDY OF A VIBRATING PISTON MOUNTED IN A RIGID BAFFLE

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Abstract: It is quite clear that the sound radiation from the piston is quite directional (except along the axis) and that it increases with frequency. There are several pressure nodes, and these results in a beam pattern of sound radiation.

The sound radiation from a vibrating piston mounted in an infinite baffle is a classical problem — one which is covered in numerous books on fundamental acoustics. The vibrating piston can be either a vibrating surface or a vibrating layer of air. The primary assumption in the analysis (one which is not strictly correct in practice for real surfaces) is that all parts of the piston vibrate in phase and with the same amplitude. Its relevance to engineering noise control is that it serves as an introduction to the sound radiation from different types of surfaces, e.g. loudspeakers; open ends of flanged pipes, plates and shells etc.

Consider a flat, circular piston of radius z which is mounted in an infinite, rigid baffle as illustrated in Figure 1. The noise radiated by the vibrating piston can be modelled in terms of numerous point monopoles (monopoles where ka <1) radiating together. Each of the monopoles is, however, radiating from a rigid, reflecting, ground plane and not from free space. The sound pressure due to any one of the baffled monopoles is therefore twice that of an equivalent monopole in free space.

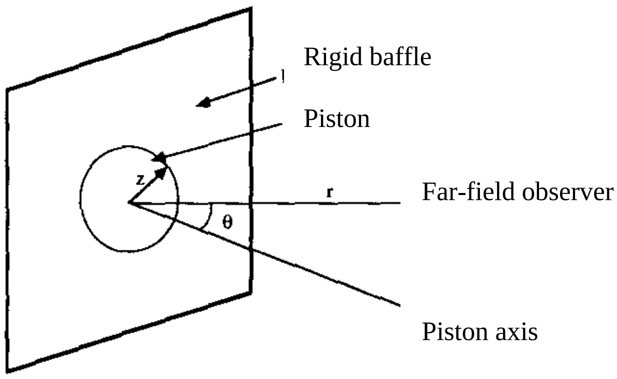

Fig. 1. Piston mounted in a rigid baffle.

It is:

$$P(r, t) = \frac{ik\rho_0 c}{2\pi r} Q_p e^{i(\omega t - kr)} \quad (1)$$

In this equation, Q represents the source strength of the elemental monopole on the piston surface and it is equal to $U_p\ \delta S$, where U_p is the peak surface velocity of the monopole and δS is the elemental surface area. The total acoustic pressure fluctuations due to the vibrating piston is simply the resultant pressure due to all the point monopoles vibrating in phase and it is obtained by integration over the whole surface area. It is:

$$p(r,\ \theta,\ t) = \frac{ik\rho_0 c\pi z^2 U_p e^{i(\omega t - kr)}}{2\pi r}\left[\frac{2J_1(kz\sin\theta)}{kz\sin\theta}\right] \quad (2)$$

In the above equation, $U_p\ e^{i\omega t}$ is the surface velocity of the piston (each of the monopoles has the same surface velocity and phase). The radiated sound pressure has a similar form to that of a monopole in a reflecting ground plane with the exception of the term in brackets which is a directivity factor. J is the first order Bessel function and it can be readily evaluated from tables.
The corresponding sound intensity in the far-field is:

$$I(r,\ \theta) = \frac{\rho_0 c k^2 U_{rms}^2 \pi^2 z^2}{4\pi^2 r^2}\left[\frac{2J_1(kz\sin\theta)}{kz\sin\theta}\right]^2 \quad (3)$$

Once again, the sound intensity has a similar form to that of a monopole in a reflecting ground plane with the exception of the term in brackets which is a directivity factor. The form of the directivity factor is presented in Figure 2.

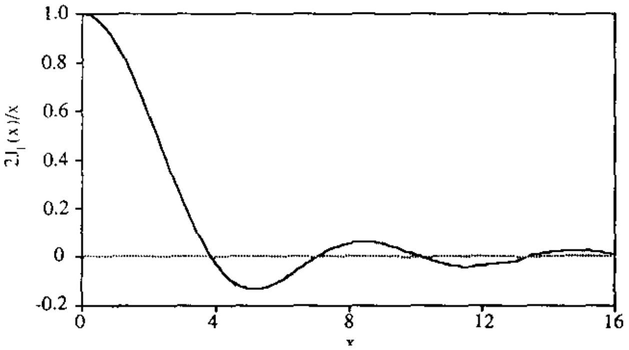

Fig- 2. Functional form of the directivity factor for a circular piston in a rigid baffle.

It is quite clear that the sound radiation from the piston is quite directional (except along the axis) and that it increases with frequency. There are several pressure nodes, and these results in a beam pattern of sound radiation. This is schematically illustrated in Figure 3. At low frequencies (kz<1) the intensity distribution is approximately constant, whereas at high frequencies there are several nodal points and corresponding lobes of radiated sound. Hence, low frequency loudspeakers can be large and still remain omni-directional whereas high frequency loudspeakers need to be small and to be relatively omni-directional. This example illustrates how a series of omni-directional sound sources can become directional when combined.

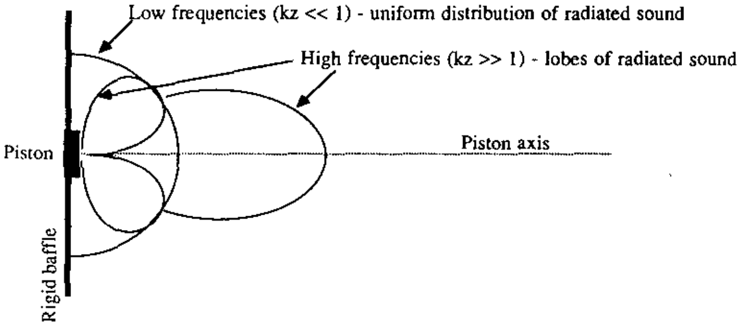

Fig. 3. Low and high frequency sound radiation patterns for a circular piston in a rigid baffle.

The preceding discussion has been restricted to the acoustic far-field. Now consider an observation point on the piston surface itself. In acoustics, when a structure radiates sound due to its vibration, another impedance term has to be included with the mechanical impedance. It is the radiation impedance of the fluid (air) in proximity to the vibrating surface — i.e. the fluid loads the vibrating surface and this alters its vibrational response. The total sound pressure at any arbitrary element on the piston surface is a sum of the pressure due to the vibrating element itself and the radiated pressures from all the other elements on the piston. The piston velocity, $U = U_p e^{i\omega t}$ is thus given by:

$$U = \frac{F_m}{Z_m + Z_r} \quad (4)$$

where F_m is the applied mechanical force, Z_m, is the mechanical impedance of the piston, and Z_r is its radiation impedance. F_m is not to be confused with the force on the piston due to the acoustic pressure fluctuations, $F_{p.}$ The mechanical impedance is associated with the mechanical driving force and the radiation impedance is associated with the acoustic driving force. It can be seen from equation (4) that the radiation impedance 'fluid-loads' the surface vibrations of the piston. Fluid loading concepts are very relevant when analysing the vibrational characteristics of structures immersed in fluids — e.g. piping systems filled with liquids, submarines etc. The radiation impedance is thus given by:

$$Z_r = \frac{F_p}{U} \quad (5)$$

The radiation impedance of a vibrating surface is sometimes defined as the ratio of the sound pressure averaged over the surface to the volume velocity through it (units of N s m^{-5} as opposed to N s m^{-1} for mechanical impedance) — i.e. some books would define Z_r as $(F_p/\pi z^2)/\ \pi z^2\ U$ or $F_p/(\pi z^2)^2 U$. In this paper, Z_r is defined, for convenience, in similar units to the mechanical impedance since the radiating surface area, πz^2 is common to both variables (pressure and volume velocity).

The radiation impedance of a piston can be obtained by integrating the elemental pressure distribution over the surface area of the piston to obtain the total sound pressure at a point and subsequently integrating this again over the surface to obtain the force, F_p The radiation impedance is thus obtained and given by:

$$Z_r = \rho_0 c \pi z^2 \{R_1(2kz) + iX(2kz)\} \quad (6)$$

where:

$$R_1(x) = \frac{x^2}{2 * 4} - \frac{x^4}{2 * 4^2 * 6} + \frac{x^6}{2 * 4_2 {*} 6^2 * 8} - \ldots \quad (7)$$

and

$$X_1(x) = \frac{4}{\pi}\left\{\frac{x}{3} - \frac{x^3}{3^2 * 5} + \frac{x^5}{3^{2*}5^2 * 7} - \ldots\right\} \quad (8)$$

The resistive function, R and the reactive function, X are plotted in Figure 4. The resistive part is real and is due to the radiated sound pressure. The imaginary part is a mass loading term due to the fluid (air or liquid) in proximity to the piston.

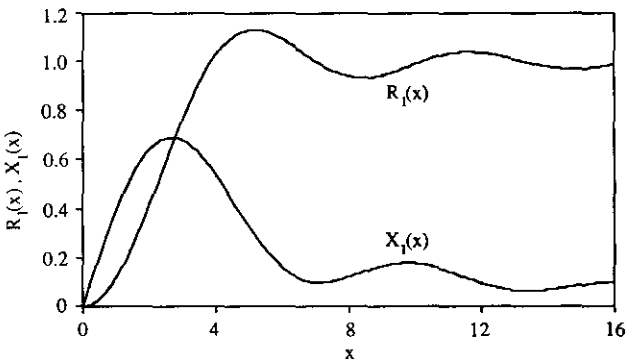

Fig. 4. Resistive and reactive functions for the radiation impedance of a circular piston.

This mass loading term can become significant when the structure is radiating into liquids.

The mechanical properties of the piston also affect its sound radiation properties. The equation of motion of the piston is (from Newton's second law)

$$M\frac{\delta U}{\delta t} + C_V U + K_s \int U dt - F_m - F_p \quad (9)$$

where M is the piston mass, C is its damping, and K, is its stiffness. Thus equation (4) can be re-written as:

$$U = \frac{F_m}{C_V + i(M\omega - k_s / \omega) + \rho_0 c \pi z^2 \{R_1(2kz) + iX(2kz)\}} \quad (10)$$

Equation (11) clearly illustrates how the vibrational velocity of the piston is a function of (i) its structural damping, (ii) its mass, (iii) its stiffness, (iv) the acoustic radiation resistance, and (v) the acoustic radiation reactance. As mentioned earlier, the acoustic radiation resistance is due to the radiated sound pressure and the acoustic radiation reactance is due to mass loading of the piston by the fluid. For low frequencies, 2kz<< I, thus

$$R_1(2kz) \approx \frac{k^2 z^2}{2} \quad (11)$$

and

$$X_1(2kz) \approx \frac{8kz}{3\pi} \quad (12)$$

For high frequencies, 2k <<1, thus

$$R_1(2kz) \approx 1 \quad (13)$$

and

$$X_1(2kz) \approx \frac{2}{\pi kz} \quad (14)$$

The effects of mass loading can now be estimated. The amplitude of the fluid loaded mass is:

$$M_r = \frac{X_r}{\omega} = \frac{\rho_0 c \pi z^2 X_1(2kz)}{\omega} = \frac{\rho_0 c \pi z^2 X_1(2kz)}{k} \quad (15)$$

Thus at low frequencies (2k << 1)

$$M_r = \frac{8\rho_0 z^3}{3} \quad (16)$$

and at high frequencies (2kz>> 1)

$$M_r = \frac{2\rho_0 z}{k^2} \quad (17)$$

The sound power radiated by the piston can now be estimated from the real part of the radiation impedance — i.e. the acoustic radiation resistance. It could also be obtained by integrating the far-field sound intensity (equation 3). From equation the real power (rate of energy flow) of the piston is:

$$\Pi = \frac{1}{2} U_p^2 \, Re[Z_m + Z_r] \quad (18)$$

Thus,

$$\Pi = \frac{1}{2} U_p^2 \{C_v + \rho_0 c \pi z^2 R_1(2kz) \quad (19)$$

where the first term inside the brackets represents the mechanical power that is dissipated and the second term represents the sound power that is radiated into the surrounding medium. Low and high frequency estimates of the radiated sound power can be readily obtained by substituting equations (11) and (13) into equation (19).

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