ANNALS of the ORADEA UNIVERSITY.

Fascicle of Management and Technological Engineering, Volume VII (XVII), 2008

AN ANALYSIS OF THE STRESS INTENSITY FACTOR MODE II VARIATION UNDER INFLUENCES OF RESIDUAL TENSIONS AND THE POSITION OF THE CRACK CENTRE FOR AN INTERNAL CRACK SITUATED IN THE HERTZIAN STRESSES FIELD OF GEAR TEETH

Claudiu Ovidiu POPA, Lucian Mircea TUDOSE
Technical University of Cluj – Napoca

Keywords: fatigue cracks, stress intensity factor K_{II}, crack centre depth, residual tensions

Abstract: The parameter that governs the crack fatigue growth in the case of compression stresses field is the stress intensity factor mode II, K_{II}. The main goal of this paper is to present the K_{II} variation with respect to the contact stresses, the residual tensions and the crack centre depth of an existent internal crack in the sub-surface of the pinion tooth in the Hertzian stresses field with friction, for digitized (with the step π/6) values of the inclination angle *α* of the crack. As result of this study, some particular factors favorable to the future development of cracks towards the surface were identified.

1. INTRODUCTION

Fracture failure of engineering structures is caused by cracks that extend beyond a safe size. It is a catastrophic event that takes place very rapidly and is preceded by crack growth which develops slowly during normal service conditions, mainly by fatigue due to cyclic loading.

Since fatigue involves failure of the “weakest link” of the material, the fatigue strength is very closely coupled with the microstructural integrity and purity of the material. For example, larger inclusions or voids can be regarded as initiated cracks (at the surface or in the substrate) and the reduced fatigue life of the component has to be evaluated by the use of a crack propagation model.

The first stage of the crack growth [8][10][14] corresponds to *nucleation* and, then, to a growth on *a small scale* with a growth rate in order of 10^{-6} mm/cycle. It depends on the material microstructure, the applied stress ratio and the environment.

For many loading conditions, the highest loads are at the surface. But even the nominal stress is constant throughout, crack tend to nucleate at the surface because deformation of each grain is allowed to concentrate on a crystallographic plan [7].

The second stage of fatigue crack propagation was named [1][2] the *stable crack growth*. Here, in concordance with the Linear Elastic Fracture Mechanics (LEFM) outlook, the linear elastic crack growth is modeled using the Paris law representation of a surface crack in a semi infinite body subjected to a constant stress cycle.

Stress intensity factor (SIF) is one the most important parameter that governs stage II. It reflects the whole stress field at the tip of the crack when the size of the plastic zone at the crack tip is small compared to the crack length.

We considered two mashing spur gears, in which any point of the material is subjected to normal and shear stresses due to Hertzian and frictional stresses which act on the contact zone. Here, the normal tensions (if no residual tensions exist) are of compression. This case corresponds to a crack growth under mode II, also named sliding mode. According to the Paris Law, the parameter that governs the stable crack growth in this case is *stress intensity factor mode II,* K_{II}.

Its values are strongly depending, apart to the mentioned tensions, with the values of residual tensions, the size of the initiated crack and the crack centre coordinates.

The last stage (instability) corresponds to a dramatic growing of the crack that has as result, eventually, the fracture of the component.

The main goal of this paper is to present the K_{II} variation with respect to the contact stresses, the residual tensions and the crack centre depth of an existent internal crack in the sub-surface of the pinion tooth in the Hertzian stresses field with friction, for digitized (with the step π/6) values of the inclination angle α of the crack.

2. CRACK GROWTH IN COMPRESSION CONDITIONS

A crack in a solid can be stressed in three different modes. Normal stresses give rise to the "opening mode" denoted as Mode I. The displacements of the crack surfaces are perpendicular to the plane of the crack. This is the case that corresponds to the tensile stresses that act on the crack faces and it has as result positive values of SIF K_I.

In-plane shear results in Mode II or "sliding-mode": the displacement of the crack surfaces is in the plane of the crack and perpendicular to the leading edge of the crack.

The "tearing mode" or Mode III is caused by out-of-plane shear. Crack surface displacements are in the plane of the crack and parallel to the leading edge of the crack [1]

From the traditional-theory point of view if the crack faces are compressed, the crack growth is obstructed, fact that contradicts the experimental observations. Pitting is one of the most important types of gear deterioration and appears even if the normal tensions are of compression.

Various suggestions have been published to solve this contradiction. Few of them consider that the defects and discontinuities in the material structure are responsible for this behavior [9].

Others [4] consider that the fracture occurs due to the loss of the local stability near the crack or [6] the roughness of the crack surfaces is the cause of fracture in the compression direction.

All these suggest [3][5][11] that the crack propagate in sliding mode driven by the cyclic stress caused by repeated rolling contact. In such case, the nature of the fracture is columnar, and separation of the solid occurs into vertical columns produced by the crack growth in the direction of the uniaxial compression [4][6][9].

In order to estimate the crack growth in compression conditions, a law similar to Paris law crack growth was proposed [12]:

$$da/dN = C \cdot \Delta K_{II}{}^{n} \tag{1}$$

where: a – crack length, mm;

N – number of loading cycles;

da/dN – crack growth per cycle, mm/cycle;

ΔK_{II} – stress intensity factor (Mode II) range, MPa·(mm)$^{1/2}$;

n – Paris law exponent;

C – Paris Law coefficient, (mm/cycle)/[MPa·(mm)^{½}]^{n};

3. STRESS FIELD AND SIF MODE II EVALUATION IN THE PINION TOOTH SUB-SURFACE

As was presented in [13] two mashing spur gears made of OLC45 (AISI 1045) with an internal inclined crack in the pinion tooth were considered (Fig.1).

Any point of material is subjected to normal and shear stresses due to Hertzian and frictional stresses.

The position of the current contact point is determined by the abscissa $X_{contact}$.

In the current contact point a local system xy is also anchored (Fig. 1) [13].

On contact zone acts simultaneously a normal force and a frictional force.

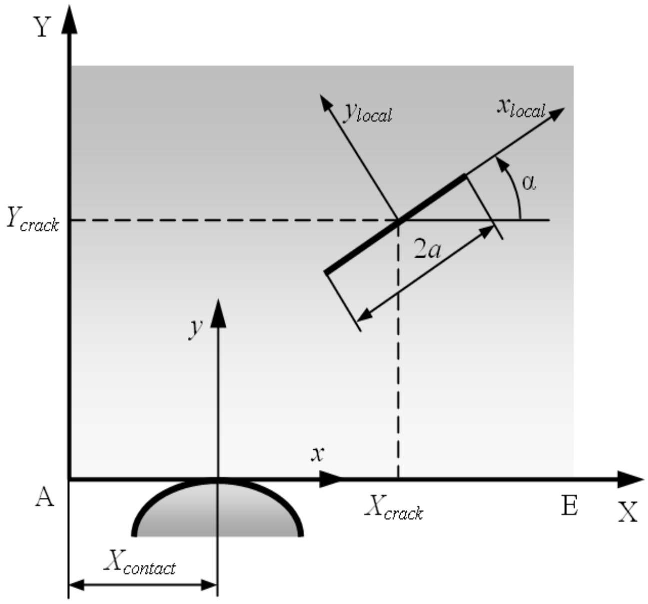


Fig.* 1: *Coordinate systems on contact zone

The position of the crack is given by the coordinates X_{crack} and Y_{crack} of its centre in the system AXY. Affined to crack, a local coordinate system $x_{local}y_{local}$ is defined. This system is originated in the centre of the crack.

The length of the crack is 2*a* and we presume that both faces are loaded by normal and tangential stresses of arbitrary intensity, as shown in Fig. 2.

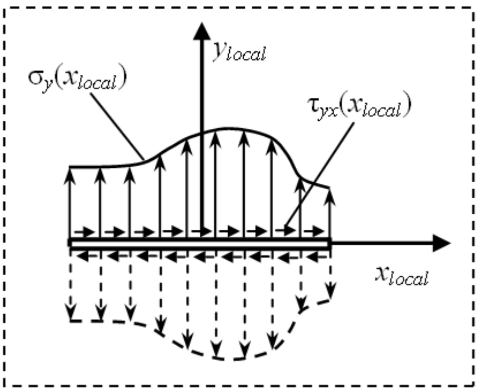


Fig.* 2: *General loading on internal crack surfaces

The most general case presume that the crack is inclined with angle α with respect of axis AX (angle between the *X* and x_{local} directions, measured counterclockwise) as shown in Fig. 1.

The stress intensity factor mode II can be determined by expression [13]:

$$K_{II} = \frac{1}{\sqrt{\pi \cdot a}} \cdot \int_{-a}^{a} \tau_{yxlocal} \cdot \sqrt{\frac{a + x_{local}}{a - x_{local}}} dx_{local} \quad (2)$$

In equation 2 the shear stresses that load both sides of the crack (Fig. 2) according to rotated system $x_{local}y_{local}$ is [13]:

$$\tau_{yxlocal} = \frac{-\sin(2\alpha)}{2}\sigma_x + \frac{\sin(2\alpha)}{2}\sigma_y + \cos(2\alpha)\tau_{yx} \quad (3)$$

where the effective stresses in any point of the active surface or of its sub-surface of the pinion tooth with taking into account the residual stresses are [13]:

$$\sigma_x = \sigma_{xH} + \sigma_{xf} + q_{xrez}$$
$$\sigma_y = \sigma_{yH} + \sigma_{yf} \qquad (4)$$
$$\tau_{yx} = \tau_{yxH} + \tau_{yxf}$$

The expressions of the normal and shear stresses due to the Hertzian load σ_{xH}, σ_{yH} and τ_{xyH} respectively, were exhaustive presented in [13]. Also here we can find the expression for the stresses due to frictional forces σ_{xf}, σ_{yf} and τ_{yxf} (if the tangential force on the contact surface is proportional to the normal acting force).

All these stresses are functions of $X_{contact}$, X_{crack}, Y_{crack}, x_{local} and inclination angle α.

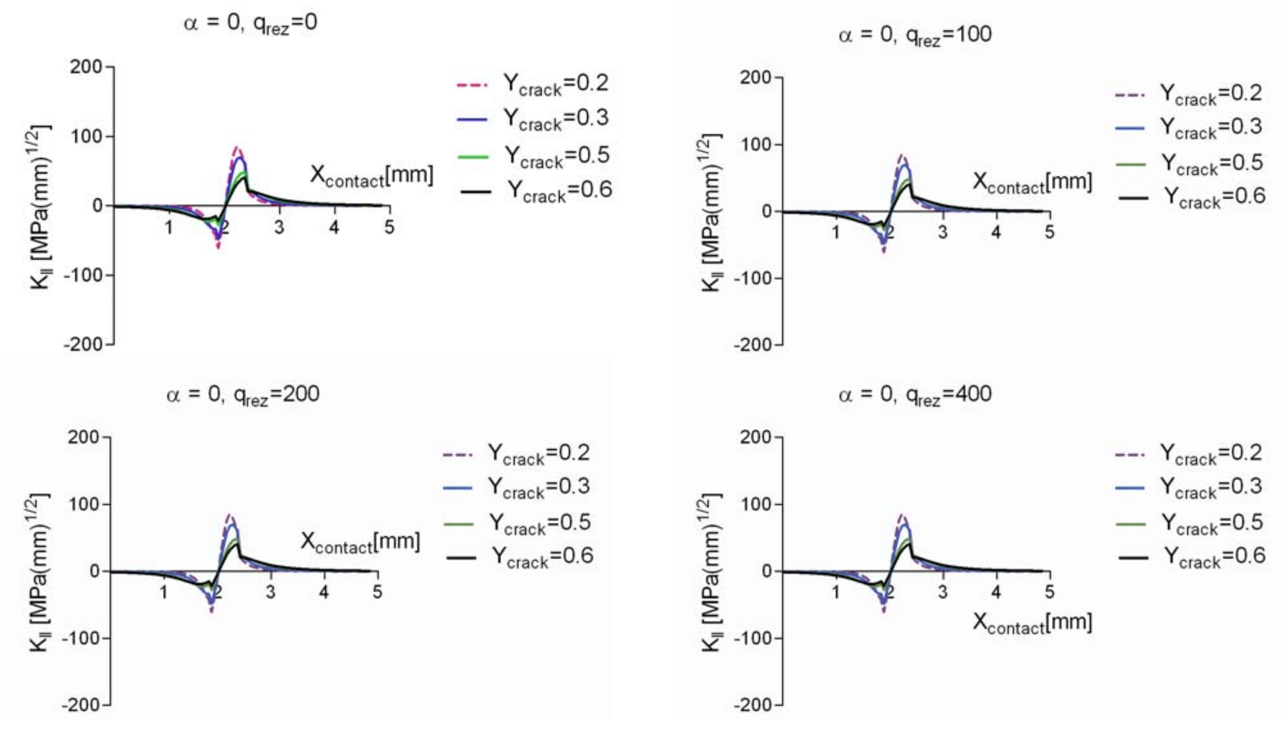
Using these equations one can determine the values of normal and shear stresses in every point belonging to the crack (in crack coordinate system), for any crack position inside the material (X_{crack}, Y_{crack}), for any length (2a) and inclination angle (α), and for any position of contact point ($X_{contact}$).

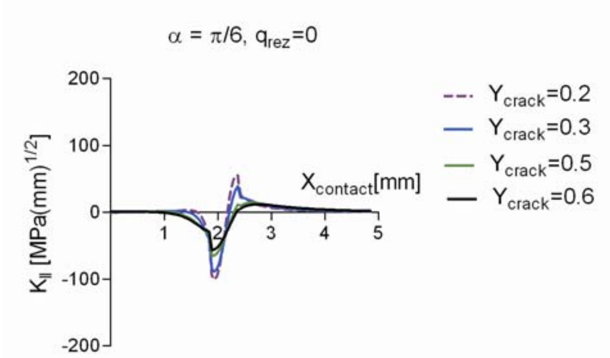
4. INFLUENCES OF CRACK CENTRE DEPTH AND RESIDUAL TENSIONS ON K_{II} VALUES

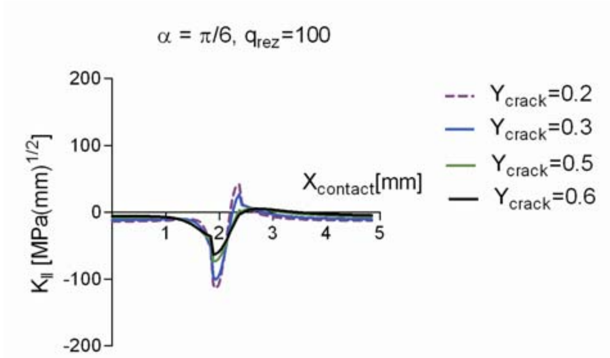
In order to study the stress intensity factor mode II variation, K_{II}, we considered an internal crack situated at the point having X_{crack} = 2.0 mm and the semilength a = 0.05 mm.

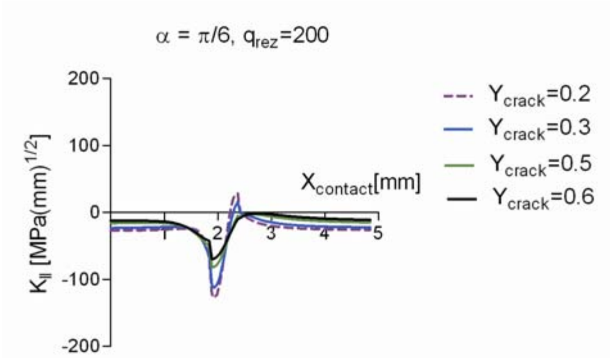
We also have considered that the centre depth is variable (Y_{crack}= 0.2, 0.3, 0.5 and 0.6 mm) and that the crack is subjected to the Hertzian and frictional stresses (determined for μ_f = 0.1) with taking account the variable residual tensions (q_{rez} = 0, 100, 200, 400 MPa). The input gearing data are the same with those established in [13].

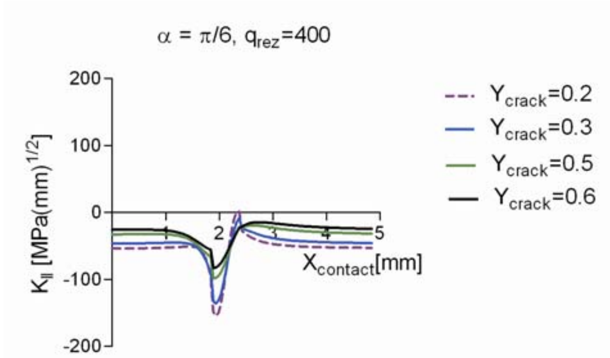
Based on equations 2 and 3 we compute the values of SIFs K_{II} for digitized (with the step π/6) values of the inclination angle α of the crack (Fig. 3).

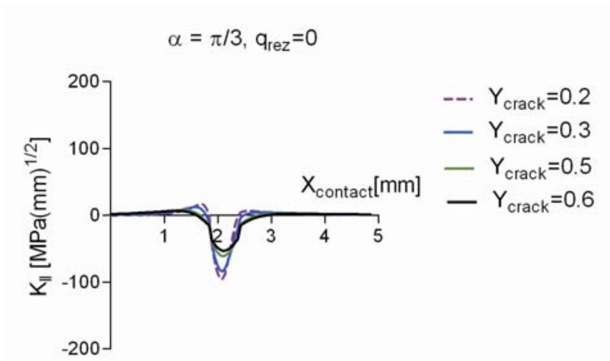


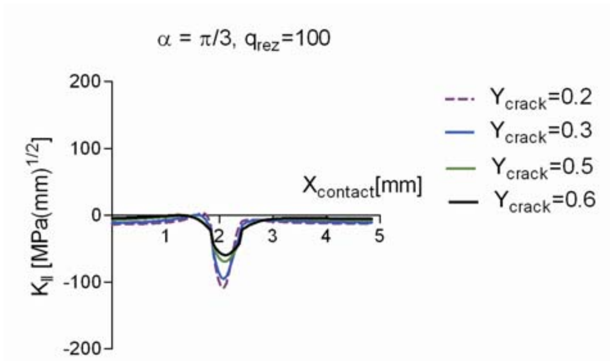

α = π/6, q_rez=0
K_II [MPa(mm)^1/2]
X_contact[mm]
Y_crack=0.2
Y_crack=0.3
Y_crack=0.5
Y_crack=0.6

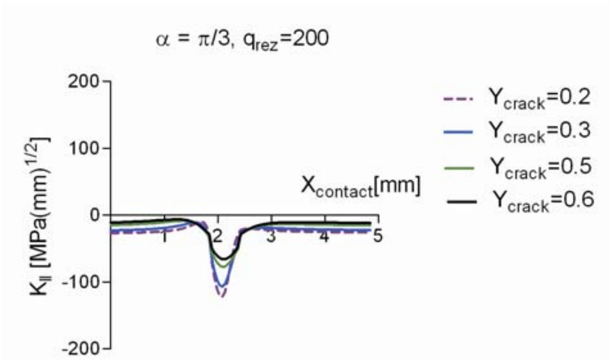

α = π/6, q_rez=100
K_II [MPa(mm)^1/2]
X_contact[mm]
Y_crack=0.2
Y_crack=0.3
Y_crack=0.5
Y_crack=0.6

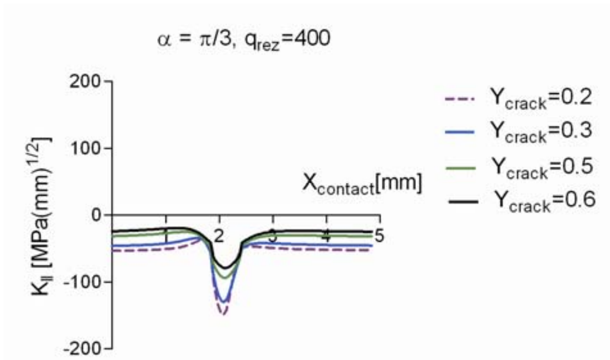

α = π/6, q_rez=200
K_II [MPa(mm)^1/2]
X_contact[mm]
Y_crack=0.2
Y_crack=0.3
Y_crack=0.5
Y_crack=0.6

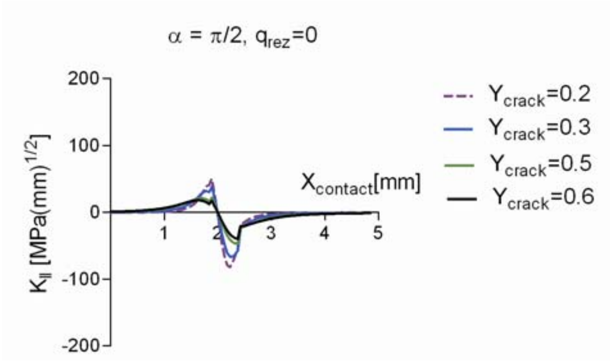

α = π/6, q_rez=400
K_II [MPa(mm)^1/2]
X_contact[mm]
Y_crack=0.2
Y_crack=0.3
Y_crack=0.5
Y_crack=0.6

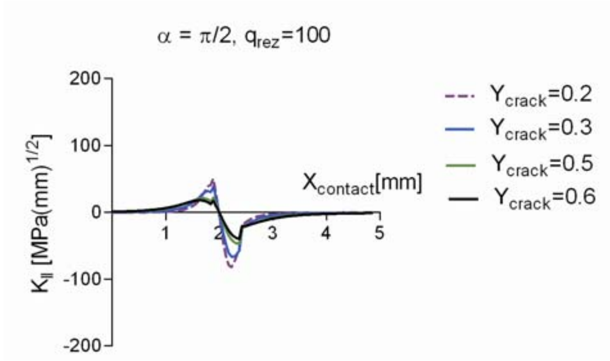

α = π/3, q_rez=0
K_II [MPa(mm)^1/2]
X_contact[mm]
Y_crack=0.2
Y_crack=0.3
Y_crack=0.5
Y_crack=0.6

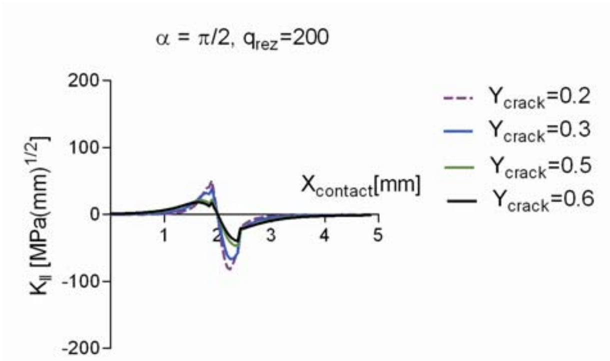

α = π/3, q_rez=100
K_II [MPa(mm)^1/2]
X_contact[mm]
Y_crack=0.2
Y_crack=0.3
Y_crack=0.5
Y_crack=0.6

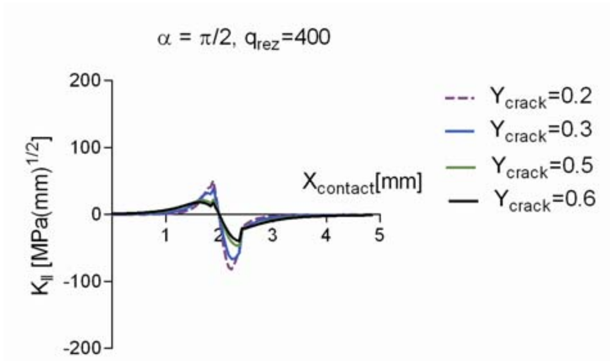

α = π/3, q_rez=200
K_II [MPa(mm)^1/2]
X_contact[mm]
Y_crack=0.2
Y_crack=0.3
Y_crack=0.5
Y_crack=0.6

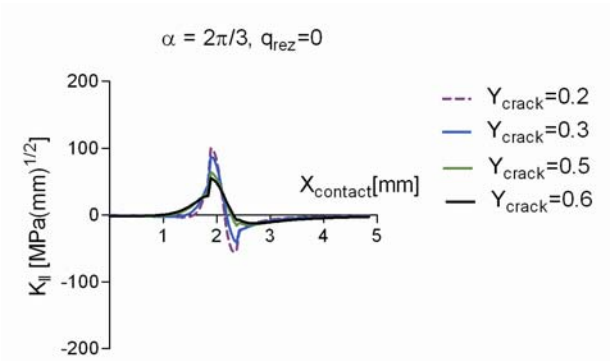

α = π/3, q_rez=400
K_II [MPa(mm)^1/2]
X_contact[mm]
Y_crack=0.2
Y_crack=0.3
Y_crack=0.5
Y_crack=0.6

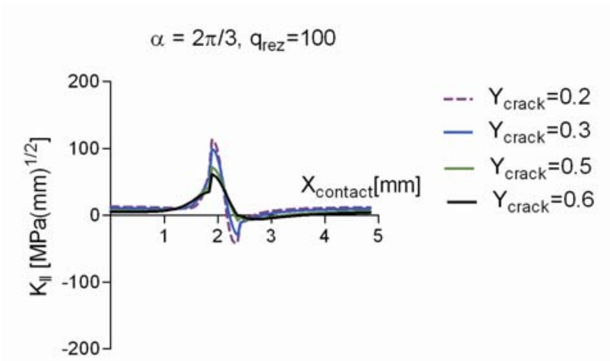

$\alpha = \pi/2$, $q_{rez}=0$
K_{II} [MPa(mm)$^{1/2}$]
$X_{contact}$[mm]
$Y_{crack}=0.2$
$Y_{crack}=0.3$
$Y_{crack}=0.5$
$Y_{crack}=0.6$

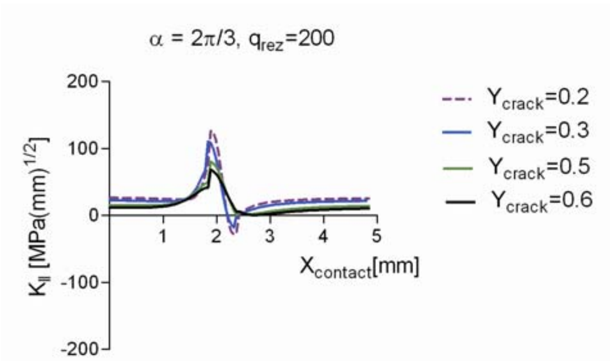

$\alpha = \pi/2$, $q_{rez}=100$
K_{II} [MPa(mm)$^{1/2}$]
$X_{contact}$[mm]
$Y_{crack}=0.2$
$Y_{crack}=0.3$
$Y_{crack}=0.5$
$Y_{crack}=0.6$

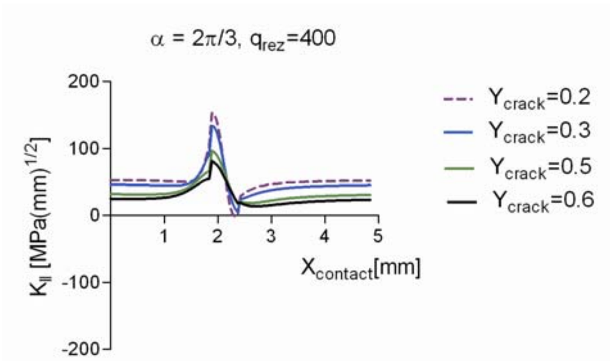

$\alpha = \pi/2$, $q_{rez}=200$
K_{II} [MPa(mm)$^{1/2}$]
$X_{contact}$[mm]
$Y_{crack}=0.2$
$Y_{crack}=0.3$
$Y_{crack}=0.5$
$Y_{crack}=0.6$


$\alpha = \pi/2$, $q_{rez}=400$
K_{II} [MPa(mm)$^{1/2}$]
$X_{contact}$[mm]
$Y_{crack}=0.2$
$Y_{crack}=0.3$
$Y_{crack}=0.5$
$Y_{crack}=0.6$


$\alpha = 2\pi/3$, $q_{rez}=0$
K_{II} [MPa(mm)$^{1/2}$]
$X_{contact}$[mm]
$Y_{crack}=0.2$
$Y_{crack}=0.3$
$Y_{crack}=0.5$
$Y_{crack}=0.6$


$\alpha = 2\pi/3$, $q_{rez}=100$
K_{II} [MPa(mm)$^{1/2}$]
$X_{contact}$[mm]
$Y_{crack}=0.2$
$Y_{crack}=0.3$
$Y_{crack}=0.5$
$Y_{crack}=0.6$


$\alpha = 2\pi/3$, $q_{rez}=200$
K_{II} [MPa(mm)$^{1/2}$]
$X_{contact}$[mm]
$Y_{crack}=0.2$
$Y_{crack}=0.3$
$Y_{crack}=0.5$
$Y_{crack}=0.6$


$\alpha = 2\pi/3$, $q_{rez}=400$
K_{II} [MPa(mm)$^{1/2}$]
$X_{contact}$[mm]
$Y_{crack}=0.2$
$Y_{crack}=0.3$
$Y_{crack}=0.5$
$Y_{crack}=0.6$

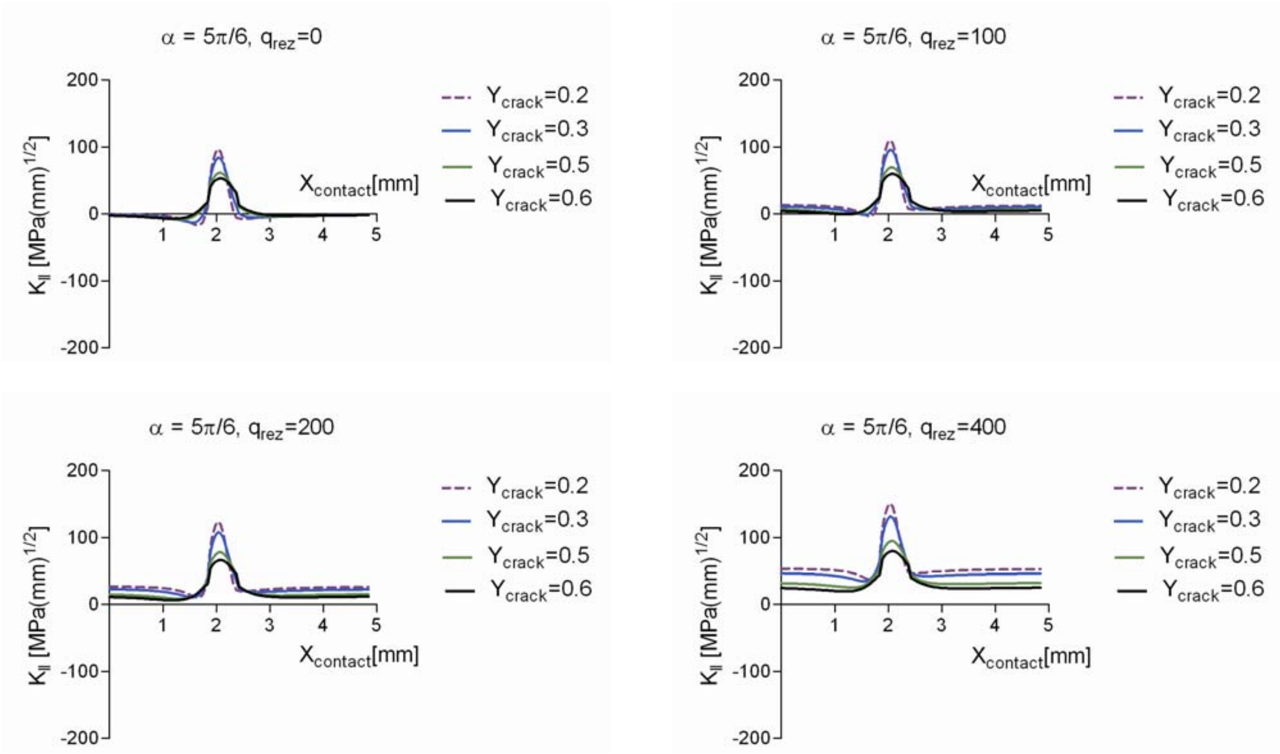


Fig. 3: SIF K_{II} variation as function of crack centre depth (Y_{crack}, mm) and residual tensions (q_{rez}, MPa)

5. CONCLUSIONS

The crack can propagate in sliding mode driven by the cyclic stress caused by repeated rolling contact even the normal stresses are of compression.

The parameter that governs the crack growth in this case is the stress intensity factor mode II, K_{II};

It depends on the normal and shear stresses which occur both from Hertzian and frictional stresses that are acting at the surface of the mating teeth;

An accurate SIF K_{II} variation as function of crack centre depth (Y_{crack} = 0.2, 0.3, 0.5, 0.6 mm) and different values of residual tensions (q_{rez} = 0, 100, 200, 400 MPa) was performed;

The values of SIFs K_{II} were computed for digitized (with the step π/6) values of the inclination angle α of the crack;

For both α = 0 and α = π/2 (corresponding to a horizontal or vertical position of the crack) residual tensions doesn't influence K_{II} values or crack growth under Mode II;

For small values of α (π/6, π/3) the growing of q_{rez} implies the growing of the negative values of K_{II};

For α > π/2 the growing of q_{rez} implies the growing of the positive values of K_{II} (the greatest values being obtained for $X_{contact} \cong 1.8$ mm), so the crack is most favored to grow under Mode II;

The maximum variation of K_{II} ($\Delta K_{II\ max}$) is obtained for Y_{crack} = 0.2 mm;

The minimum variation of K_{II} ($\Delta K_{II\ min}$) is obtained for the deepest Y_{crack} (0.6 mm);

We can observe that the K_{II} maximum positive values are increasing with inclination angle α and $X_{contact}$, so the probability of crack growth under Mode II is the greatest in these conditions.

6. REFERENCES

[1] Broek, D., (1984), *Elementary engineering fracture mechanics*, Martinus Nijhoff Publishers, Hague.

[2] Cioclov, D., (1977), *Mecanica ruperii materialelor*, Editura Academiei RSR, Bucureşti (in Romanian).

[3] Fan, H., Keer, L.M., Cheng, W., Cheng, H.S., (1993), *Competition between Fatigue Crack Propagation and Wear*, ASME J. of Tribology, 115, p. 141-147.

[4] Guz, A. N., (1983), *Brittle Fracture Mechanics for Materials with Internal Stresses*, Naukova Dumka, Kiev (in Russian).

[5] Keer, L.M., Bryant, M.D., (1983), *A Pitting Model of Contact Fatigue*, ASME J. of Lubrication Technology, 105, p. 198-205.

[6] Lavrov, N.A., Slepian, L.I., (1991), *To the theory of tensile fracture of solids under compression*, Arch. Leningrad Mining Inst. 125, p. 48-54 (in Russian).

[7] Murakami, M., (2002), *Metal Fatigue: Effects of Small Defects and Nonmetallic Inclusions*, Elsevier,

[8] Murakami, K., (1987), *Stress Intensity Factor Handbook*, vols. I and II, Pergamon Press.

[9] Obert, L., (1968), *Brittle fracture of rock*, vol. 7, in "*Fracture: an advanced treatise*", H. Liebowitz Editor, Academic Press, N.Y.

[10] Saxena A., Muhlstein C.L., (1996), *Fatigue Crack Growth Testing*, ASM Handbook, Fatigue and Fracture, vol. 19, ASM International, p. 410-452.

[11] Sih, G.C., Liebowitz, H., (1968), *Mathematical Theories of Brittle Fracture*, vol. 2, in "*Fracture: an advanced treatise*", H. Liebowitz Editor, Academic Press, N.Y.

[12] Tudose, L.M., Popa C.O., (2007), *Fatigue Crack Growth Simulation in the Hertzian Stresses Field of Teeth Gears*, ROTRIB 07, RO-066-1-9, ISSN 1843-6501.

[13] Tudose, L.M., Popa C.O., (2007), *Stress Intensity Factors Analysis on Cracks in the Hertzian Stresses Field of Teeth Gears*, ROTRIB 07, RO-118-1-8, ISSN 1843-6501.

[14] Tyron, R.G., (1997), *Probabilistic Mesomechanical Fatigue Model*, Grant NGT-51053, Lewis Research Center.