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APPLICATIONS OF THE UNCERTAINTY REGIONS BASED WORKSPACE DEFORMATION MODELING

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Abstract: The mechanical vibrations influence the precision of any mass system, specifically altering the motion parameters. The robotic applications are mechanical by essence, thus disturbed by the presence of vibrations. Before taking the appropriate actions to reduce the undesired effect, the behavior of the vibrations must be known. This paper presents two examples of workspace deformation based on the uncertainty regions modeling, briefly described at the beginning of the text.

1. INTRODUCTION

In the world of mechanical systems vibrations are omnipresent. Their effect is either wanted, in the cases of vibration processing, but mostly unwanted, as they alter the parameters of the system, by introducing perturbations. The most important parameter of motion that is affected by vibrations is the position itself.

Robots are precision machines that must meet the design requirements. Main robot's quality parameters are position, velocity and acceleration. All these parameters are influenced by the unwanted internally or externally caused mechanical vibrations. This implies that the designer of the robot should take into consideration the effect of vibrations in order to assure that the robot will discard them.

The vibrations are both deterministic and stochastic. Deterministic vibrations can be modeled using differential equations and their behavior can be predicted with satisfactory precision and therefore the robot can be designed to compensate their effect. Stochastic vibrations, however, are not predictable and their effect must be eliminated using other solutions.

One way to model the effect of stochastic vibrations is to use the uncertainty regions around the end-points. This paper briefly introduces the theoretical considerations and presents two examples of workspace deformation.

2. UNCERTAINTY REGIONS BASED WORKSPACE DEFORMATION MODELING

Uncertainty regions based vibration modeling states that the real position of the end-point of the mechanical element is actually located inside a spatial region around the ideal position. The problems focus on describing this region when taking into consideration the various structural and dynamic parameters of the elements.

The simplest approach describes the region as a spherical surface centered into the ideal position. This description is very restrictive and can be improved by considering the whole spherical volume, instead of only the surface. The distribution of probability inside the spherical region can be normal, uniform or of any other form. One interesting function is the normal (Gaussian) distribution centered in the ideal position, as seen in fig. 1. While the probability of finding the end-point close to the surface of the sphere is low, it considerably increases in the vicinity of the ideal position.

The spherical region is still very restrictive, therefore a disc shaped region can be used, as seen in fig. 2. This approach is based on the elastic behavior of a solid bar, which deforms stronger along transversal axis rather than along the longitudinal one. The volumetric probability distribution can also be normal, uniform or of other form.

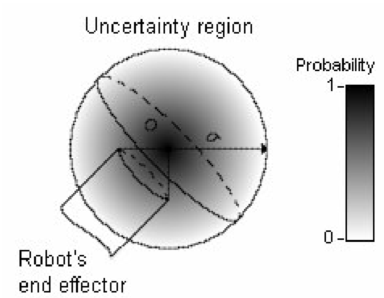


Fig. 1 – Spherical volume normal distribution of probability uncertainty region

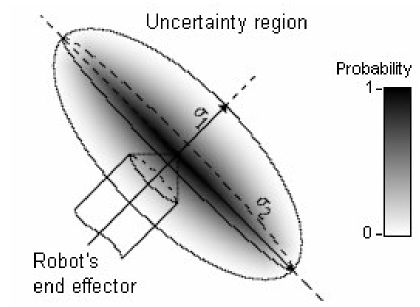


Fig. 2 – Disc shaped volume normal distribution of probability uncertainty region

However, this approach does not restrict the model to these two strict shapes, any other more appropriate uncertainty region geometry being usable.

The next step after having the appropriate vibration model is to insert it in a forward kinematics model. As all modern robots are driven by digital computers, a numerical kinematics model is best suited. For example, an *n* components robot with no translational degrees of freedom but only rotational ones should be considered. Its end effector position can be described by the following set of equations:

$$\begin{aligned} x_f &= \sum_{i=1}^{n} (l_i \cdot \sin\phi_i \cdot \cos\theta_i) \\ y_f &= \sum_{i=1}^{n} (l_i \cdot \sin\phi_i \cdot \sin\theta_i) \\ z_f &= \sum_{i=1}^{n} (l_i \cdot \cos\phi_i) \end{aligned} \tag{1}$$

where,
l_i – the length of the i^{th} component;
Φ_i – the angle between the i^{th} component and the fixed Oz axis;
θ_i – the angle between OP' and the fixed Ox axis, where P' is the projection of the end effector on the Oxy plane;
$P(x_f, y_f, z_f)$ – the Cartesian position of the end effector's characteristic point in the orthogonal Oxyz fixed system.

This system can be disturbed by a stochastic sequence given by the following vector $(\varepsilon_i)_n$ where $\varepsilon_i \in R^3$ The resulting model is given by the equations in (2).

$$\begin{aligned} x_f &= \sum_{i=1}^{n} (l_i \cdot \sin\phi_i \cdot \cos\theta_i + \varepsilon_{ix}) \\ y_f &= \sum_{i=1}^{n} (l_i \cdot \sin\phi_i \cdot \sin\theta_i + \varepsilon_{iy}) \\ z_f &= \sum_{i=1}^{n} (l_i \cdot \cos\phi_i + \varepsilon_{iz}) \end{aligned} \tag{2}$$

where, ε_{ix}, ε_{iy}, ε_{iz} are the projections of ε_i point on the three axis.

3. WORKSPACE DEFORMATION MODELING OF A TARGETING SYSTEM

The first example in this paper is a two degrees of freedom robot with the structure in fig. 3. The forward kinematics equations are described in (3).

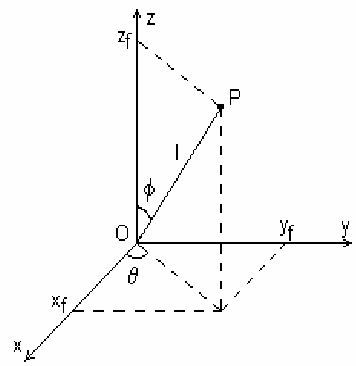


Fig. 3 – Example of targeting system

$$\begin{aligned} x_f &= l \cdot \sin\phi \cdot \cos\theta + \varepsilon_x \\ y_f &= l \cdot \sin\phi \cdot \sin\theta + \varepsilon_y \\ z_f &= l \cdot \cos\phi + \varepsilon_z \end{aligned} \tag{3}$$

The rotational axis are driven by stepper motors with a 1.8º and 3.6º step angles. This leads to a discretization of the continuous trajectory and consequently to a discrete workspace. First axis that provides the θ angle has a 0-360º opening, while the Φ axis has a 0-80º opening. The length of the element is 30cm.

The mathematical model of the uncertainty regions based deformation of the workspace is implemented and simulated by an Octave program the authors created. The Octave syntax is similar to Matlab syntax, therefore the code is portable. To have a comparison point, the ideal workspace for this system has also been generated and is presented in fig. 4. The top view (Oxy plane) and left view (Oxz plane) are shown in fig. 5.

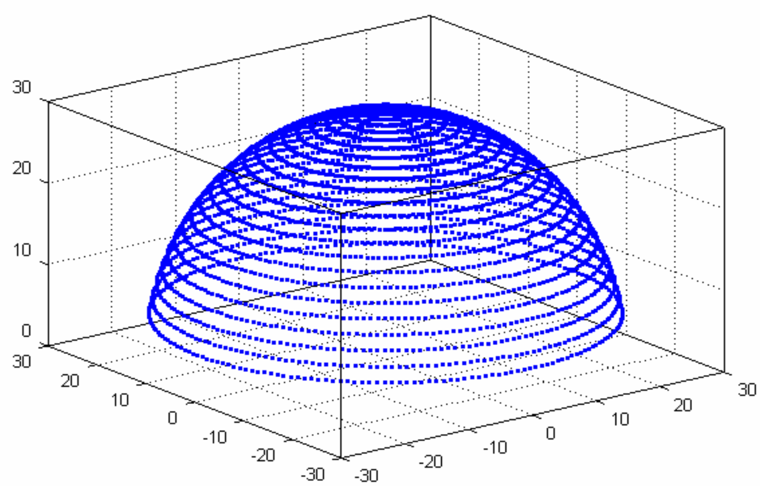
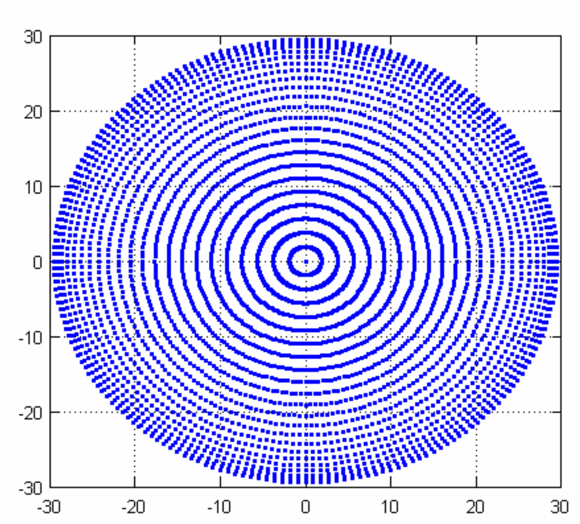
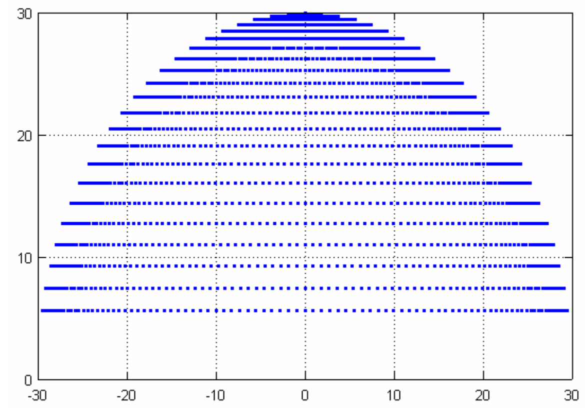


Fig. 4 – Targeting system's ideal workspace



a) Oxy plane



b) Oxz plane

Fig. 5 – Targeting system's ideal workspace planar projections

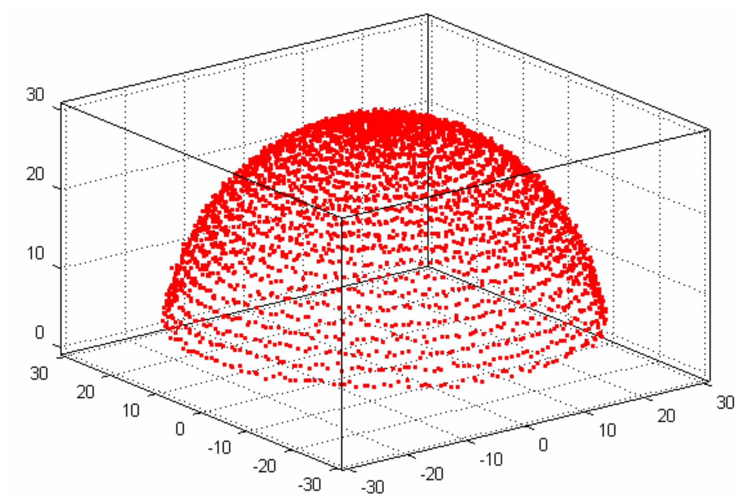
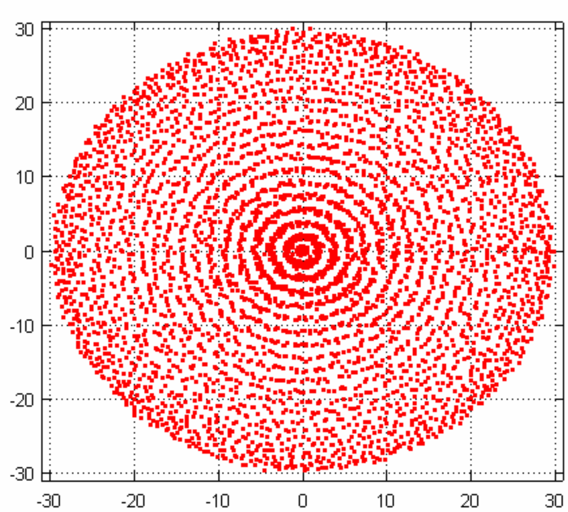
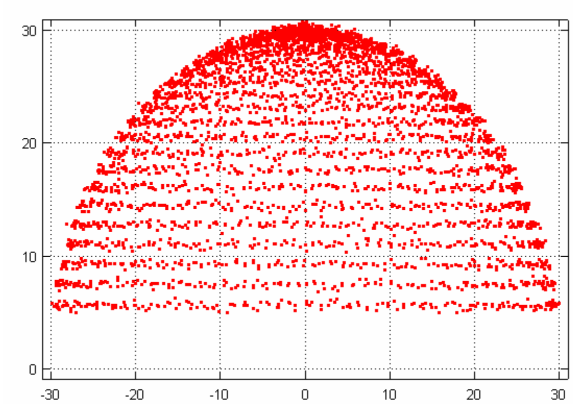


Fig. 6 – Targeting system's normally distributed uncertainty regions workspace deformation



a) Oxy plane



b) Oxz plane

Fig. 7 – Targeting system's normally distributed uncertainty regions workspace deformation planar projections

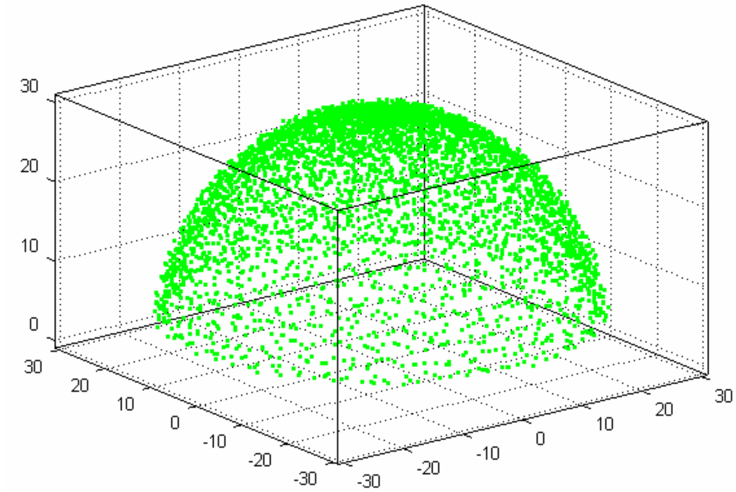
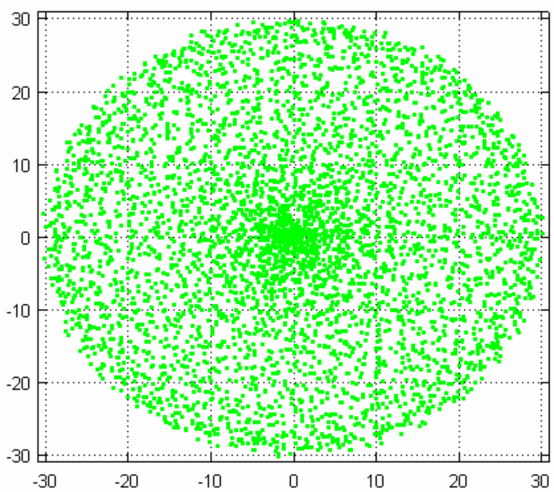
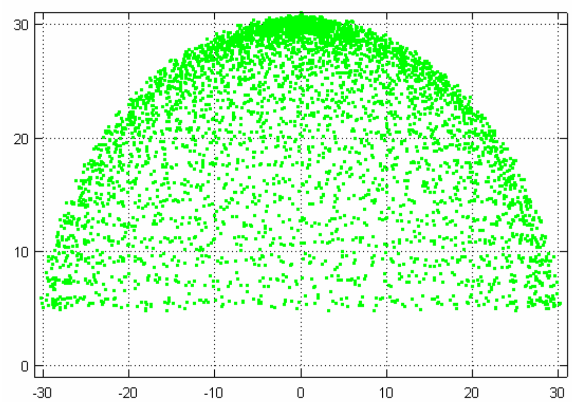


Fig. 8 – Targeting system's uniformly distributed uncertainty regions workspace deformation



a) Oxy plane



b) Oxz plane

Fig. 9 – Targeting system's uniformly distributed uncertainty regions workspace deformation planar projections

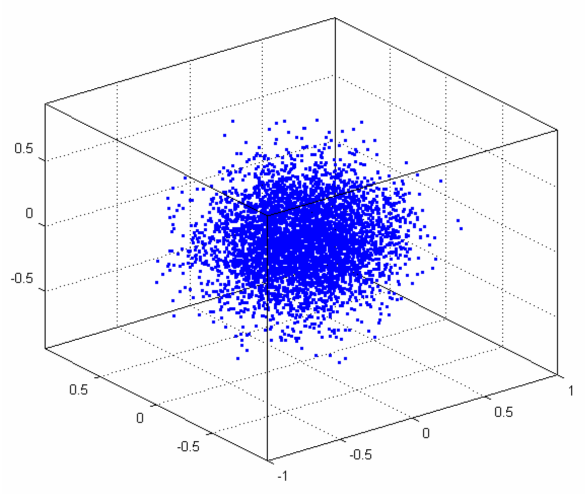
To obtain the deformed workspaces two types of perturbations were added: normally distributed and uniformly distributed probability density inside the uncertainty regions.

The normally distributed uncertainty regions deformed workspace is shown in fig. 6 and fig. 7, while the uniformly distributed uncertainty regions deformed workspace in fig. 8 and fig. 9.

As seen in these figures, the normal distribution tends to maintain the position of the end effector closer to its ideal position, while the uniform distribution places it into an equal chances sphere.

The normally distributed uncertainty regions deformed workspace resembles to the ideal workspace more than the uniformly distributed uncertainty regions deformed workspace does. This is the consequence of the distributions' mathematical functions.

In order to have a more detailed view of how different distributions generate the uncertainty region, two normally and uniformly distributed uncertainty regions have been drawn in fig. 10 and fig. 11. Please note that both regions have a spherical 1cm radius shape.



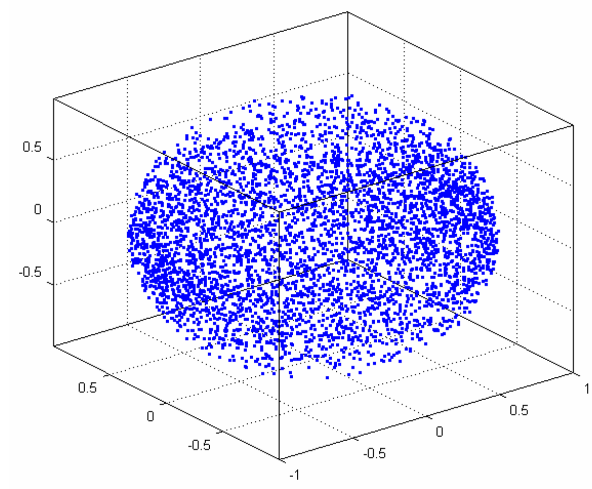


Fig. 10 – Normally distributed uncertainty region ***Fig. 11 – Uniformly distributed uncertainty region***

4. WORKSPACE DEFORMATION MODELING OF A PROCESSING SYSTEM

To introduce the cumulative effect of the intermediate elements end points position alteration as a consequence of vibrations presence, a more complex robotic system must be considered.

In this case a rather simple processing system connects two elements with a total of three degrees of freedom (rotation), as shown in fig. 12.

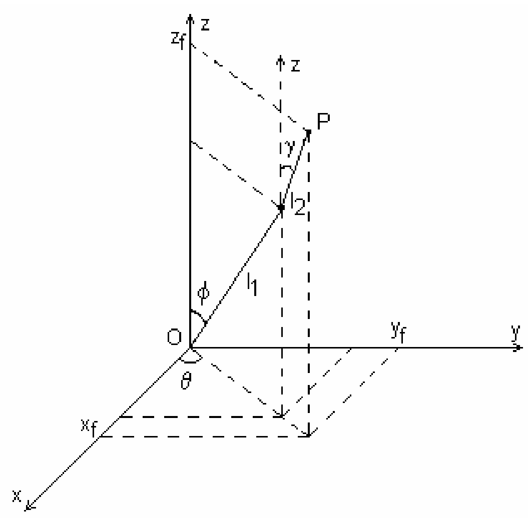


Fig. 12 – Example of processing system

For the processing system, the forward kinematics equations are presented in (4)

$$
\begin{aligned}
x_f &= l_1 \cdot \sin\phi \cdot \cos\theta + l_2 \cdot \sin\gamma \cdot \cos\theta + \varepsilon_{1x} + \varepsilon_{2x} \\
y_f &= l_1 \cdot \sin\phi \cdot \sin\theta + l_2 \cdot \sin\gamma \cdot \sin\theta + \varepsilon_{1y} + \varepsilon_{2y} \\
z_f &= l_1 \cdot \cos\phi + l_2 \cdot \cos\gamma + \varepsilon_{1z} + \varepsilon_{2z}
\end{aligned} \tag{4}
$$

The axis are driven by three stepper motors, from which two with 1.8º step angle and on with 20º step angle. These step angles have been chosen for demonstration purposes. θ axis has a 0-360º opening, Φ axis has a 0-80º opening and the γ axis has a -150-150º opening. The length of the first element is 30cm and the length of the second 5cm.

For demonstration purposes, the Φ angle has been frozen to 40º and the workspaces have been generated for this value.

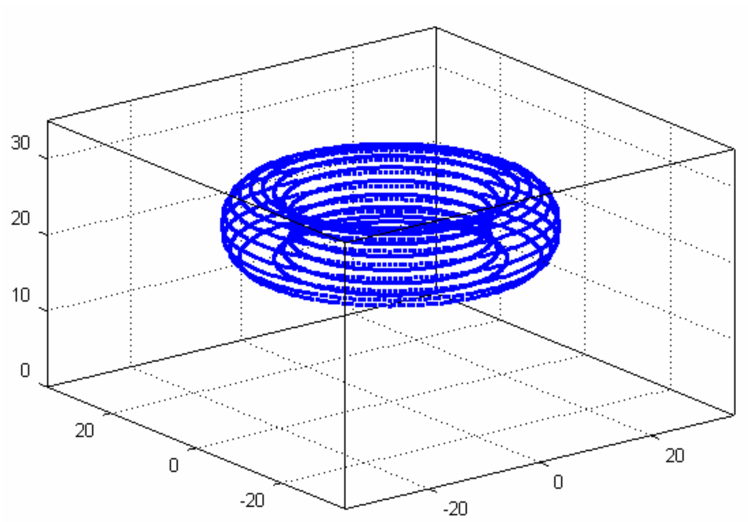
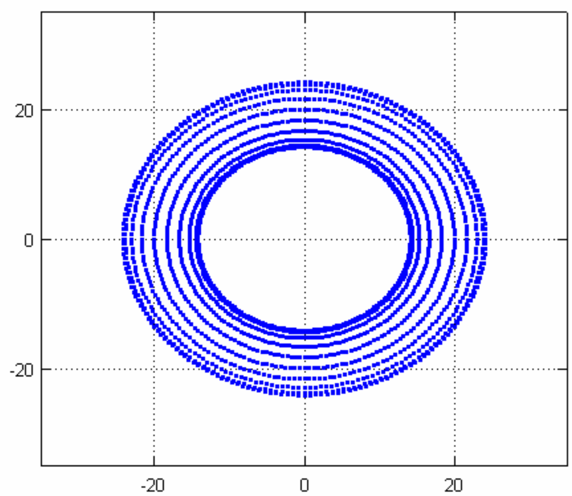
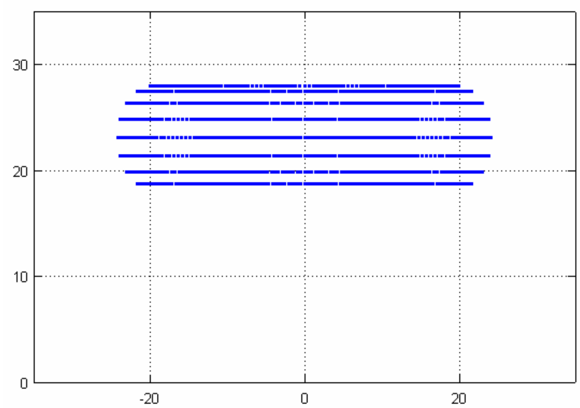


Fig. 13 – Processing system's ideal workspace



a) Oxy plane



b) Oxz plane

Fig. 14 – Processing system's ideal workspace planar projections

As the previous targeting system example, after applying the perturbations the real workspace deforms. In this case, the end effector's uncertainty region is obtained as a result of the composition of intermediate uncertainty regions.

The final result preserves the characteristics of the intermediate regions.

The normally distributed uncertainty regions workspace deformation is shown in fig. 15 and fig. 16 and the uniformly distributed uncertainty regions workspace deformation in fig. 17 and fig. 18.

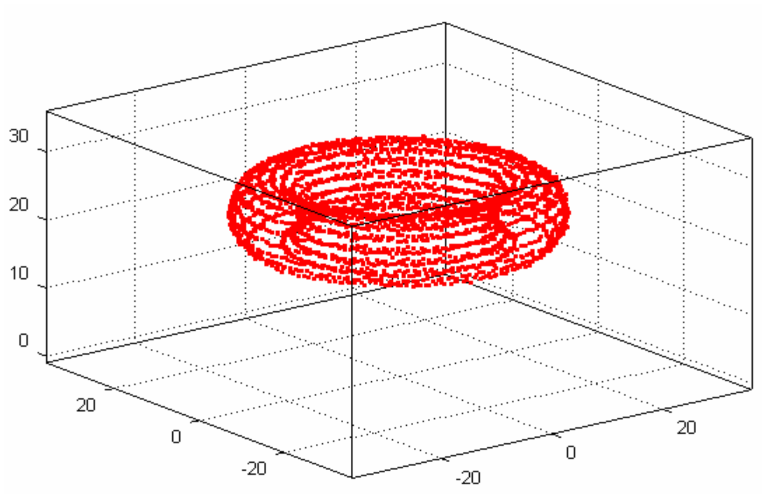
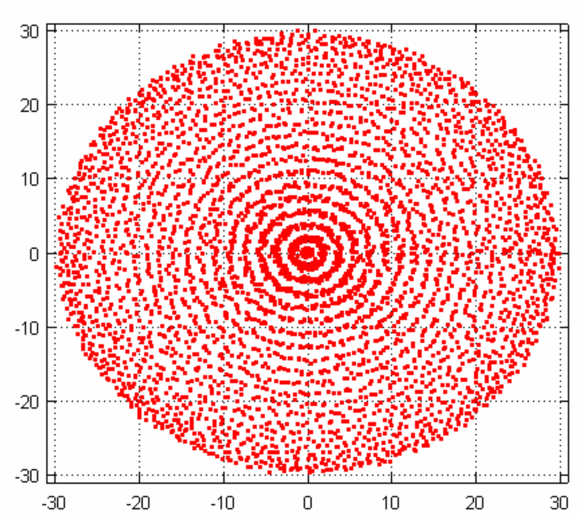
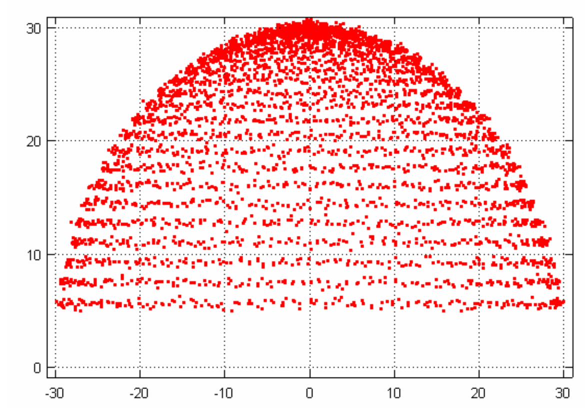


Fig. 15 – Processing system's normally distributed uncertainty regions workspace deformation



a) Oxy plane



b) Oxz plane

Fig. 16 – Processing system's normally distributed uncertainty regions workspace deformation planar projections

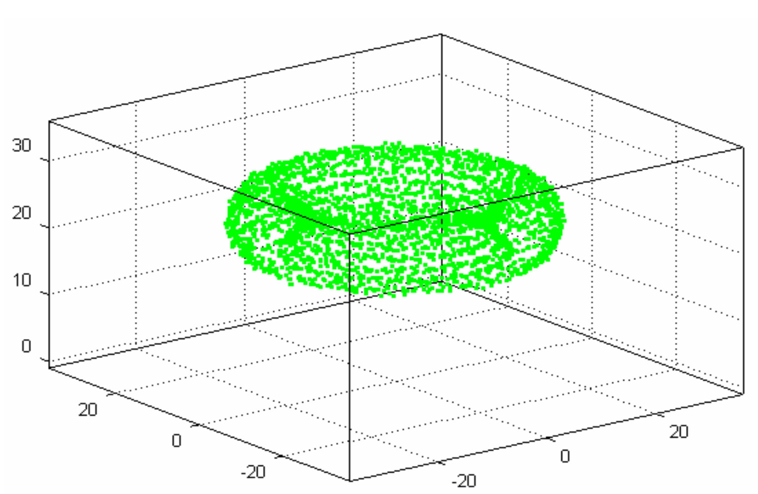
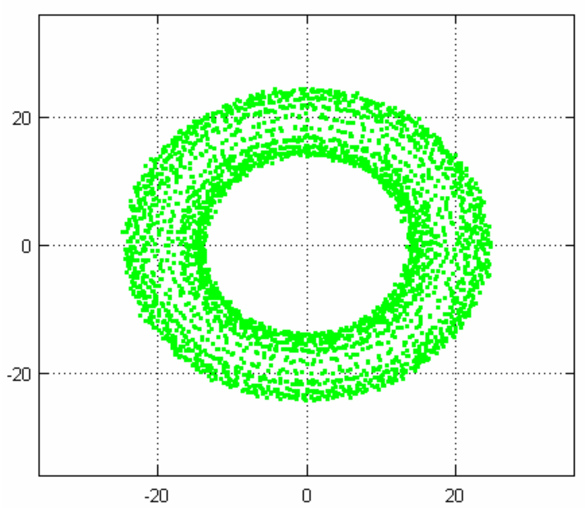
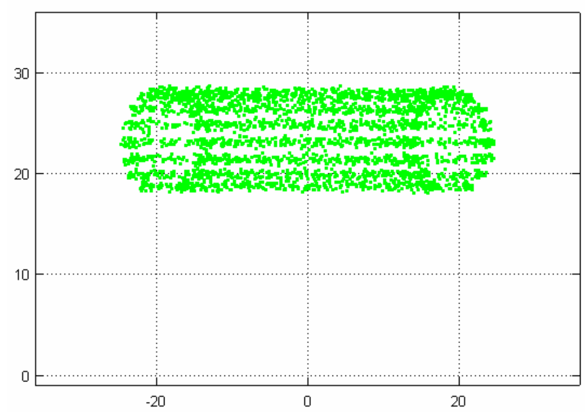


Fig. 17 – Processing system's uniformly distributed uncertainty regions workspace deformation



a) Oxy plane



b) Oxz plane

Fig. 18 – Processing system's uniformly distributed uncertainty regions workspace deformation planar projections

As seen in the simulations, even if the deviation parameters are the same in both normal and uniform distributions, the closest model to the ideal is the one based on normally distributed uncertainty regions.

5. CONCLUSIONS

The uncertainty regions based vibrations effect modeling requires very little and not highly accurate informations about the perturbations in the system. Therefore, this modeling can be applied in the robot's design phases to correctly choose and tune the axis control in order to have a higher precision of positioning.

This model can also be improved by adding parametric and even structural information from the robot itself, and therefore obtaining a differently shaped and more suited uncertainty region.

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