

## THE STUDY OF RELATIVE MOVEMENTS FROM KINEMATICS COUPLES OF ROTATION AND TRANSLATION THROUGH HOMOGENEOUS OPERATORS

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### **Abstract:**

From the structural point of view, the industrial robots mechanisms are formed from kinematics elements connected through kinematics couples which permit the movement and the power transmission from the action engines to a final elector.

The paper shows the study of a robot movement using homogeneous operators. It is presented a description of parameters and the way in which these define the robot in space 3D, and also a description of the obtained results from the algorithm introduction in MatLab medium.

### **1. MatLab language**

MatLab is a programmed medium and a language of a high performance used frequently in technology which included the numerical calculation, advanced graphics, the programming and visualization.

This offers to the researchers, engineers and every men of science an interaction system powerful and easy to use, in which the problems and solutions are expressed in a natural way.

His typical utilisation include the mathematical calculation (numerical and symbolical), development algorithms, modelling and stimulation, data analysis, exploration and visualization of these, scientific and engineering graphics, development application, including the realization of graphic interferences.

The MatLab language offers a modelling and stimulation medium of linear and unlinear systems. It operates with rectangular matrixes of real and complex numbers.

### **2. The obtained results from the simulation**

The programmes realised in MatLab medium, follow the analysis kinematics analysis of R.T.T. robot. The realised is a study of the space – temporal movement in which are resolved the trajectory problem, the speed and the point acceleration and also the connection from these dements.

To determine the position of the robots in space, it is used the matrix calculation, with the instruction of the rotation matrixes and those of homogeneous transformation.

The positioning analysis of the industrial robots mechanisms, are using frequently the notation of homogeneous coordinates and homogeneous transformation matrixes.

The position representation of a point P in homogeneous coordinates in report with a mark, it is made by using the column vector with 4 elements,  $[x \ y \ z \ 1]$ , where x, y, z are Cartesian coordinates of the point P in the P mark.

The elementary homogeneous operators, express the relative positions between two coordinates systems in the case is which the passing from a system to another is realised with an elementary movements of rotation on translation.

In the moment of realizing the programme, the matrix of translation was considered to be a matrix of pure transformation of whose expression is giving by the relation:

$${}^A T_B = \text{Trans}(a,b,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where a,b,c are the components of the mark translation  $T_B$  in the length of the mark axes  $T_A$ . The used notes in the inside of the programme went this (a,b,c) but the meaning is the same. The transportation of the matrix in the programme lines was made corresponding to the relations offered by Craig:

```
%Translate link (Craig)
x=-7;
y=-1.5;
z=-3;
t=[1 0 0 x; 0 1 0 y; 0 0 1 z; 0 0 0 1];
%end translate
```

For the rotation modelling it was used the matrix 4x4:

$$R(z, \theta) = \begin{bmatrix} c & -s & 0 & 0 \\ s & c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotation matrix is written to realise the programme, in this way:

```
%Rotate link (Craig)
THETA=90;
THETA = THETA*pi/180; %Note: THETA is in radians.
c = cos(THETA);
s = sin(THETA);
Rz = [c -s 0 0; s c 0 0; 0 0 1 0; 0 0 0 1];
%end rotate
```

The successive application of these operators permits the calculation of the coordinates for every modification of the coordinates system.

For realising the positional analysis, are associated every kinematics element, and also the objects from the medium in witch robot is working orthogonal marks, defined in generally so is origins and his axes to correspond some particular directions from the geometrical kinematics element point of view of the manipulator object.

It is known that the trajectory is the geometrical place of the successive positions occupied by the point during the movement. The trajectory can be plane or curve.

With the help of MatLab language, it was made a simulation programme of a robot which in movement describes a plane trajectory.

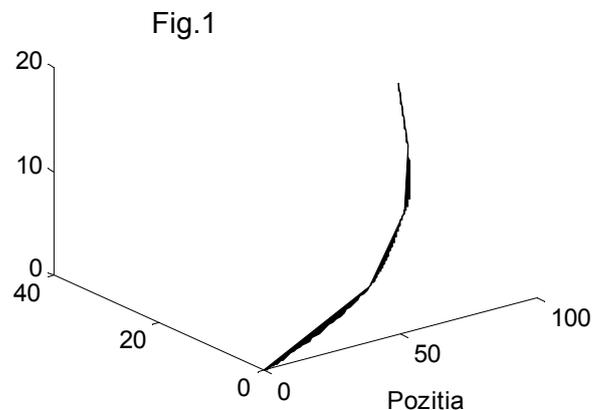
The simulation programme begins with the treatments of three important parameters: position, velocity and acceleration.

The point position is giving in every moment. The problem is resolved with the elimination of the time parameters between of the three lows:  $x(t)$ ,  $y(t)$ , and  $z(t)$  is obtaining the position vectors:  $\vec{r} = \vec{r}(t)$  of which expression will be function in the used coordinates system. With the help of MatLab soft for RTT robot case, it was obtained the variation of the position from figure 1. the programme lines which generate this graphic of the successive variation position occupied by the robot's arm were written in this way:

```

.....
tha1=[t11,t21,t31,t41];
ddd2=[t12,t22,t32,t42];
ddd3=[t13,t23,t33,t43];
subplot(221)
fill3(t11,t12,t13,'k',t21,t22,t23,'k',t31,t32,t33,'k',t41,t42,t43,'k');
xlabel ('Position')

```



Where: tha1, ddd2, ddd3, describes the lines of a matrix 4x4. This matrix describes every point of which the robot's joint is occupied in his movement.

The velocity, studied parameters give information about the way in which variation, the position vector  $\Delta \vec{r}$  in the time  $\Delta t$ , following the relation:

$$\Delta \vec{r} = \vec{r}(t + \Delta t) - \vec{r}(t)$$

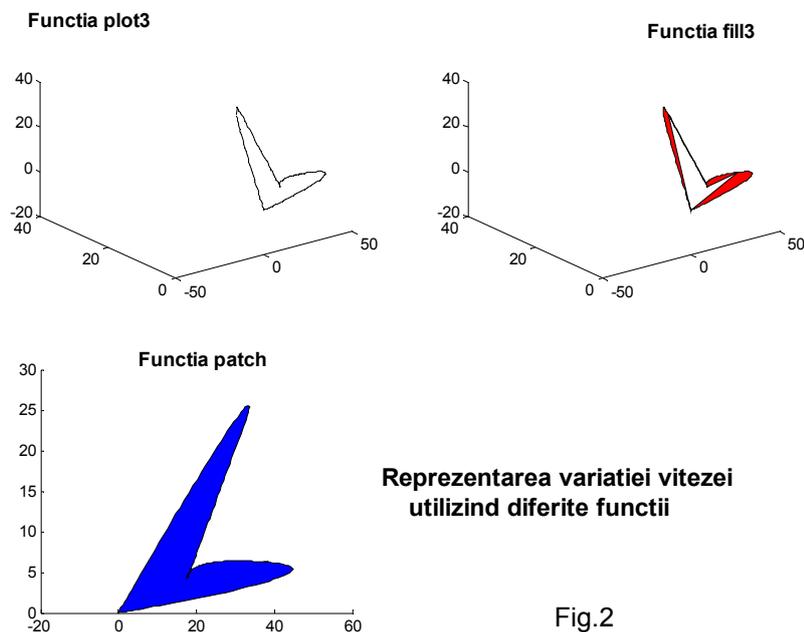
In define the velocity of a point in the first derivation in report with time of the position velocity:

$$\vec{r}: \vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}.$$

The modelling and simulation of the speed variation using different functions of plot, it was obtained the graphic of the velocity variation time function (figure 2).

The graphic of velocity variation touch by the robot's arm can be described in MatLab in this way:

```
.....
vtha1=[v11,v21,v31,v41]; vd2=[v12,v22,v32,v42]; vd3=[v13,v23,v33,v43];
subplot(221)
plot3(v11,v12,v13,':k',v21,v22,v23,':k',v31,v32,v33,':k',v41,v42,v43,':k')
title ('Reprezentarea varietii vitezei, utilizind diferite functii');
ylabel ('Functia plot3');
subplot(222);
fill3(v11,v12,v13,'r',v21,v22,v23,'r',v31,v32,v33,'r',v41,v42,v43,'r');
ylabel ('Functia fill3');
subplot(223):
patch(vtha1,vd2,vd3,'b');
xlabel ('Functia patch');
```



The acceleration, the third dimension which interests us, shows the way in which variation the point velocity. So, the acceleration of a point is the second derivation in report with the time of the position vector, or the first derivation in the report with the velocity time:

$$\bar{a} = \ddot{x}\bar{i} + \ddot{y}\bar{j} + \ddot{z}\bar{k}$$

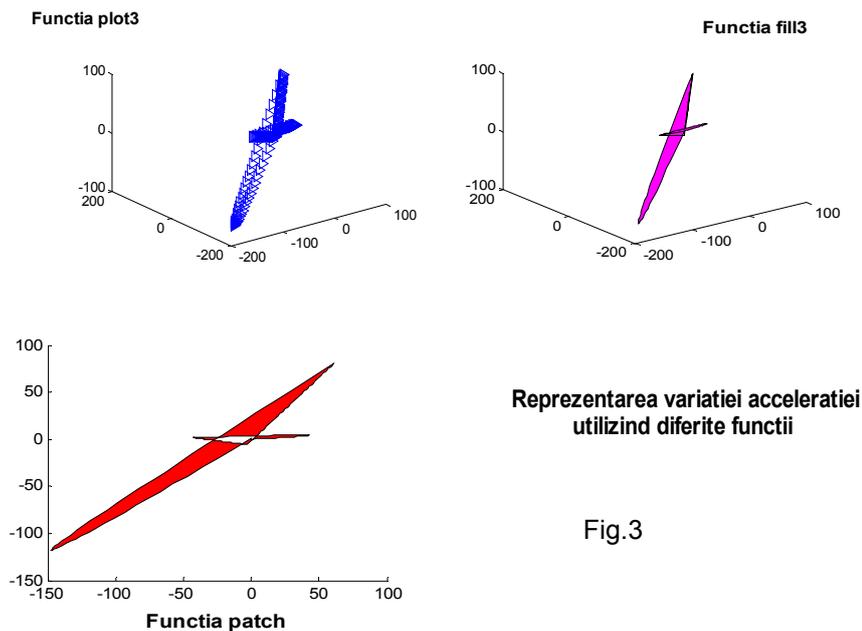
Consequently of the simulation:

```

.....
atha1=[a11,a21,a31,a41]; ad2=[a12,a22,a32,a42]; ad3=[a13,a23,a33,a43];
subplot(221)
plot3(a11,a12,a13,'>b',a21,a22,a23,'>b',a31,a32,a33,'>b',a41,a42,a43,'>b');
ylabel ('Functia plot3')
subplot(222)
fill3(a11,a12,a13,'m',a21,a22,a23,'m',a31,a32,a33,'m',a41,a42,a43,'m');
ylabel ('Functia fill3')
subplot(223)

```

it is generated the graphic of the accelerations achieved by the robot's arm time function (figure 3).



### 3. The simulation of the trajectory and the errors which appear during his crossing

The elaborated software based on the algorithm presented below, it permits not only the generation of a linear trajectory and also the errors simulation which can be produced in the time of RTT working robot, referring at the frequent: the exception from different reasons of the pre – established trajectory (figure 4).The software is simple and created just for didactic application.

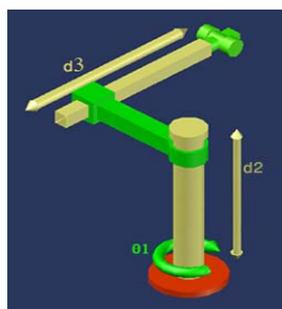
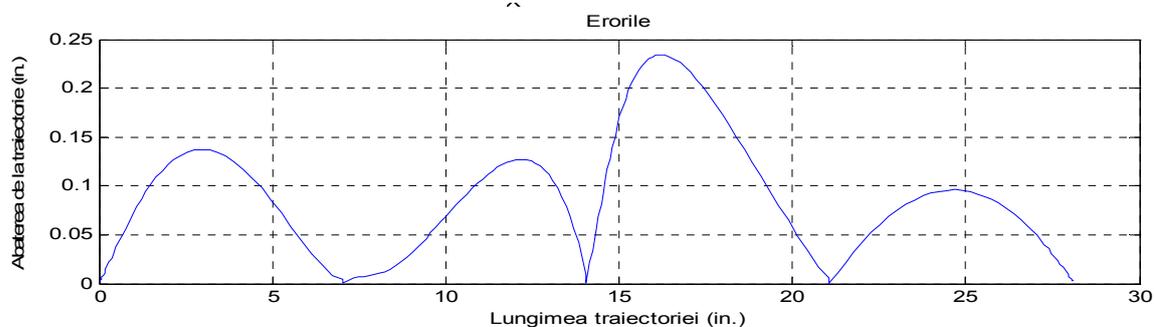


Fig. 4



The obtained results, achieved the purpose which we were interested: modelling, simulation, processing for the distinguish trajectory and the derivations of the wanted trajectory.

### References

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- [2] Ghinea Marin, Firețeanu Virgiliu, "Matlab", ed. Teora, 1997