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METHODS OF ESTABLISHING THE MOST PROPICIOUS NUMBER OF NECESSARY OPERATORS IN THE MAINTENANCE ACTIVITY

Marius, ROMOCEA
University of Oradea,mromocea@yahoo.com

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Abstract. Approaching the maintenance problems we face some waiting phenomena, due to the aleatory character of the breakdowns. This leads to the necessity of conceiving a waiting plan as convenient as possible from the economic point of view . The paper presents the relations which allow us to establish the probability of existence in the waiting system of "n" units at a certain moment, and the average number of the existing units in the waiting system as well, which is so important in practice. The paper ends with an investigation on a given case.

1. INTRODUCTION

The costs for manufacturing the products are directly influenced by the costs for maintenance activitities, each maintenance system being characterized by specific costs. The lowest costs can be found within the preventive maintenance strategies, the corrective strategies proving to be very expensive.

Among the factors which influence the costs of the maintenance activity, with an impact upon the competitiveness of the company, we can mention:

- the aims and the strategies of the company
- the quality standards imposed to the products and to the correspondent maintenance activities
- the amount of technology involved in production
- the amount of technology involved in the maintenance activity
- the qualification of the production and maintenance staff
- the staff's motivation

A big part of the costs for the maintenance activity cannot be found in the company's book-keeping documents, because they are difficult to identify and measure, being included in the category of the maintenance hidden costs [6].

The budget for the maintenance department is made up by the financial resources earmarked by the company for sustaining this activity, resources which must cover, among other things, the payment for the work of the maintenance operators. That leads us to the necessity to establish the most appropriate number of operators, necessary in every production process, in order to solve the problems related to the rational operation, the upkeep and the repairing of the machines and equipments in advantageous economic conditions. As the breakdowns' occurance has an aleatory character, establishing the necessary number of operators can be solved using the waiting models [3].

The waiting theory, as a chapter of the operational research, studies the basic characteristics of the systems that are expected to produce waiting phenomena, due to the fact that a certain service is required by a number of "units", the occurance of which has an aleatory character.

The classic mathematic model used to describe the functioning in time of the systems which are discret in space and discret or continuous in time – such as the electronic systems etc – are the Markov processes, particularly the homogeneous Poisson processes.

The method for establishing the reliability function of the systems by describing their functioning through the Markov processes is one of the precise methods of modelling.

2. THEORETICAL GROUND

Waiting models describe servicing systems and processes of a mass character, which can be met in many other fields of practical activities: industry, transportation, communications, trade etc.

The main problem occuring in applying the waiting theory consists in establishing and justifying the expenses necessary to reach a certain level of the servicing quality within the waiting phenomena of a mass character. This leads us to the very important role played by the quantitative indicators of the servicing quality : the length of the waiting line, the volume of the services done in a time unit etc. The main characteristic, common to many waiting models, is the existence of a flow of requirements for servicing, called entrance flow, characterized by the number of requirements that enter the system in a time unit. Even when a detailed checking of the equipment is done before starting to work, there can occur breakdowns and so, requirements for unpredicted repairs, that's why we say that the entrance flow has an aleatory character.

In every waiting system there are elements that perform the services, elements that are called servicing stations or channels. The maintenance operator is a servicing station.

In order to use the results of the waiting theory in the reliability theory, it is enough to replace the term "requirement" with the term "breakdown", and the "servicing" time with "repairing" time.

An entrance flow is regular or determined (D) if the events follow each other in certain lapses of time. The main characteristic of an unstationary flow of events is the momentary density $\lambda(t)$ defined by the equality

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{m(t+\Delta t) - m(t)}{\Delta t} = m'(t) \tag{1}$$

where, by m(t) we mean the average number of events (breakdowns) in the lapse of time (0,t).

It is shown [3] that for a poissonian unstationary flow, the number of events (breakdowns) occured in the lapse of time τ, which has its origin in the point t_0, follows a Poisson law:

$$P_k(\tau, t_0) = \frac{a^k}{k!} e^{-a}, \quad k \in N \tag{2}$$

where „a" represents the average value of the number of the events within the lapse of time (t_0, $t_0+\tau$), that is :

$$a = \int_{t_0}^{t_0+\tau} \lambda(t)\, dt \tag{3}$$

In the case of the poissonian unstationary flow we have :

$$P(\xi \geq t) = e^{-a} = e^{-\int_{t_0}^{t_0+t} \lambda(t) dt} \quad (t>0) \tag{4}$$

and the repartition function is expressed by:

$$F_{t_a}(t)=1-e^{-a}=1-e^{-\int_{t_0}^{t_0+t}\lambda(t)dt} \quad (t>0) \tag{5}$$

the repartition density of the aleatory lapse of time ξ between two consecutive events, from which the first occurs in the moment t_0, it is given by the relation :

$$f_{t_0}(t)=\lambda(t_0+t)e^{-\int_{t_0}^{t_0+t}\lambda t\,dt} \quad (t>0) \tag{6}$$

An ordinary flow of homogenous events, in which the lapses of time $\xi_1,\xi_2,\ldots,\xi_n$ between the successive events are independent aleatory variables, is called flow of limited postaction or Palm flow.

The simple flow which preserves one point from k+1 points while the rest of them are omitted, is called Erlang flow of k order and it is written as (E_k). The Erlang flow represents a case of a particular Palm flow.

In an Erlang flow (E_k) the probability of the first event is λdt, the probability of the second is

$$P_k(t)=\frac{(\lambda t)^k}{k!}e^{-\lambda t} \tag{7}$$

and the density of probability

$$f_k(t)=\frac{\lambda(\lambda t)^k}{k!}e^{-\lambda t}, \quad (t>0) \tag{8}$$

and it corresponds to the Erlang repartition law of k order.

A waiting model is completely described by the following elements: the entrance flow, the waiting line, the servicing station and the exit flow. The entrances in the system are supposed to be at random and independent.

The servicing time can be : determined (D), poissonian or markovian (M), of Erlang type (E_k), or general (G), if there is no hypothesis expressed about the servicing time repartition function.

Considering these notes and using the symbol (S) for the number of the servicing stations, each waiting model can be characterized by a symbol having the form •/•/•, which indicates: the type of the entrance flow, the type of the repartition of the servicing time and the number of the servicing stations in the system. Thus, we can meet the following types of waiting models: D/D/S, M/D/S, M/M/S, M/E_k/S, E_k/M/S, E_k/E_k/S, M/G/S etc.

3. THE APPLICATIVE ASPECT

Let's suppose that in an enterprise there are N machines functioning independently one from another, having the same technological characteristics, the average number of the machines that go out of order in a certain period of time being λ. The periods of time needed for repairs are independent aleatory variables, following a negative exponential repartition of parameter μ. We have a case of a M/M/1 model with a number of limited units (N).

Let's consider :

$$P_j(t)=\{\xi(t)=j\} \tag{9}$$

where $\xi(t)$ represents the number of the units (broken machines) in the system, at the moment $t \geq 0$. Analizing the changes that may occur in the system in the lapse of time ($t, t + \Delta t$) we obtain :

$$\mu \, p_n = \lambda \, p_{n-1} \qquad (10)$$

where

$$p_j = \lim_{t \to \infty} P_j(t), \quad j = 0,1,\cdots,N$$

The average number of machines in the system is given by :

$$U_M = N - \frac{1}{\rho}(1 - p_0), \qquad \rho = \frac{\lambda}{\mu} \qquad (11)$$

The average value of the mechanic's inactivity is:

$$\sum_{j=0}^{1} (1 - j)\, p_0 = p_0$$

The average number of the machines that are not in the system is given by :

$$N - U_M = \frac{1}{\rho}(1 - p_0) \qquad (13)$$

The probability that a broken machine will wait a time w, can be expressed by:

$$p(w > 0) = \sum_{j=1}^{N} P_j = 1 - p_0 \qquad (14)$$

For a case of S mechanics we use the model M/M/S. We use $\xi(t)$ for the aleatory variable which represents the number of machines that are not functioning at the moment t. We say that the waiting system is in the state E_J, at the moment t, if j machines are not functioning.

If j < S, no waiting line appears and there are S – j inactive mechanics, if j > S will represent the machines waiting for being repaired.

Analysing the tranzitions that may take place within the lapse of time $(t, t + \Delta t)$, we obtain a system of differential equations from which we infer that:

$$\frac{E(L)}{E(T)} = \frac{\mu}{\lambda} \qquad (15)$$

Where

E(L) represents the average number of machines in a functioning state

E(T) represents the average number of the machines that are being repaired

If we use q for the probability that a machine will break, then the probability that the same machine will stay in a waiting line of j machines is given by the relation:

$$q_j = q \cdot p_j \qquad (16)$$

4. STUDY CASE

1. The investigation has been realized in an local enterprise specialized in shoes manufacturing, having only one qualified operator for repairing the ADLER sewing machines. Within 8 hours there is an average of six requirements for repairing the broken machines. The mechanic can repair on an average 0,9 machines/hour. Considering what we mentioned above we can answer the following questions:

a. How many broken machines are waiting for being repaired?

b. Which is the average time of waiting?

c. Knowing that a machine occupies 1,2 m^2 , we can determine the space necessary for storing the broken machines

d. The number of the broken machines that are brought in the workshop to be repaired

$$\lambda = \frac{6}{8} = 0{,}75$$

a. The number of machines that the mechanic can repair within an hour is $\mu = 0{,}9$.
The upkeep factor has the following value (in service)

$$\rho = \frac{\lambda}{\mu} = \frac{0{,}75}{0{,}9} = \frac{5}{6}$$

The average number of the wating machines in the workshop is

$$U_M = \frac{\rho}{1-\rho} = \frac{\frac{5}{6}}{1-\frac{5}{6}} = 5 \text{ machines}$$

b. The average waiting time for a machine will be :

$$w^* = \frac{\rho}{\mu(1-\rho)} = \frac{\frac{5}{6}}{0{,}9\left(1-\frac{5}{6}\right)} = 4\frac{17}{27} = 4{,}63 \text{ hours}$$

c. As 4 < 4,63 < 5, the area intended to be used for storage must be of minimum 1,2 × 5 = 6 m^2

2. Another department of the same enterprise is endowed with N=5 machines for bending the front sides of the shoes, whose upkeep factor (in service) is $\rho = 0{,}1$. Knowing that the repartition of probability for the functioning time of a machine before it breaks is exponential, and considering those mentioned above, we intend to establish the following:

a. To determine the probability that, in the repairing workshop where a single mechanic works, there could be j machines (j = 0,1,2,3,4,5).

b. To find out the average number of the machines that are in the repairing workshop

a. We apply the relation:

$$p_j = \left[1 + \sum_{j=1}^{N} \frac{n!\rho^j}{(N-j)!}\right]^{-1}$$

p_0 = the probability that in the repairing workshop there could be no machines waiting for being repaired

$$p_0 = \frac{1}{1+5!\left[\frac{0{,}1}{4!} + \frac{(0{,}1)^2}{3!} + \frac{(0{,}1)^3}{2!} + \frac{(0{,}1)^4}{1!} + \frac{(0{,}1)^5}{0!}\right]} = 0{,}564$$

We use p_j for the probability that in the workshop there could j machines to be repaired. In this case :

$$p_1 = \frac{5!\cdot 0{,}1}{4!} \cdot 0{,}564 = 0{,}282$$

$$p_2 = \frac{5! \cdot (0,1)^2}{3!} \cdot 0,564 = 0,1128$$

$$p_3 = \frac{5! \cdot (0,1)^3}{2!} \cdot 0,564 = 0,03384$$

$$p_4 = \frac{5! \cdot (0,1)^4}{1!} \cdot 0,564 = 0,006768$$

$$p_5 = \frac{5! \cdot (0,1)^5}{0!} \cdot 0,564 = 0,0006768$$

b. By using the relation :

$$U_M = N - \frac{1}{\rho}(1 - p_0)$$

we determine the average number of machines that are not in the system (in repairs)

$$U_M = 5 - \frac{1}{0,1}(1 - 0,564) = 0,64$$

and the average number of machines that are not in the repairings workshop will be :

$$U_M^* = U_M - (1 - p_0) = 0,64 - (1 - 0,564) = 0,204$$

5. CONCLUSIONS :

1. The maintenance activity is part of a large category of practical problems regarding production and taking decisions, where it is important to study the aleatory occurance of some units at a certain point, occurance which follow certain theoretical repartition laws, in other words, the maintenance activity produce waiting problems.
2. Being aware of the elements mentioned above and of the length of the waiting line, helps us to determine the best number of necessary maintenance operators, in order to improve the direct and indirect costs of this activity.
3. The number of the accidental breakdowns can be diminished by following the systematic maintenance programme

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