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STRESS CONCENTRATION FACTOR OF DENTAL FILLING

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Abstract. The aim of the paper is to obtain the stress state and stress concentration factor from a dental matrix with a circular filling. The elastic characteristics of matrix material are considered constant, and those of the filling are assumed variable. As a study model, the uni-axial stretched matrix is considered infinite. The filling is considered perfectly bonded to the matrix and there are not relative displacements at the matrix-filling interface. The results are presented as graphics for principal stresses and hoop stresses from the matrix-filling interface.

1. INTRODUCTION

In dental practice, the stress concentration factor of a filling upon dental matrix is extremely important and it must be considered as it is possible that a fill up of decay could harm more than the lack of it. It is really significant to specify the type of filling material in order to obtain a stress concentration factor as small as possible.

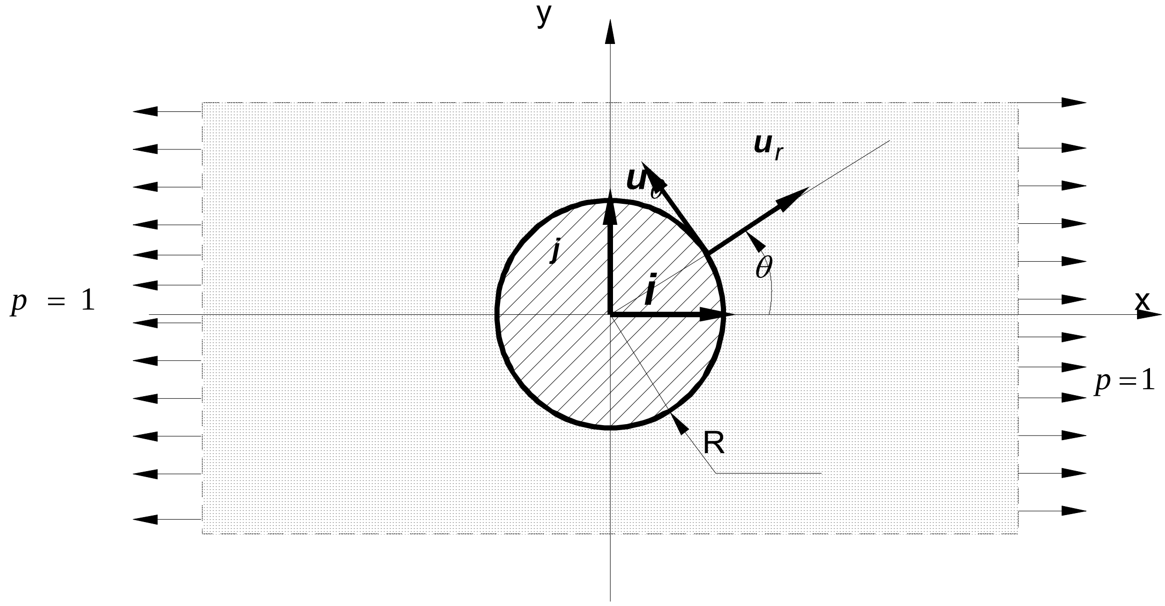


Fig. 1. A circular elastic inclusion inserted in an infinite elastic matrix

During mastication movements, the teeth loading is extremely complicated, especially due to the complicated shape of the oclusal surfaces. The shape of the filling also takes part in founding the stress concentration factor. In order to obtain analytical expressions for stresses both in dental matrix and fillings, the uni-axial stretched holed plane with unity circular cavity is modelled, Fig.1.

2. MATHEMATICAL BACKGROUND

In order to study the stress and strain state the method of elastic potentials of Muskhelishvili, [1], is applied. The stress state from an elastic material is described by the Kolosov-Muskhelishvili, equations, (1)

$$\begin{aligned} \sigma_x + \sigma_y &= 2[\Phi(z) + \overline{\Phi(z)}]; \\ \sigma_y - \sigma_x + 2i\tau_{xy} &= 2[\bar{z}\Phi'(z) + \Psi(z)]; \\ 2\mu[u + iv] &= \kappa\varphi(z) - z\overline{\varphi'(z)} - \psi(z), \end{aligned} \tag{1}$$

where $\sigma_x, \sigma_y, \tau_{xy}$ are the Cartesian stresses and u,v are Cartesian components of displacement vector. In relations (1), z, represents the affix of a point from the complex plane and Φ, Ψ, ϕ, ψ are complex potentials; between these there are the following relations:

$$\Phi(z) = \varphi'(z), \Psi(z) = \psi'(z). \tag{2}$$

Superstriking of a complex number means the complex conjugate and $i = \sqrt{-1}$. In relations (1) μ represents the shear modulus and κ is a parameter characterising the elastic properties of the material, having the form:

$$\kappa = 3 - 4\nu \tag{3}$$

for the plane stress state – the case of thin plates, and

$$\kappa = \frac{3-\nu}{1+\nu} \tag{4}$$

for the plane strain state, where ν is the Poisson ratio. The circular shape of the hole and the absence of tangential motion between matrix and inclusion allows the use of polar co-ordinates, (r, θ):

$$\begin{aligned} \sigma_x + \sigma_y &= \sigma_r + \sigma_\theta = 2[\Phi(z) + \overline{\Phi(z)}]; \\ \sigma_y - \sigma_x + 2i\tau_{xy} &= \sigma_\theta - \sigma_r + 2i\tau_{\rho\theta} = 2[\bar{z}\Phi'(z) + \Psi(z)]e^{2i\theta}; \\ u + iv &= (v_r + iv_q)e^{i\theta} \end{aligned} \tag{5}$$

The boundary conditions needed are: on the contour of the junction tooth-filling, the radial stresses from matrix must be equal to the radial stresses from filling, σ_r and similar to the tangential stresses, $\tau_{r\theta}$. The displacement components in polar co-ordinates must be the same in matrix and filling. These conditions are written bellow, with the superscript *m* for matrix and *o* for circular filling:

$$\left.\begin{aligned}\sigma_r^m &= \sigma_r^o,\\ \tau_{r\theta}^m &= \tau_{r\theta}^o,\\ v_r^m &= v_r^o,\\ v_\theta^m &= v_\theta^o,\end{aligned}\right| la\ \ r = R \tag{6}$$

Muskhelishvili defines for matrix the following material parameters:

$$\beta^m = -\frac{2(\mu^o - \mu)}{\mu^m + \mu^o\kappa}, \quad \gamma^m = \frac{\kappa(\kappa^o - 1) - \mu^o(\kappa^m - 1)}{2\mu^o + \mu(\kappa^o - 1)}, \delta = \frac{\mu^o - \mu^m}{\mu^m + \mu^o\kappa}, \tag{7}$$

and for the inclusion:

$$\beta^o = \frac{\mu^o(\kappa + 1)}{2\mu^o + \mu^m(\kappa^o - 1)}, \ \gamma = 0, \delta^o = \frac{\mu^o(\kappa + 1)}{\mu + \mu_o\kappa} \tag{8}$$

For the matrix it was found:

$$\varphi^m(z) = \frac{p}{4}\left(z + \frac{\beta R^2}{z}\right), \psi^m(z) = -\frac{p}{2}\left(z + \frac{\gamma^m R^2}{z} + \frac{\delta^m R^4}{z^3}\right), \tag{9}$$

and for the inclusion:

$$\varphi^o(z) = \frac{p}{4}\left(\beta^o z + \frac{\gamma^o z^3}{R^2}\right), \psi^o(z) = -\frac{p}{2}\delta^o z \tag{10}$$

3. STRESS FINDING

By denoting:

$$\begin{gathered} S^{m,o} = 2[\Phi^{m,o}(z) + \overline{\Phi^{m,o}(z)}] \\ T^{m,o}(z) = 2[\bar{z}\,\Phi'^{m,o} + \Psi^{m,o}] \end{gathered} \tag{11}$$

The Cartesian expressions for the stresses are:

$$\begin{gathered} \sigma_x^{m,0}(z) = \frac{Re[S^{m,o}(z)] - Re[T^{m,o}(z)]}{2} \\ \sigma_y^{m,0}(z) = \frac{Re[S^{m,o}(z)] + Re[T^{m,o}(z)]}{2}, \\ \tau_{xy}^{m,0}(z) = \frac{Im[T^{m,o}(z)]}{2} \end{gathered} \tag{12}$$

Where $Re(z)$ and $Im(z)$ represent the real and imaginary part, respectively, of the complex number z.

The relations between the stresses in polar and Cartesian co-ordinates are:

$$\sigma_r^{m,o}(z) = cos^2\theta \cdot \sigma_x^{m,o}(z) + sin^2\theta \cdot \sigma_y^{m,o}(z) + sin 2\theta \cdot \tau_{xy}^{m,o}(z),$$
$$\sigma_\theta^{m,o}(z) = sin^2\theta \cdot \sigma_x^{m,o}(z) + cos^2\theta \cdot \sigma_y^{m,o}(z) - sin 2\theta \cdot \tau_{xy}^{m,o}(z) \quad (13)$$
$$\tau_{r\theta}^{m,o}(z) = \frac{\sigma_y^{m,o}(z - \sigma_x^{m,o}(z)}{2} sin 2\theta + cos 2\theta \cdot \tau_{xy}^{m,o}$$

Finally, the principal stresses from matrix and inclusion respectively are found with the relations:

$$\sigma_{max}^{m,o}(z) = \frac{\sigma_x^{m,o}(z) + \sigma_y^{m,o}(z)}{2} + \sqrt{\left(\frac{\sigma_x^{m,o}(z) - \sigma_y^{m,o}(z)}{2}\right)^2 + [\tau_{xy}^{m,o}(z)]^2}$$
$$\sigma_{min}^{m,o}(z) = \frac{\sigma_x^{m,o}(z) + \sigma_y^{m,o}(z)}{2} - \sqrt{\left(\frac{\sigma_x^{m,o}(z) - \sigma_y^{m,o}(z)}{2}\right)^2 + [\tau_{xy}^{m,o}(z)]^2} \quad (14)$$
$$\tau_{max}^{m,o}(z) = \sqrt{\left(\frac{\sigma_x^{m,o}(z) - \sigma_y^{m,o}(z)}{2}\right)^2 + [\tau_{xy}^{m,o}(z)]^2}$$

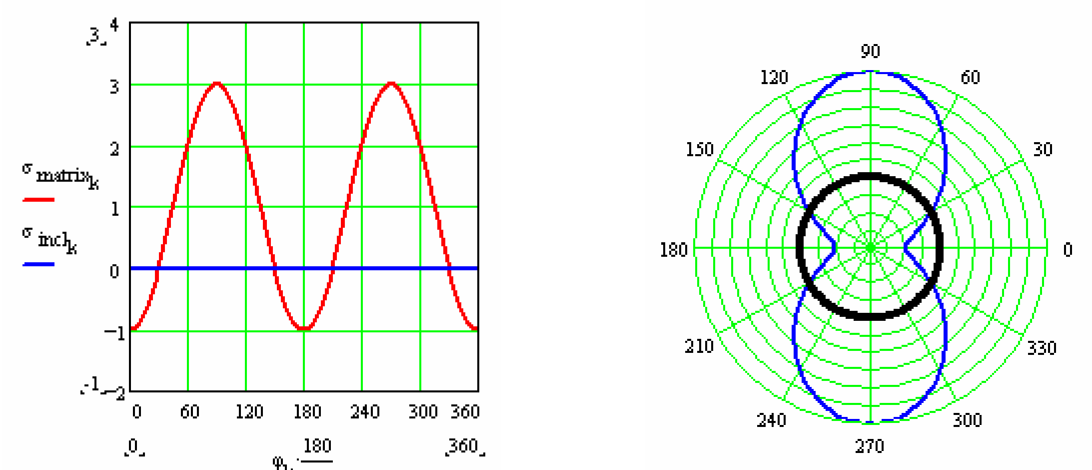
For photoelastic studies, the maximum shear stress is needed as it allows the plotting of the isochromatics patterns and the geometrical loci where principal stresses have constant direction and whose representation gives the isoclinics, as shown by Frocht, [2]. The equation of the isoclinics pattern is:

$$tan[\alpha^{m,o}(z)] = \frac{2\tau_{xy}^{m,o}(z)}{\sigma_x^{m,o}(z) - \sigma_y^{m,o}(z)}. \quad (15)$$

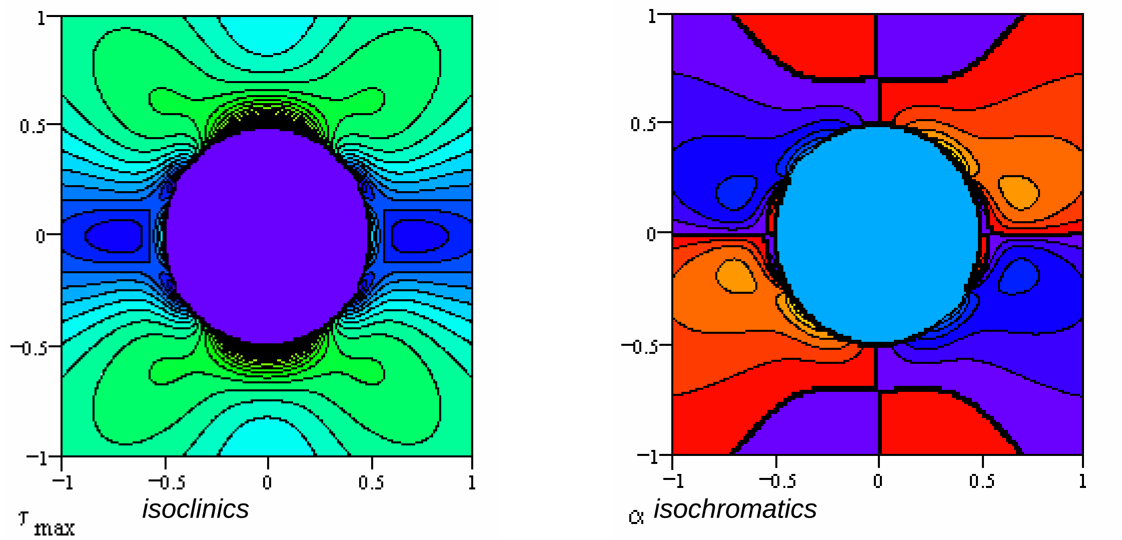
4. THEORETICAL AND EXPERIMENTAL RESULTS

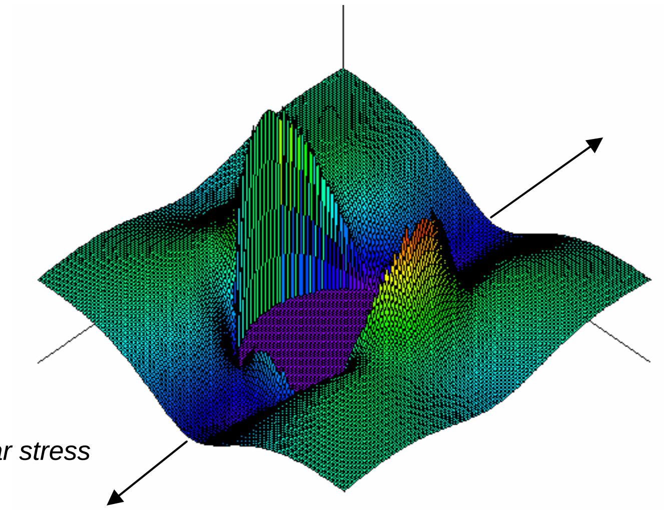
Next, the variation of the hoop stress σ_θ is presented, at the limit between the inclusion (blue line) and dental matrix (red line), the isoclinics and isochromatics patterns and the 3D graphical representation of the maximum shear stress.
The stretching load is $p = 1$ and thus, on the contour of the junction, the stress concentration factor is the hoop stress, [3].

a) Plane matrix with circular hole uni-axial stretched at infinity, (Kirsch's problem), Fig. 2, Rekach, [4].
$E_m = 2{,}5 \cdot 10^9 Pa, \nu_m = 0.38,\ E_0 = 0, \nu_0 = 0$.



hoop stress around matrix –inclusion junction





principal shear stress

Fig.2 Holed plane, $E_m = 2{,}5 \cdot 10^9\, Pa, \nu_m = 0.38,\; E_0 = 0, \nu_0 = 0$

b) Matrix stiffer than the inclusion, Fig. 3:

$E_m = 2{,}5 \cdot 10^9\, Pa, \nu_m = 0.38,\; E_0 = 10^9\, Pa, \nu_0 = 0.3$.

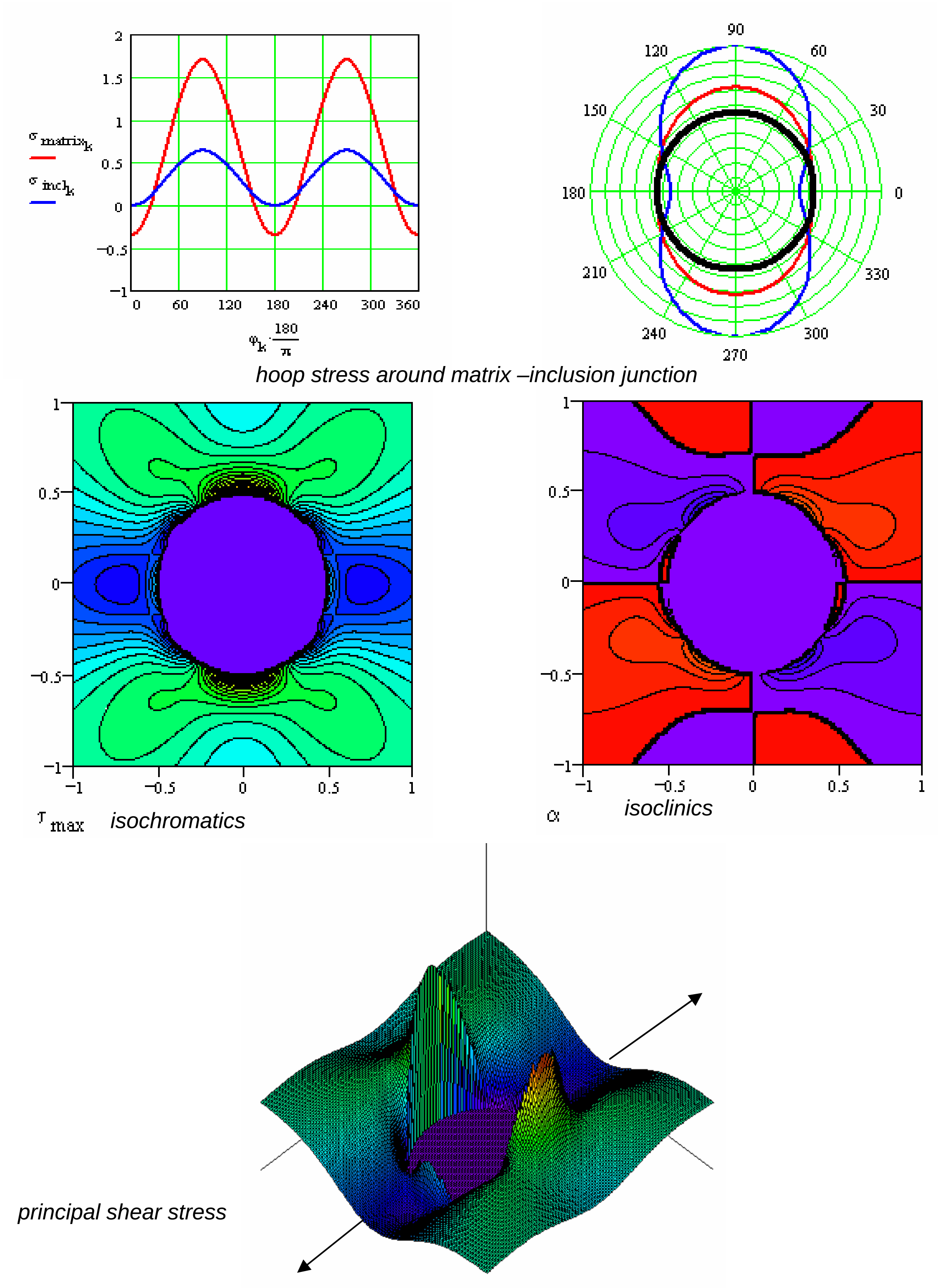


Fig.3. Elastic inclusion, $E_m = 2{,}5 \cdot 10^9 \, Pa, \nu_m = 0.38, \; E_0 = 10^9 \, Pa, \nu_0 = 0.3$

c) Elastic inclusion stiffer than the matrix, Fig.4:
$E_m = 2.5 \cdot 10^9 \, Pa, \nu_m = 0.38, \; E_0 = 5 \cdot 10^9 \, Pa, \nu_0 = 0.3$.

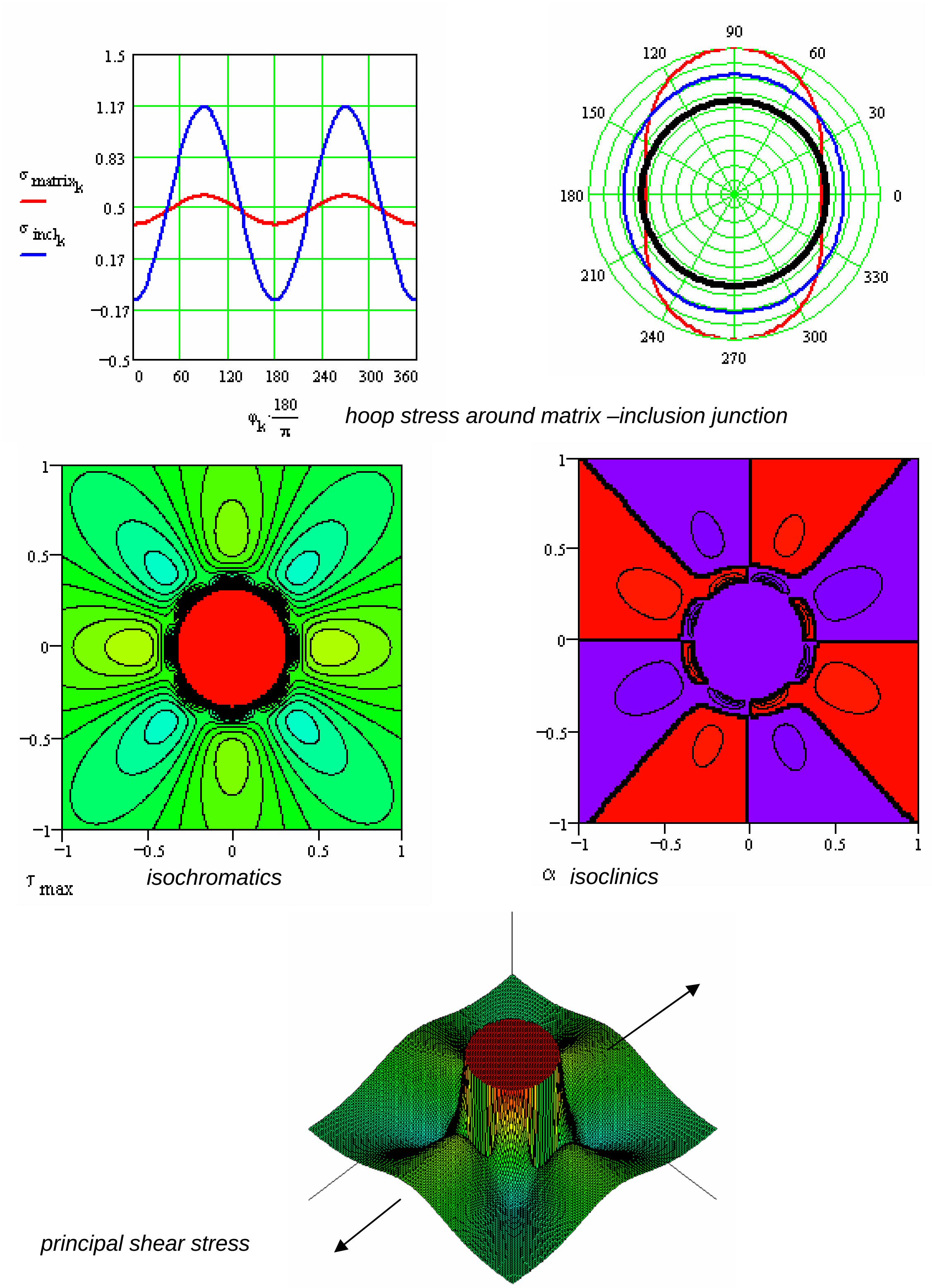


Fig.4. Elastic inclusion $E_m = 2.5 \cdot 10^9\, Pa, \nu_m = 0.38,\;\; E_0 = 5 \cdot 10^9\, Pa, \nu_0 = 0.3$

d) Rigid inclusion, Fig.5:

$E_m = 2.5 \cdot 10^9\, Pa, \nu_m = 0.38,\;\; E_0 = 10^{20}\, Pa, \nu_0 = 0.3$.

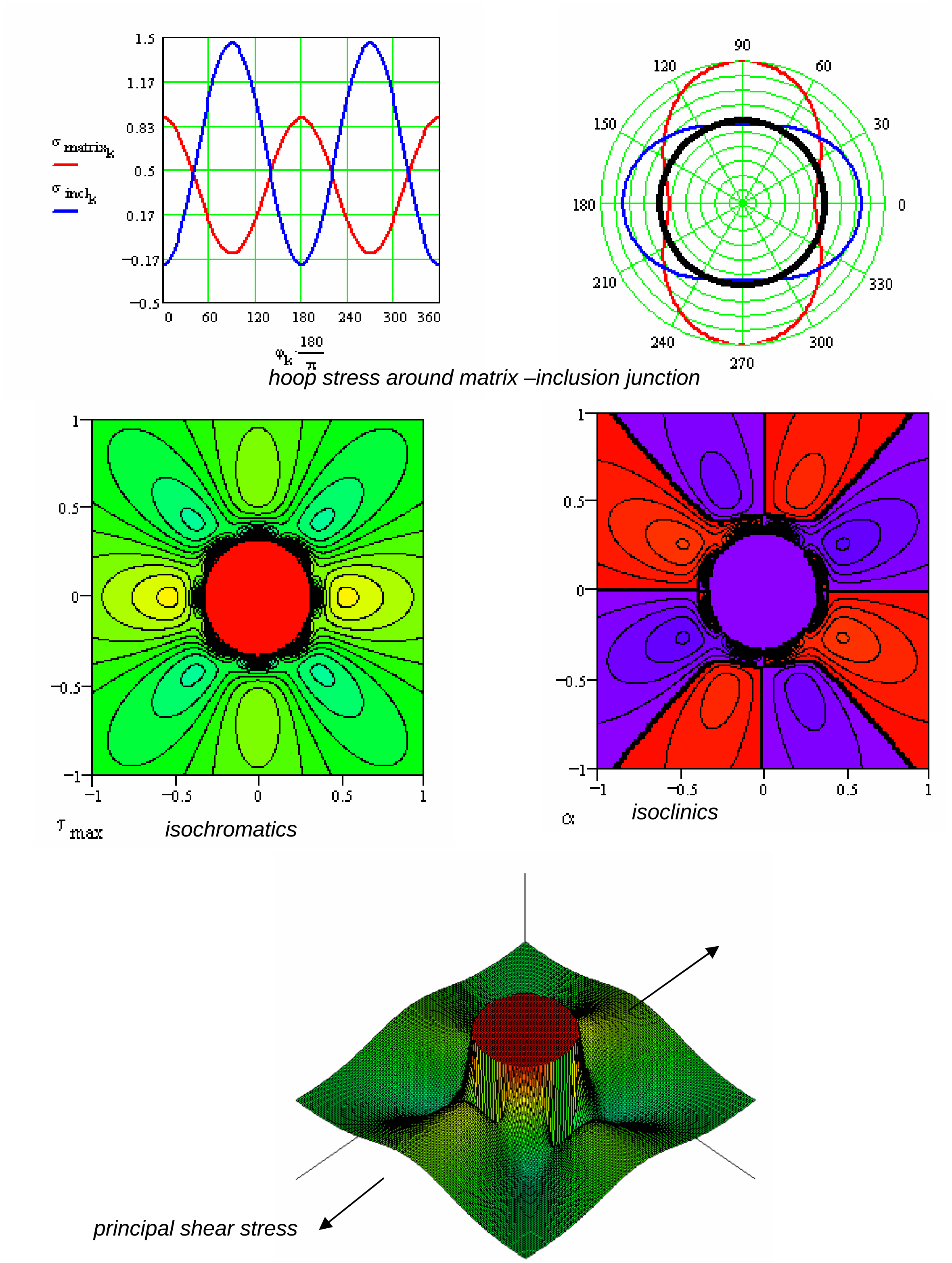


Fig.5. Rigid inclusion, $E_m = 2.5 \cdot 10^9 \, Pa, \nu_m = 0.38, \; E_0 = 10^{20} \, Pa, \nu_0 = 0.3$

In order to validate the theoretical results, photoelastic techniques were applied. A plate made of photoelastic material, uni-axially stretched, with a hole filled with steel inclusion bonded to the hole, was used. The photoelastic isochromatic pattern obtained is shown in Fig 6. As assumed, the maximum stresses in matrix appear at the ends of the diameter parallel to the stretching direction. The experimental tests confirm the theoretical results obtained for the matrix stress state. Since the inclusion is made of opaque material, the stress state cannot be shown via photoelasticity, but can be obtained analytically by the relations deduced above.

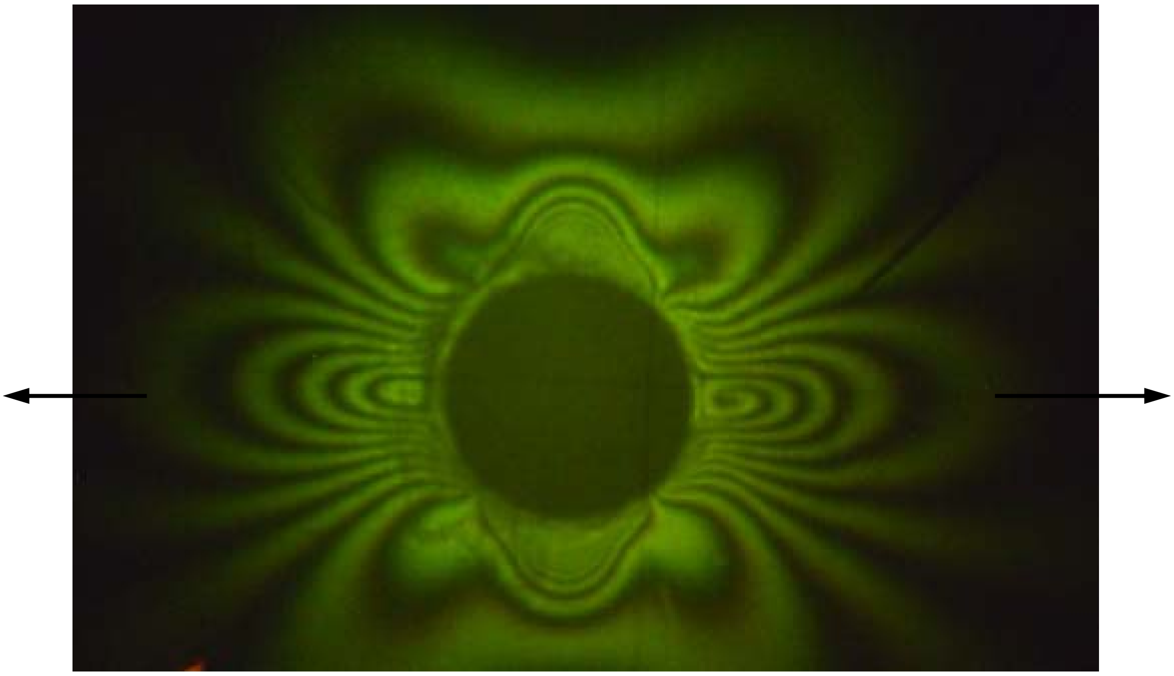


Fig.6. Experimental isochromatic pattern

CONCLUSIONS

1. The maximum stress occurs on the contact between the dental matrix and the inclusion, for any elastic parameters of the materials.
2. The stress in the filling has a less pronounced variation compared to the variation in the matrix.
3. For a stiffer matrix than the inclusion, the maximum stress occurs in the matrix; when the filling is stiffer than the matrix, the maximum stresses arise in the filling.
4. The most favourable situation is when the elastic parameters of the filling are visibly greater than the matrix elastic parameters. In this case, the extreme hoop stresses from dental matrix and inclusion present the lowest amplitude.

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