**ANNALS of the ORADEA UNIVERSITY.**

**Fascicle of Management and Technological Engineering, Volume VII (XVII), 2008**

# ON THE ADVANTAGES OF INVOLUTES ASYMMETRICAL TEETH BY COMPARISON WITH THE INVOLUTE SYMMETRICAL TEETH AT SPUR GEARS

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**Key words:** asymmetric gear, asymmetric involute profiles, bending stress, contact stress, efficiency

**Abstract:** A method for increasing the performances of the transmission of involute gears is to use the asymmetric profiles in order to obtain better function parameters for the active profile. This paper is focused on the comparison of the quality of asymmetric teeth gears with those with symmetrical teeth. For emphasize the advantage of the asymmetrical gears it has been developed an application in Matlab for determine the values and compare in the same graphical representation some parameters of the booth cases of spur gears transmission. It has been performed the comparison between asymmetrical gears with different coefficient of asymmetry and with the symmetric gears generated with the gear rack with the 20 degrees profile angle.

## 1 INTRODUCTION

Asymmetric gears characterized by asymmetric involute profiles of the tooth are used only in the last few years. Because the great numbers of geometrical parameters, in relation with the classical gears, designing and manufacture those special gears it is more difficult. By using the computational method this inconvenient can be solved without significant supplementary costs. There are authors that emphasize the advantage of using gears with bigger mesh angle on the active flank, having the coefficient of asymmetry bigger than the unit. Others emphasize the advantage of using smaller mesh angle on the active flank, having the coefficient of asymmetry smaller than the unit. The coefficient of asymmetry is defined as the ratio between the base circle diameter of the inactive flank and the base circle diameter of the active flank [4], [5].

In this paper authors approach it is considered that the booth variant of asymmetric gears have specific advantages. The variant of using of an asymmetric gear must be established taking in consideration the evaluation criteria chose by the beneficiary. Depending on the profile used as active profile, there are different forces of incidence to the profile of the tooth, a different bending stress, contact stress, displacements and efficiency.

An asymmetric gear with a certain geometry established in the first stage of the design, can be used in two ways which, from a functional point of view, represent two different gears. In this paper are analyzed booths: the “direct asymmetric gears”, with the asymmetry coefficient higher than 1; the “inverted asymmetric gears”, with the asymmetry coefficient lower than 1. Booth cases are compared with the symmetrical one, which can be considered a particular case of asymmetric gear having the coefficient of asymmetry equals to the unit.

For booth cases the profile with a high gearing angle is called “direct profile” and the profile with a low gearing angle is called “inverted profile”. Considering those names if the direct profile is the active one there are speaking about direct gear and if the inverted gear is the active one there are speaking about inverted gears [1], [2].

In order to have the possibility to compare two gears that from geometrical point of view have the same level of asymmetry, but with different asymmetry coefficient depending of the choosing of the active flank, it has been defined the degree of asymmetry:

$$As = \cos\alpha_{wi} / \cos\alpha_{wd} - 1 \quad (1)$$

where $\alpha_{wd}, \alpha_{wi}$ are the mesh angles on the direct respectively inverted profiles.

## 2. THE FUNCTIONAL PARAMETERS USED AS QUALITY INDICATORES

The behaviour of the designed asymmetric gear under the load one can rapid evaluate by the calculus algorithms and the MATLAB applications, many routines that permit to approach the following aspects [3]:

- Mathematical modelling the meshing of the asymmetric gears by determining the profile angles for the pinion tooth and for the gear tooth for an established number of contact points.
- The elasticity, implicitly the rigidity, of the asymmetric tooth and of the pairs of teeth in contact and the variation for these parameters during the meshing cycle.
- On the base of the tooth elasticity it can be solved the statically unknown problem of load distribution between the two pairs of teeth in meshing and so it was determined the diagram of variation of the normal force.
- The relative sliding speed and the variation during the meshing period.
- The instantaneous power loses the variation of the instantaneous efficiency and than the medium efficiency for a meshing period.
- The bending stress at the bottom of the tooth in relation with the number of the contact point. The variation during the meshing cycle of the bending stress it was so resulting.
- The contact stress has been determined also for every the contact point and represented the diagram of variation for a meshing cycle.

In this paper the comparison between the asymmetric and symmetric gears will be performed using as quality indicators the contact stress, the bending stress and the medium efficiency. Decreasing the bending stress and contact stress and increasing the efficiency are multiple objectives for any toothed gears transmission design.

The contact stress has been determined as a function depending on the pressure angle corresponding to the contact point on the pinion active profile:

$$\sigma_{Hjd,i} = \sqrt{\frac{0{,}3 \cdot E_1 \cdot E_2 \cdot F_{njd,i} \cdot tg\alpha_{wd,i} \cdot (1+u)}{(E_1+E_2) \cdot b \cdot r_{b1d,i} \cdot tg\alpha_{j1d,i} \cdot \left[ tg\alpha_{wd,i} \cdot (1+u) - tg\alpha_{j1d,i} \right]}}, \quad (2)$$

where:

$E_1, E_2$ are the elasticity modules of the pinion and gear materials;

$F_{njd,i}$ $(F_{njd}, F_{nji})$ is the normal force on the tooth profile corresponding to the “j” point of contact (for direct gear, for inverted gear);

$u = z_2 / z_1$ is the ratio of the number of teeth of the gear and pinion;

$r_{b1d,i}$ $(r_{b1d}, r_{b1i})$ is the radius of the base circle of the active profile of the pinion (for direct gear, for inverted gear);

$\alpha_{j1d,i}$ $(\alpha_{j1d}, \alpha_{j1i})$ is the pressure angle corresponding to the “j” contact point on the pinion active profile.

The relation 2 permit to calculate the contact stress for any contact point of the pinion profile, implicitly of the gear, defined by the parameter $\alpha_{j1d,i}$, which vary in the range $(\alpha_{p1d}, \alpha_{a1d})$ for direct gear and for inverted gear in the range $(\alpha_{p1i}, \alpha_{a1i})$. The notations $(\alpha_{p1d,i}, \alpha_{a1d,i})$ are referring to the pressure angles corresponding to the first point of contact from the bottom of the tooth, respectively to the point of contact from the addendum circle of the pinion. The maximum values of the mentioned function of variation of the contact stress, during the time of the meshing cycle it is an important performance indicator.

The bending stress at the bottom of the tooth it has been determined considering the tooth as a short beam fixed in the gear body. It was established a method for obtain the maximum cross section of the tooth, because those used for the symmetrical gears can't be applied.

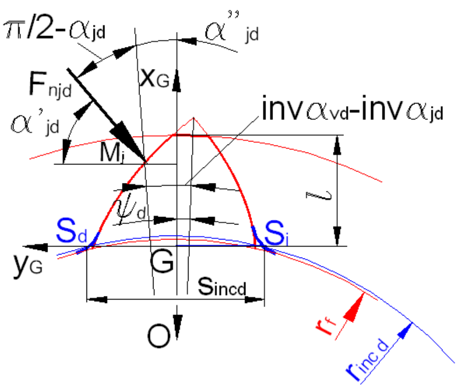



*Fig. 1 Load position for direct gear*

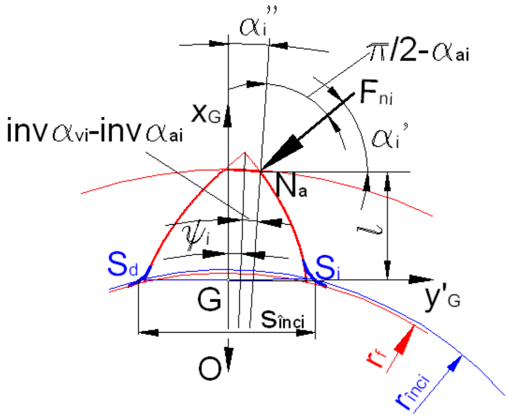



*Fig. 2 Load position for inverted gear*

In the figure 1 is represented the position of load for the j point of contact on the direct profile for the directed gear. With the notation from the figure are determined the efforts in the maximum cross section corresponding to the j contact point:, axial force, shear force and bending moment:

$$N_{jd} = F_{njd} \cdot \sin\alpha'_{jd} \; ; \quad (3)$$

$$T_{jd} = F_{njd} \cdot \cos\alpha'_{jd} \; ; \quad (4)$$

$$M_{ijd} = F_{njd} \cdot \cos\alpha'_{jd} \cdot x_G^{Mj} - F_{njd} \cdot \sin\alpha'_{jd} \cdot y_G^{Mj} \, . \quad (5)$$

The normal stresses result by the addition of the axial force and bending moment:

$$\sigma_{jd-} = -\sigma_{M_{ijd}} - \sigma_{Njd} = -\frac{M_{ijd}}{b \cdot s_{\hat{i}ncd}^{2}/6} - \frac{N_{jd}}{b \cdot s_{\hat{i}ncd}} \; ; \quad (6)$$

$$\sigma_{jd+} = \sigma_{M_{ijd}} - \sigma_{Njd} = \frac{M_{ijd}}{b \cdot s_{\hat{i}ncd}^{2}/6} - \frac{N_{jd}}{b \cdot s_{\hat{i}ncd}} \, . \quad (7)$$

The shear stress and the equivalent stress resulting from:

$$\tau_{jd} = T_{jd} / b \cdot s_{\hat{i}ncd} \, , \quad (8)$$

$$\sigma_{ejd} = \sqrt{\left(\sigma_{jd+}\right)^2 + \left(2{,}5 \cdot \tau_{jd}\right)^2} \, . \quad (9)$$

where $s_{\hat{i}ncd}$ is the dimension in the frontal plane of the gear of the maximum cross section.

In the figure 2 it is represented the load on the inverted profile considering the entire load applied on the exterior point of the profile, $N_a$. In the first stage of the research it has been used this approximation, but considering that the accuracy it is not enough the efforts have been determined with the relations:

$$N_{ji} = F_{nji} \cdot \sin\alpha'_{ji} \; ; \quad (10)$$

$$T_{ji} = F_{nji} \cdot \cos\alpha'_{ji} \; ; \quad (11)$$

$$M_{iji} = F_{nji} \cdot \cos\alpha'_{ji} \cdot x_G^{Nj} - F_{nji} \cdot \sin\alpha'_{ji} \cdot y'^{Nj}_G \, . \quad (12)$$

where are used the normal force and the geometrical parameters for the Nj point of contact with the number j, on the inverted profile that this time is the active one.
In the figures 3 and 4 are given examples of variation diagrams for the contact stress and bending stress to the pinion tooth respectively the gear tooth.

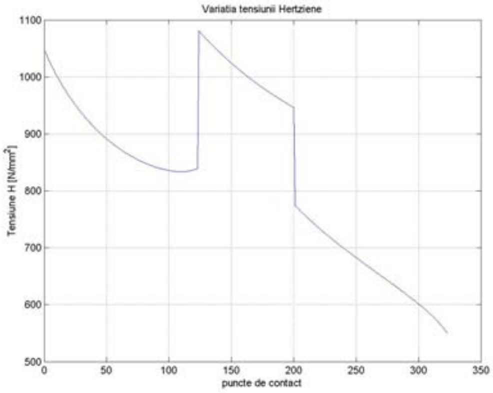



*Fig. 3 Variation of the contact stress*

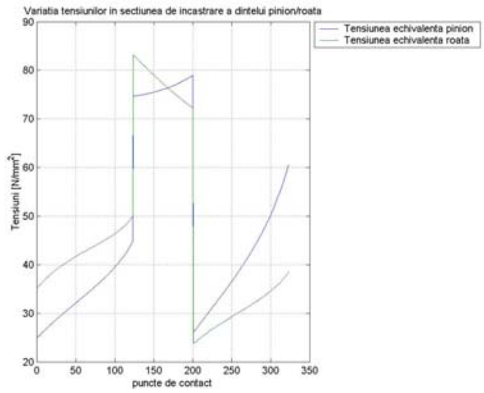



*Fig. 4 Variation of the bending stress*

The comparison of the symmetric and asymmetric variants or solutions it is made on the base of the maximum value of the contact stress, maximum value of the bending stress at the pinion tooth and gear tooth, that can result during the function.

The power losses by friction is equals with the multiplication of the friction force with the relative sliding speed of the mating flanks of teeth which are in contact:

$$P_{fj} = \mu \cdot F_{n\,jd} \cdot \Delta v_j = \mu \cdot F_{n\,jd} \cdot r_{b1d} \cdot \left|\tan\alpha_{wd} - \tan\alpha_{j1d}\right| \cdot (\omega_1 + \omega_2) \qquad (13)$$

The active power is the multiplication of the moment of the driving wheel and the angular speed:

$$P_j = M_1 \cdot \omega_1 = \left(F_{njd} \cdot r_{b1d} - \mu \cdot F_{njd} \cdot r_{b1d} \cdot \tan\alpha_{j1d}\right) \cdot \omega_1 . \qquad (14)$$

Knowing the power losses due to friction forces and the active power, there can be determined the instantaneous power losses and the instantaneous efficiency [3]:

$$\psi_{fj} = P_{fj} / P_j ; \quad \eta_j = 1 - \psi_{fj} . \qquad (15)$$

The instantaneous power losses, determined by the corresponding pressure angle $\alpha_{j1d}$ on the pinion tooth profile and the angle of mesh on the direct profile $\alpha_{wd}$, results:

$$\psi_{fj} = \left|\mu \cdot \left(\tan\alpha_{wd} - \tan\alpha_{j1d}\right) / \left(1 - \mu \cdot \tan\alpha_{j1d}\right)\right| \cdot (1 + z_1 / z_2) . \qquad (16)$$

In the case of two pair of teeth in contact:

$$P_{fj} = \mu \cdot r_{b1d} \cdot (\omega_1 + \omega_2)\left(F_{njd(1)} \cdot \left|\tan\alpha_{wd} - \tan\alpha_{j11d}\right| + F_{njd(2)} \cdot \left|\tan\alpha_{wd} - \tan\alpha_{j12d}\right|\right), \qquad (17)$$

$$P_j = r_{b1d} \cdot \omega_1 \left(F_{njd(1)} \cdot \left(1 - \mu \cdot \tan\alpha_{j11d}\right) + F_{njd(2)} \cdot \left(1 - \mu \cdot \tan\alpha_{j12d}\right)\right) \qquad (18)$$

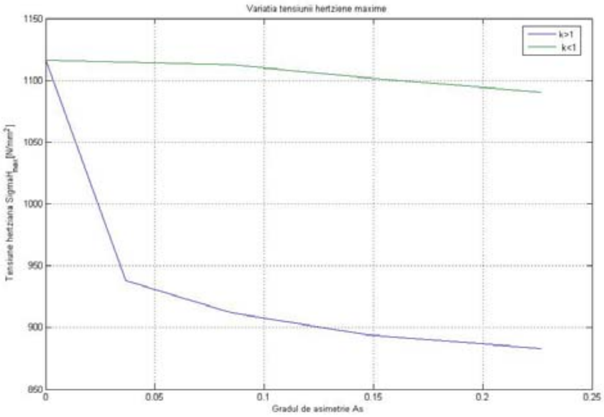
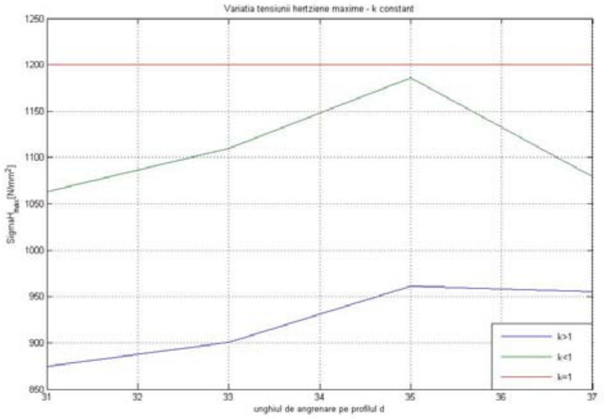
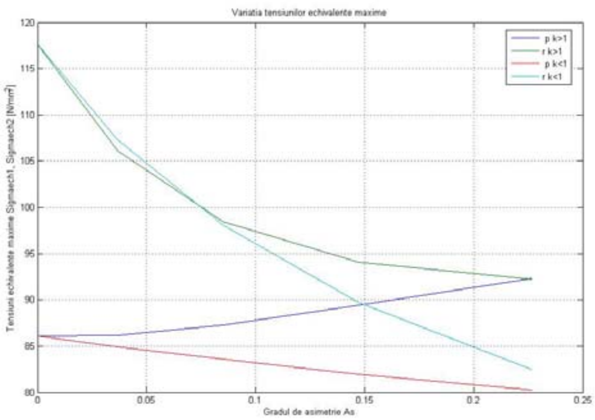
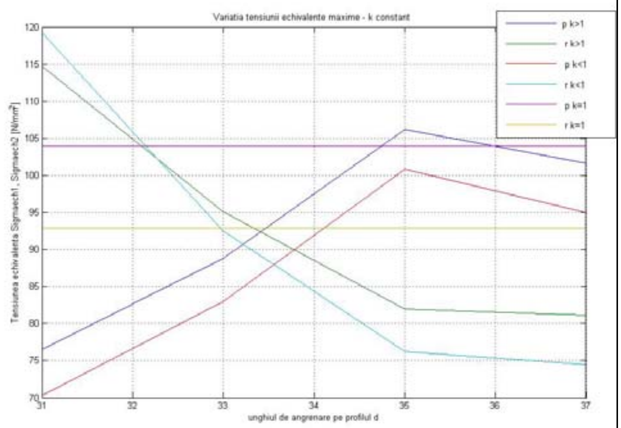
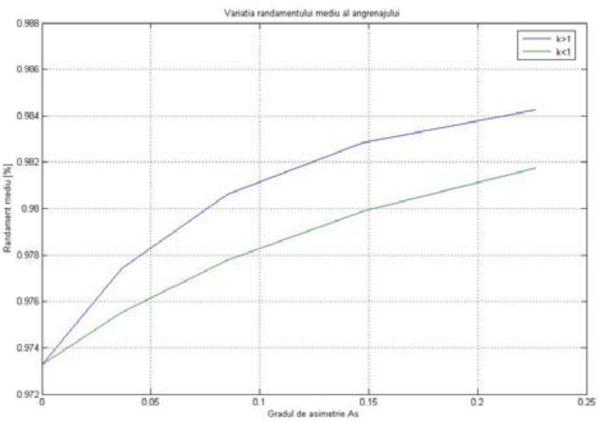
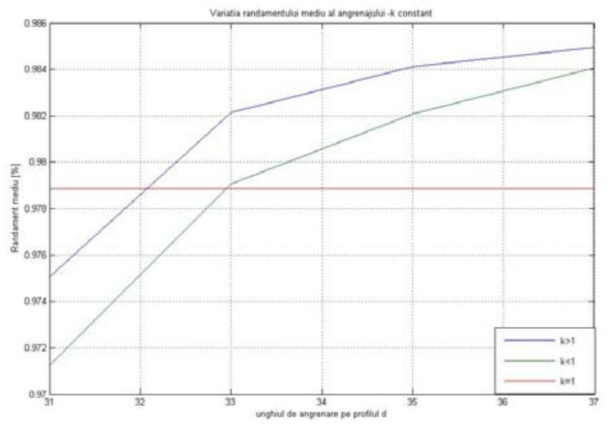
where $F_{njd(1)}$ is the normal force to the direct profile of the tooth which acts on the first pair of teeth, and $F_{njd(2)}$ is the normal force to the direct profile of the tooth which acts on the second pair of teeth.

If there are been calculated the instantaneous power losses for "n" points of contact, the length of the base pitch it is divided by the number "n", and so the medium power losses and the medium efficiency for a meshing period result:

$$\psi_f = \left(\sum_{j=1}^{n} \psi_{fj}\right) / n \ ; \ \eta = 1 - \psi_f \qquad (19)$$

## 3. THE COMPARISON BETWEEN ASYMMETRIC AND SYMMETRIC GEARS

***Table1 Variation diagrams of the functional parameters***

<table>
<tr><td>Variable coefficient of asymmetry</td><td>Constant coefficient of asymmetry</td></tr>
<tr><td>$\sigma_H = \sigma_{Hmax}(As)$</td><td>$\sigma_H = \sigma_{Hmax}(\alpha_{wd})$</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$\sigma_{ech} = \sigma_{echmax}(As)$</td><td>$\sigma_{ech} = \sigma_{echmax}(\alpha_{wd})$</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$\eta = \eta_{med}(As)$</td><td>$\eta = \eta_{med}(\alpha_{wd})$</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

First have been analysed gears with different coefficients of asymmetry, which geometrical parameters are given in table 2. The results are presented in the first column of the table 1. Second have been analysed asymmetric gears having constant coefficient of asymmetry but with different mesh angles on the asymmetric profiles, which geometrical parameters are given in table 3. Those have been compared with the symmetric gear generated with the gear rack with 20 degrees profile angle. The results are presented in the second column of the table 1. The values obtained for the symmetric gear there are represented as horizontal line for easy comparing with those obtained for different asymmetric gears.

***Table 2 Asymmetric gears with different coefficient of asymmetry***

| Gear | 1 | 2d/2i | 3d/3i | 4d/4i | 5d/5i |
|---|---|---|---|---|---|
| Initial date | $z_1 = 16$ ; $z_2 = 57$ ; $a = 120 mm$ | | | | |
| $\alpha_{wd}$ (degrees) | 20 | 25 | 30 | 35 | 40 |
| $\alpha_{wi}$ (degrees) | 20 | 20 | 20 | 20 | 20 |
| $k \geq 1$ Direct asymmetric gears | 1 | 1,0368 | 1,0850 | 1,1471 | 1,2266 |
| $k \leq 1$ Inverted asymmetric gears | 1 | 0,9645 | 0,9216 | 0,8717 | 0,8152 |
| As | 0 | 0,0368 | 0,0850 | 0.1471 | 0,2266 |

***Table 3 Asymmetric gears with the same coefficient of asymmetry but different mesh angles***

| Gear | 1 $\alpha_{dc} = 20$ | 2d/2i | 3d/3i | 4d/4i | 5d/5i |
|---|---|---|---|---|---|
| Initial date | $z_1 = 16$ ; $z_2 = 57$ ; $a = 120 mm$ ; $k = 1{,}12 \geq 1$ ; $k = 0{,}89 \leq 1$ | | | | |
| $\alpha_{wd}$ (degrees) | 23 | 31 | 33 | 35 | 37 |
| $\alpha_{wi}$ (degrees) | 23 | 16.25 | 20.06 | 23.44 | 26.55 |

## 4 CONCLUSIONS

The study performed for variable coefficient of asymmetry emphasize that the performances are improved for booth variant of asymmetric gear in relation with the classical one by increasing the degree of asymmetry.

From the variation diagrams obtained for the gears with the same coefficient of asymmetry, it can be observed that for great values of the mesh angle on the direct profile the parameters are better for the asymmetric gears. It is interesting that with asymmetric gears the bending stress on the pinion tooth and on the gear tooth can be balanced.

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