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THE STUDY OF THE STRESS AND DISPLACEMENTS AT THE DIRECT ASYMMETRIC GEARS IN RELATION WITH THE COEFFICIENT OF ASYMETRY USING THE FINITE ELEMENTS METHOD

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Key words: asymmetric gear, asymmetric tooth, coefficient of asymmetry, stress, displacements

Abstract: The aim of this paper is to presents the analysis of the influence on the maximum values of stress and displacements of the asymmetric teeth in meshing, in relation with the coefficient of asymmetry. There are analyzed five gears, with different coefficient of asymmetry, using the 2D models of the asymmetric gears, obtained in AUTOCAD MECHANICAL with a generating program in AUTOLISP. The coefficient of asymmetry is different because the gears analyzed have been designed for the same center distance, numbers of teeth, the same mesh angle on the inverted profile but different mesh angle on the direct profile.

1 INTRODUCTION

The asymmetric gears formed of toothed gears with asymmetric involutes teeth are used with the aim to improve the performances, already known, of the classical involutes gears [8], [9]. The direct asymmetric gears are characterized by bigger pressure angles on the profile of the active flank, in relation with the inactive one, and values of the coefficient of asymmetry bigger than one. The coefficient of asymmetry is defined as the ratio between the base circle diameter of the inactive profile and the base circle diameter of the active profile. These gears have the advantage of smaller stresses and displacements of the teeth in the meshing zone in comparison with the classical symmetric involutes gears. It is necessary to be analyzed the influence of the coefficient of asymmetry on the values of stress and displacements for ensure to the designing engineer all the date necessary for choosing the grade of asymmetry for obtain the optimal performances. The study with the finite elements method need in the first stage to determine the geometrical parameters and the 2D models of all gears that are the object of the analysis [6], [7], [10]. There are used, for generating the models and for verify the results, parameters obtained with a packet of Programs for Designing, Modeling and Analyzing Asymmetric Gears, developed as applications in MATLAB [4].

2. THE GEOMETRICAL MODEL

For representing the asymmetric gears components of the transmission, have been developed two applications in AUTOLISP. For determiner the geometrical parameters, some of theme being initial date at 2D model generation, it has been used the developed modeling program for the asymmetric gears. [2], [3], [4], [5]. Also it was necessary to determine the dimension, in frontal plane, of the surface of contact, the pressure angles and the diameters corresponding to the extreme points of the segment of line of action with one pair of teeth in contact.

Than has been separated, for the pinion and also for the gear, the close contour limited by the asymmetric profiles of the tooth and the segment between the two points of the filet profiles situated on the dedendum circles. By this segment is represented the maximum cross section of the tooth, considered as a fixed beam in the body of the gear (figure1).

The elliptic distribution of the pressure of contact it has been replaced with a step distribution because the routine 2D FEA of AUTOCAD program admit only uniform distribution of the loads (figure 2). Also the routine 2D FEA of AUTOCAD program considers that the pressure of contact is constant on the direction perpendicular on the frontal plane, in which the analysis has been performed.

The width of the gears, which is perpendicular on the frontal plane in which is performed the finite element analysis, it has been b = 30 mm. The same value was used in the calculus of the stress and displacements performed with the MATLAB application. There has been considered as material of the gears, which has been choosing from the material tables of the FEA routine, 31CrMo12.

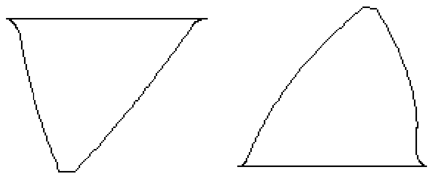


Fig. 1 Determination of the geometrical model

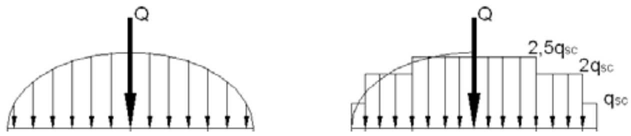


Fig. 2 The elliptic / uniform distribution of the load

The maximum values of the stress and displacements corresponding to the extreme points of the segment of the line of action with one pair of teeth in contact, the load it has been applied on the corresponding profile points, to the point Bd for the tooth of the gear, and to the point Dd for the tooth of the pinion (considering Ad, Bd, C, Dd, Ed the significant points of the line of action for the direct profile).

The most important geometrical parameters and a part of the load values used for established the geometrical model and the distribution of the load are shown in the table 1.

Table 1 Date necessary for the finite elements analysis determined with the application in Matlab

$z_1 = 16$; $z_2 = 57$; $a = 120\,mm$; $\alpha_{wi} = 20^o$; $f = 0$ ($\alpha_{wd} = \alpha_{dc}$), different gear rack generation					
Gears	1 (k=1,23)	2 (k=1,15)	3 (k=1,085)	4 (k=1,04)	5 (k=1)
α_{wd} [grade]	40	35	30	25	20
α_{wd} [rad]	0.6981	0.6109	0.5236	0.4363	0.3491
α_{wi}, α_{ic} [rad]	0.3491				
α_{a1d} [rad]	0.8471	0.7990	0.7576	0.7234	0.6969
α_{a2d} [rad]	0.7288	0.6483	0.5696	0.4932	0.4207
α_{dc} [rad]	0.6981	0.6109	0.5236	0.4363	0.3491
X_1 [mm]	1.0600	1.3900	1.7800	2.2500	2.8200
X_2 [mm]	-1.0600	-1.3900	-1.7800	-2.2500	-2.8200
d_{b1d} [mm]	40.2960	43.0896	45.5553	47.6743	49.4304
d_{b1i} [mm]	49.4304				
d_{b2d} [mm]	143.5546	153.5068	162.2908	169.8396	176.0958
d_{b2i} [mm]	176.0958				
d_{1Bd} [mm]	50.1075	51.0406	52.0593	53.0997	54.0907
d_{1Dd} [mm]	58.1634	57.6418	56.8250	55.7302	54.4010
d_{2Bd} [mm]	190.0128	189.0255	187.9519	186.9135	186.0758
d_{2Dd} [mm]	182.2775	182.8637	183.6842	184.6926	185.8337
P=18kW Q[N]	8531.9	7978.7	7546.9	7211.4	6955.2
P=30kW Q[N]	14220	13298	12578	12019	11592

3. STRESSES AND DISPLACEMENTS OBTAINED WITH THE FINITE ELEMENTS METHOD

For one can evaluate, using the finite elements method, the influence of the asymmetry coefficient on the values of stresses and displacements of the asymmetric teeth, it has been effected the analysis for five gears with coefficients of asymmetry bigger than one for a two values of the transmitted power 18 kW, respectively 30 kW:
A_as_16_57_120_40_20_0_2_1, A_as_16_57_120_35_20_0_2_1, A_as_16_57_120_30_20_0_2_1, A_as_16_57_120_25_20_0_2_1, A_as_16_57_120_20_20_0_2_1 (table1).
The code of the gear indicate the initial date used in the design program: the numbers of teeth (16, 57), the centre distance (120), the mesh angles, in degrees, for the asymmetric involutes profiles (40, 20 for the first gear), the angles of the gear rack profiles equals, in the analyzed cases, with the mesh angles (0), the generation of pinion and gear with different gear rack (2) and the last number indicate the variant of using the asymmetric gear (1) for the direct asymmetric gears.

The tension Von Mises is an equivalent stress that results from the normal and shear components of the stress, components that determine the stresses estate in one point, node, of the stressed solid:

$$\mathrm{VON} = \sqrt{\frac{1}{2}\left[(\mathrm{SX}-\mathrm{SY})^2+(\mathrm{SX}-\mathrm{SZ})^2+(\mathrm{SY}-\mathrm{SZ})^2\right]+3\left(\mathrm{TXY}^2+\mathrm{TXZ}^2+\mathrm{TYZ}^2\right)},$$

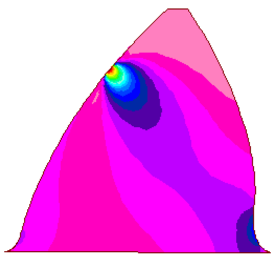
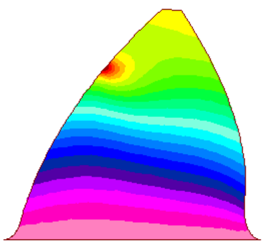
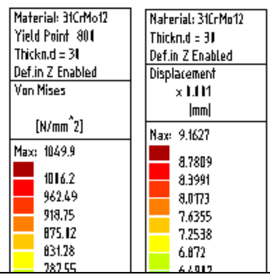
where: VON - is Von Mises equivalent stress;

$\mathrm{SX}, \mathrm{SY}, \mathrm{SZ}$ - are normal stresses on the $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ directions;

$\mathrm{TXY}, \mathrm{TXZ}, \mathrm{TYZ}$ - are shear stresses on $\mathrm{XY}, \mathrm{XZ}, \mathrm{YZ}$ planes.

Although the Von Mises equivalent stress in one node, being a value with any associate direction, can't define uniquely the stress estate in the node, for ductile materials it can be used for evaluate the level of stress. The displacements determined with the finite elements method are also resultant values. [11]. It has been obtained, for each analyzed gear, results with the form of those presented in the 2, 3 and 4 tables: the distributions of stresses, the distributions of displacements, the mesh type and the values associated with the used colors and the maximum values of the stress and displacements.

Table 2 The results of the pinion tooth analysis of the gear A_as_16_57_120_35_20_0_2_1, (30kW)

The exterior point of contact on the line of action segment with one pair of tooth in contact ($\mathrm{D_d}$)		
Stress Von Mises	Displacements	
		

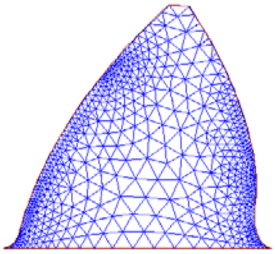
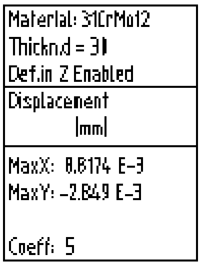
The mesh type		
		

Table 3 The results of the gear tooth analysis of the gear A_as_16_57_120_35_20_0_2_1, (30kW)

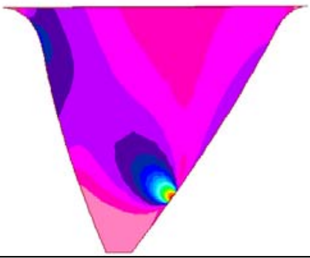
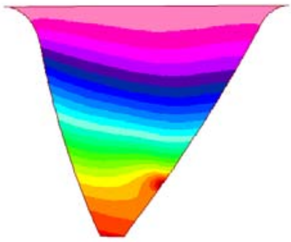
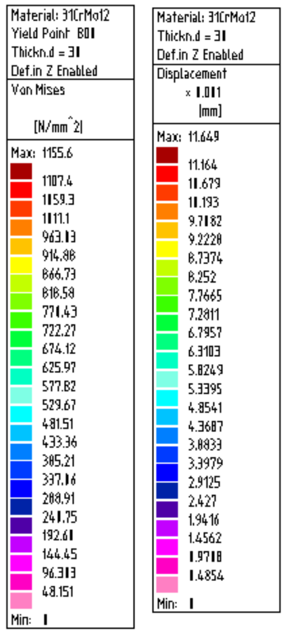
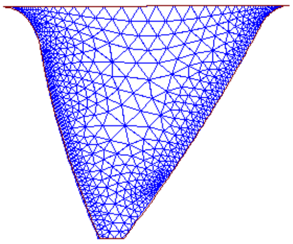
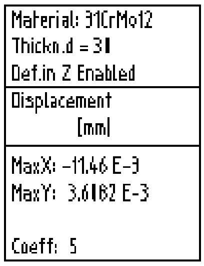
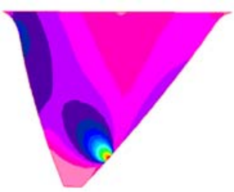
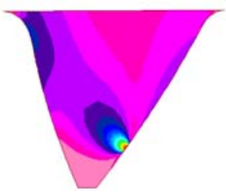
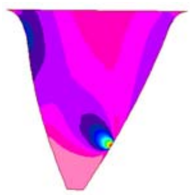
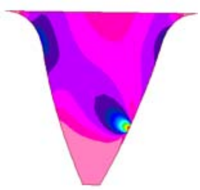
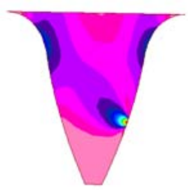
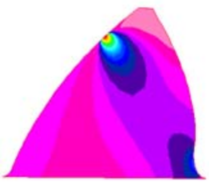
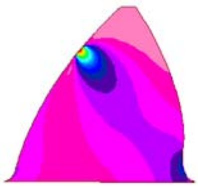
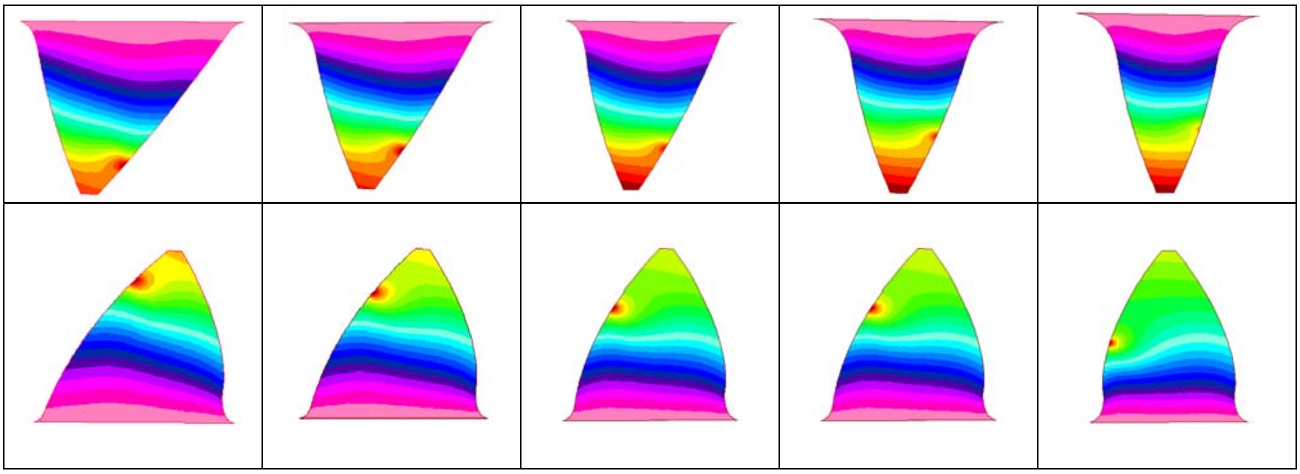
The exterior point of contact on the line of action segment with one pair of tooth in contact (B_d)		
Stress Von Mises 	Displacements 	
The mesh type 		

Table 4 The pinion and gear teeth stress and displacements for points with maximum load

Gear1: 40/20	Gear 2: 35/20	Gear 3: 30/20	Gear 4: 25/20	Gear 5: 20/20
				
				



3. THE COMPARISON OF THE RESULTS OBTAINED WITH DIFFERENT METHODS

The results obtained with the routine FEA 2D of the AUTOCAD program, and also those obtained with the modeling program developed in MATLAB have inexactitudes that accompanies any calculus methods based on some hypothesis that approximate the real phenomena. The comparative analysis of the values obtained with two different method of modeling the study phenomenon is an accessible and without additional costs possibility for verify and confirm the results. Considering the time necessary for perform the analysis one can establish the opportunity of using one or other taking in consideration the aim of research.

In the table 5 are indicated values, carry out by the mentioned calculus method, of the stress and displacements corresponding to the ending points of the meshing segment with one pair of teeth in contact, those being the points where the load are maximal.

Table 5 The values resulted with different methods

Gears A_as_16_57_120...	$\sigma_{H(Ba)}$ $\left[N/mm^2\right]$	$\sigma_{VonMises}$ $\left[N/mm^2\right]$	δ_{1D_Matlab} $\left[mm \cdot 10^{-3}\right]$	δ_{1D_FEA} $\left[mm \cdot 10^{-3}\right]$	δ_{2B_Matlab} $\left[mm \cdot 10^{-3}\right]$	δ_{2B_FEA} $\left[mm \cdot 10^{-3}\right]$
The transmitted power P=18kW						
...40_20_0_2_1	882	893	6.30	6.24	7.40	7.39
...35_20_0_2_1	893	905	5.70	5.60	7.10	7.14
...30_20_0_2_1	912	951	5.10	5.22	7.10	9.17
...25_20_0_2_1	937	969	4.60	4.85	7.40	8.04
...20_20_0_2_1	973	995	4.10	4.59	8.30	10.32
The transmitted power P=30kW						
...40_20_0_2_1	1139	1158	10.30	10.17	12.10	11.87
...35_20_0_2_1	1153	1155	9.40	9.16	11.60	11.64
...30_20_0_2_1	1177	1197	8.40	8.39	11.70	13.40
...25_20_0_2_1	1210	1233	7.50	7.75	12.10	12.52
...20_20_0_2_1	1256	1280	6.60	7.37	13.60	14.42

It can be observe that the results are very nearly, so, from the point of view of accuracy, there are useful the booth method of study. Considering the time necessary for carry out the significant values the finite elements method it is not favorable. For perform all fazes imposed by the finite elements method the time necessary is great.

The values of the stress in the zone of contact are smaller for all the direct asymmetric gear in relation with thee symmetrical one, the stress decrease with the coefficient of asymmetry. For the teeth of the gear the displacements are also smaller to the asymmetrical gears. The bigger values of the displacements of the pinion teeth in meshing indicate that the bending stress is bigger in this case. For the same transmitted power, the same centre distance and meshing ratio, considered as initial date, the normal force to the tooth profile is greater.

The finite elements method analyze confirm that from the point of view of the contact stresses the direct asymmetric gears are superior to the classical symmetrical involutes gears generated with the gear rack with the profile angles equals with 20 degrees. For obtain smaller value of the contact stress must choose a gear with bigger coefficient of asymmetry. If for the aim of the designed transmission it is important also the precision of the transmission it must take in consideration the transmission error that is the result of the displacements of the pinion and of the gear.

6. CONCLUSIONS

For evaluate the variation of stress and displacements, with the coefficient of asymmetry, for the direct asymmetric gear, using the finite element analysis method, it is necessary before to have the 2D models of a number of gears on the base of which it is performed the analysis.

All geometrical parameters can be obtained, with the modeling programs developed as application in MATLAB for the study of the asymmetric gears. For obtain the 2D model, and implicitly the 3D model, of pinion and of the gear, can be used the programs developed in AUTOLISP for AUTOCAD that permit the rapid representation of the asymmetric gears.

The finite element method being verified in many applications can be considered as a method for determine but more for verify and confirm the results obtained with the MATLAB application, that is more efficient and easily to use.

If the aim of the designing engineer is to reduce the contact stress, booth method indicates the advantage of using the direct asymmetric gear with bigger coefficient of asymmetry.

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