THE DETERMINATION OF THE THERMIC FIELD EQUATIONS AT THE LASER DEPOSITION

Florin Ciofu, Alin Nioață
The University „ Constantin Brâncuşi" – Tg.Jiu; florin.ciofu@utgjiu.ro; nalin@utgjiu.ro

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Abstract : By laser direct deposition, the parts are manufactured by injecting powder in a melt metal bath with the help of the laser. During the manufacture it is deployed a complex " thermic history" in the different built regions. This includes re-melting and many thermic cycles at low temperatures (of hundreds of degrees). The phenomena of thermic transmission are variable in time, being by excellence reversible phenomena, too, as the temperature difference which intervenes cannot be ever small infinite. In the most general case, the temperature is a function of space coordinates, and of the time ح.

1. The defining of the bidimensional network of the surface on which it is made the deposition.

The surface on which the laser acts is considered under the form of a node bidimensional net (fig) 1. Attaching a coordinates system Oxy can be established the network constants Δx, and Δy respectively, and each node can be identified by its coordinates (m, n). Around each point it is delimited a control parallelepiped volume, whose section is represented by the pointed rectangle from the figure 1. The dimension rectangular on the figure plan is considered as being unitary.

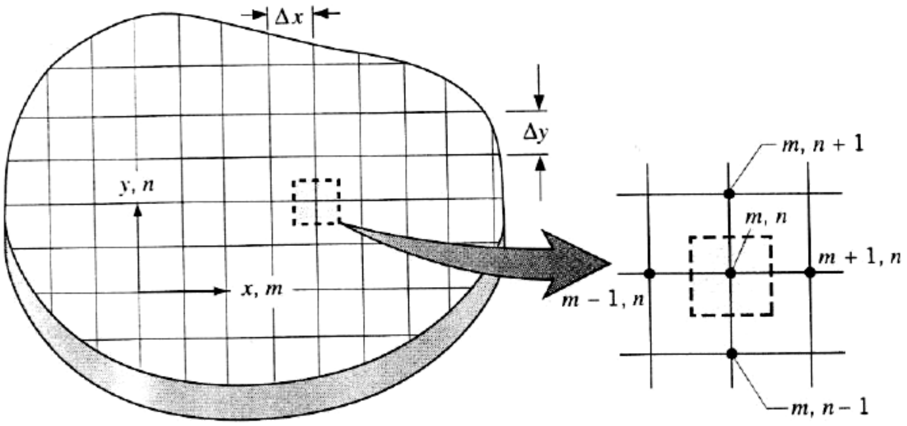

Fig. 1 The node bidimensional network used for modeling of the thermic transfer

2. The case of an interior node

In this case, the temperature of the central node (m, n) is influenced by the temperatures of the 4 nodes situated in the immediate neighborhood of this one (fig.2).

The calculation of the temperature is performed by an iterative process, using as entering data, together with the thermic features of the material, the density of the thermic flux absorbed from the source and the temperature in the point (m,n) at a previous moment. The time steps are noted with the superior index p, p+1 etc.

The thermic fluxes transmitted to the central node from the four neighbor nodes, can be expressed by the formulas:

$$
\begin{aligned}
q_{(m,n+1)\to(m,n)} &= kA\frac{\left(T_{m,n+1}-T_{m,n}\right)}{\Delta y},\ A=\Delta x\cdot 1\\
q_{(m+1,n)\to(m,n)} &= kA\frac{\left(T_{m+1,n}-T_{m,n}\right)}{\Delta x},\ A=\Delta y\cdot 1\\
q_{(m,n-1)\to(m,n)} &= kA\frac{\left(T_{m,n-1}-T_{m,n}\right)}{\Delta y},\ A=\Delta x\cdot 1\\
q_{(m-1,n)\to(m,n)} &= kA\frac{\left(T_{m-1,n}-T_{m,n}\right)}{\Delta x},\ A=\Delta y\cdot 1
\end{aligned}
\tag{1}
$$

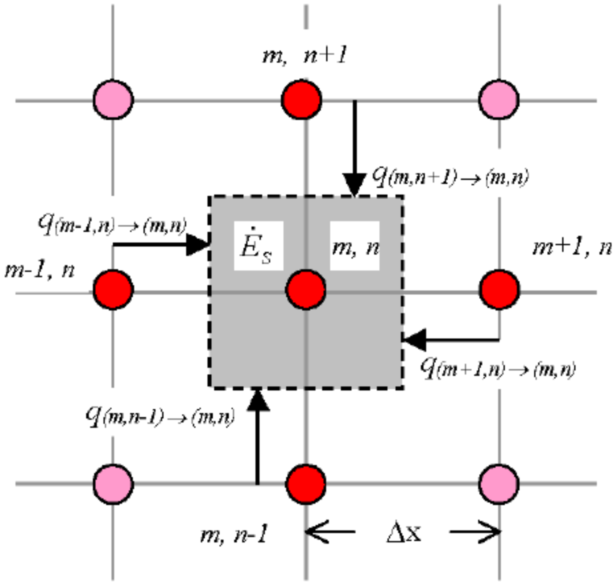

Fig 2. Interior Node

Summing up the corresponding formulas of the four nodes, and applying the energy conservation law, it can be written :

$$
\begin{aligned}
&kA\frac{\left(T^{p+1}_{m,n+1}-T^{p+1}_{m,n}\right)}{\Delta y}+kA\frac{\left(T^{p+1}_{m+1,n}-T^{p+1}_{m,n}\right)}{\Delta x}+kA\frac{\left(T^{p+1}_{m,n-1}-T^{p+1}_{m,n}\right)}{\Delta y}+kA\frac{\left(T^{p+1}_{m-1,n}-T^{p+1}_{m,n}\right)}{\Delta x}+\dot{q}\Delta x\cdot A=\\
&=\rho CA\cdot\Delta x\frac{\left(T^{p+1}_{m,n}-T^{p}_{m,n}\right)}{\Delta t}
\end{aligned}
\tag{2}
$$

After performing the calculations it is obtained :

$$
\left(T^{p+1}_{m,n+1}-T^{p+1}_{m,n}\right)+\left(T^{p+1}_{m+1,n}-T^{p+1}_{m,n}\right)+\left(T^{p+1}_{m,n-1}-T^{p+1}_{m,n}\right)+\left(T^{p+1}_{m-1,n}-T^{p+1}_{m,n}\right)+\dot{q}\Delta x^2\frac{1}{k}=\frac{1}{\alpha}\frac{\Delta x^2}{\Delta t}\left(T^{p+1}_{m,n}-T^{p}_{m,n}\right) \tag{3}
$$

Noting:

$$
Fo=\frac{\alpha\Delta t}{\Delta x^2} \tag{4}
$$

It results :

$$T_{m,n+1}^{p+1} + T_{m+1,n}^{p+1} + T_{m,n-1}^{p+1} + T_{m-1,n}^{p+1} - 4T_{m,n}^{p+1} + \dot{q}\Delta x^2 \frac{1}{k} = \frac{1}{Fo}\left(T_{m,n}^{p+1} - T_{m,n}^{p}\right) \tag{5}$$

It results the relationship of recurrence between $T^{p}_{m,n}$ and $T^{p+1}_{m,n}$ under the expression :

$$(1+4Fo)T_{m,n}^{p+1} - Fo\left(T_{m,n+1}^{p+1} + T_{m+1,n}^{p+1} + T_{m,n-1}^{p+1} + T_{m-1,n}^{p+1}\right) - \frac{\dot{q}Fo\Delta x^2}{k} = T_{m,n}^{p} \tag{6}$$

As the invariant F_o and the parameter k depend on the temperature, it is proposed the calculation of these values for the previous known temperature $T^{p-1}_{m,n}$ and $T^{p}_{m,n}$ followed by the interpolation of the values for the next interval.

3. The case of a marginal point

In this case the node is positioned on edge (fig 3).

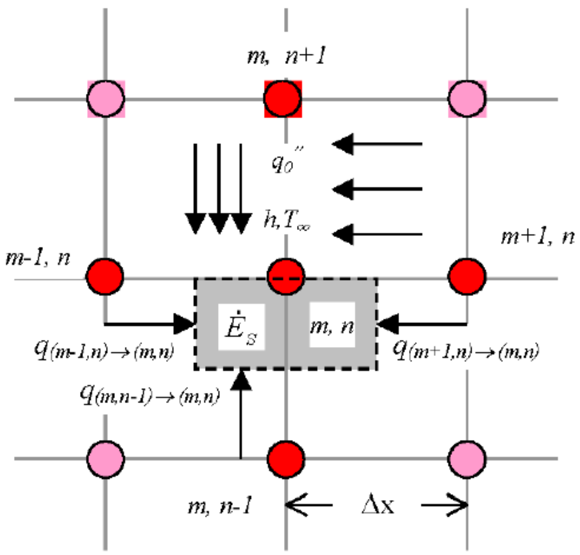

Fig. 3 Marginal Point

q_0'' [W/m²]

$$\dot{E}_{in} = \dot{E}_{st} \tag{7}$$

$$\dot{E}_{st} = \rho CV \frac{T_{m,n}^{p+1} - T_{m,n}^{p}}{\Delta t}, iar V = \Delta x \cdot \frac{\Delta x}{2} \cdot 1 = \frac{(\Delta x)^2}{2} \cdot 1 \quad [m^3] \tag{8}$$

$$\dot{E}_{in} = \sum_{i=1}^{4} q''_{(i \to m,n)} \cdot A \tag{9}$$

$$\begin{aligned} q_{(m,n+1)\to(m,n)} &= q_0'' \cdot A,\ A = \Delta x \cdot 1 \\ q_{(m+1,n)\to(m,n)} &= kA\frac{\left(T_{m+1,n} - T_{m,n}\right)}{\Delta x},\ A = \frac{\Delta y}{2}\cdot 1 \\ q_{(m,n-1)\to(m,n)} &= kA\frac{\left(T_{m,n-1} - T_{m,n}\right)}{\Delta y}, A = \Delta x \cdot 1 \\ q_{(m-1,n)\to(m,n)} &= kA\frac{\left(T_{m-1,n} - T_{m,n}\right)}{\Delta x},\ A = \frac{\Delta y}{2}\cdot 1 \end{aligned} \tag{10}$$

$$T_{m,n}^{p} = \left(1+4Fo\right)T_{m,n}^{p+1} - Fo\left(T_{m+1,n}^{p+1} + 2T_{m,n-1}^{p+1} + T_{m-1,n}^{p+1}\right) - \frac{q_0'' Fo \Delta x^2}{k} \tag{11}$$

If we suppose that on the free surface appears a convection heat transfer having the value :

$$q_{(m,n+1)\to(m,n)} = hA\left(T_\infty - T_{m,n}^{p+1}\right), A = \Delta x \cdot 1 \tag{12}$$

it results:

$$\begin{aligned} & hA\left(T_\infty - T_{m,n}^{p+1}\right)(\Delta x \cdot 1) + k\frac{\Delta x}{2}\frac{\left(T_{m+1,n}^{p+1} - T_{m,n}^{p+1}\right)}{\Delta x} + k\Delta x\frac{\left(T_{m,n-1}^{p+1} - T_{m,n}^{p+1}\right)}{\Delta y} + k\frac{\Delta x}{2}\frac{\left(T_{m-1,n}^{p+1} - T_{m,n}^{p+1}\right)}{\Delta x} = \\ & = \rho C \cdot \frac{(\Delta x)^2}{2}\frac{\left(T_{m,n}^{p+1} - T_{m,n}^{p}\right)}{\Delta t} \end{aligned} \tag{13}$$

Noting:

$$\alpha = \frac{k}{\rho C} \tag{14}$$

$$2hA\left(T_\infty - T_{m,n}^{p+1}\right)\frac{\Delta x}{k} + \left(T_{m+1,n}^{p+1} - T_{m,n}^{p+1}\right) + 2\left(T_{m,n-1}^{p+1} - T_{m,n}^{p+1}\right) + \left(T_{m-1,n}^{p+1} - T_{m,n}^{p+1}\right) = \frac{1}{\alpha}\cdot\frac{(\Delta x)^2}{\Delta t}\left(T_{m,n}^{p+1} - T_{m,n}^{p}\right) \tag{15}$$

using the expressions of the invariants :

$$Fo = \frac{\alpha \Delta t}{(\Delta x)^2} \tag{16}$$

$$Bi = \frac{h\Delta x}{k} \tag{17}$$

it results :

$$2Bi\left(T_\infty - T_{m,n}^{p+1}\right) + T_{m+1,n}^{p+1} + 2T_{m,n-1}^{p+1} + T_{m-1,n}^{p+1} - 4T_{m,n}^{p+1} = \frac{1}{Fo}\left(T_{m,n}^{p+1} - T_{m,n}^{p}\right) \tag{18}$$

respectively,

$$2BiT_\infty + T_{m+1,n}^{p+1} + 2T_{m,n-1}^{p+1} + T_{m-1,n}^{p+1} - 2T_{m,n}^{p+1}\left(Bi+2\right) = \frac{1}{Fo}\left(T_{m,n}^{p+1} - T_{m,n}^{p}\right) \tag{19}$$

and it results the temperature (T m,n) of the node as:

$$(1+4Fo+2Bi\cdot Fo)T_{m,n}^{p+1}-Fo\left(T_{m+1,n}^{p+1}+2T_{m,n-1}^{p+1}+T_{m-1,n}^{p+1}\right)-2BiFoT_{\infty}=T_{m,n}^{p} \quad (20)$$

4. The case of a node situated under an interior angle.

In this case the deposition is performed under an interior angle (fig 4)

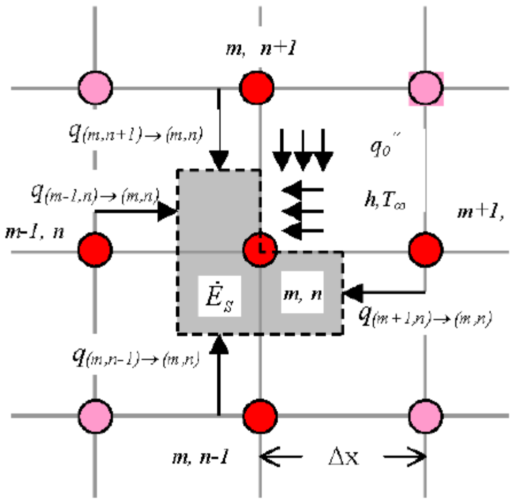

Fig 4. Node situated under an interior angle

The thermic fluxes transmitted by the neighbor nodes are written with the formulas:

$$\begin{aligned}
q_{(m,n+1)\to(m,n)}&=kA\frac{\left(T_{m,n+1}-T_{m,n}\right)}{\Delta y},\ A=\frac{\Delta x}{2}\cdot 1\\
q_{(m+1,n)\to(m,n)}&=kA\frac{\left(T_{m+1,n}-T_{m,n}\right)}{\Delta x},\ A=\frac{\Delta y}{2}\cdot 1\\
q_{(m,n-1)\to(m,n)}&=kA\frac{\left(T_{m,n-1}-T_{m,n}\right)}{\Delta y},\ A=\Delta x\cdot 1\\
q_{(m-1,n)\to(m,n)}&=kA\frac{\left(T_{m-1,n}-T_{m,n}\right)}{\Delta x},\ A=\Delta y\cdot 1
\end{aligned} \quad (21)$$

the heat flux received by convection on the exterior surfaces will be:

$$q_0=q_{0,x}^{"}A_x+q_{0,y}^{"}A_y,\ A_x=\frac{\Delta y}{2}\cdot 1,\ A_y=\frac{\Delta x}{2}\cdot 1 \quad (22)$$

$$q_{0,x}^{"}=q_{0,y}^{"};\ q_0=q_0^{"}\Delta x\cdot 1 \quad (23)$$

After performing the calculations it is obtained the temperature $T^{p+1}_{m,n}$ of the node (m, n):

$$\left(1+4Fo+\frac{4}{3}Bi\cdot Fo\right)T_{m,n}^{p+1}-\frac{2}{3}Fo\left(T_{m,n+1}^{p+1}+T_{m+1,n}^{p+1}+2T_{m,n-1}^{p+1}+2T_{m-1,n}^{p+1}\right)-\frac{4}{3}BiFoT_{\infty}=T_{m,n}^{p} \quad (24)$$

5. The case of a node situated in an exterior corner.

In the figure 5 it is given the case of an exterior corner. The thermic fluxes transmitted by the neighbor nodes have the values:

$$\begin{aligned} q_{(m,n+1)\to(m,n)} &= 0 \\ q_{(m+1,n)\to(m,n)} &= 0 \\ q_{(m,n-1)\to(m,n)} &= kA\frac{\left(T_{m,n-1}-T_{m,n}\right)}{\Delta y},\ A=\frac{\Delta x}{2}\cdot 1 \\ q_{(m-1,n)\to(m,n)} &= kA\frac{\left(T_{m-1,n}-T_{m,n}\right)}{\Delta x},\ A=\frac{\Delta y}{2}\cdot 1 \end{aligned} \quad (25)$$

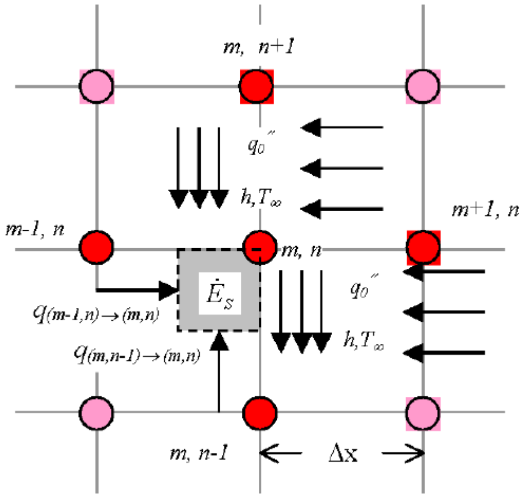

Fig 5 Node situated in an exterior corner

The thermic flux transmitted via convection is:

$$q_0 = q''_{0,x}A_x + q''_{0,y}A_y,\ A_x = \frac{\Delta x}{2}\cdot 1,\ A_y = \frac{\Delta y}{2}\cdot 1 \quad (26)$$

The temperature ($T^{p+1}_{m,n}$) of the node (m, n) is calculated by the formula:

$$T^{p}_{m,n} = \left(1+4Fo+4Bi\cdot Fo\right)T^{p+1}_{m,n} - 2Fo\left(T^{p+1}_{m,n-1}+T^{p+1}_{m-1,n}\right) - 4BiFoT_{\infty} \quad (27)$$

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