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SOFTWARE FOR MULTI-CRITERIA OPTIMISATION OF EXTERNAL CYLINDRICAL GEARS

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Abstract: This paper presents a software for multi-criteria optimization of external cylindrical gears. The outputs of the software are different solutions ordered depending on their approach to the imposed criteria. The optimization method is based on taking decision in condition of certainty and allows the selection of an optimal solution from a *k* number of possible solutions considering quantity and quality criteria of optimization.

1. INTRODUCTION

The durability and safe functioning of an external cylindrical gear that is part of a mechanical transmission are influenced by a great number of factors. Designing a gear that is able to transmit an imposed torque at an imposed rotation speed – in certain functioning and size conditions – is an undetermined problem which can be, in fact, described by the dimensioning equations and is satisfied by a large number of solutions. During design, using some correlations and intervals of the variables – based on designing experience or theoretical research – makes the solutions number decrease, but it still remains big enough. As a result, an optimal solution cannot be obtained without imposing certain optimizing criteria depending on constructive, technological and functional particularities of the gearing and using dedicated software.

This paper presents software for multi-criteria optimization of external cylindrical gears; the outputs are several solutions ordered depending on their approach to the imposed criteria.

2. THEORETICAL BASES

For choosing the optimal solution for an external cylindrical gearing, a method that is based on taking decisions in certainty states is used. This method allows choosing the optimal solution from *k* possible solutions, taking account of all the influencing quantitative and qualitative criteria. The decision is based on favorable and unfavorable spaces of the different solutions.

The optimizing criteria have been chosen by authors and the marks N_i, for each criterion and each solution *i* from the *k* possible solutions ($i \in [1...k]$), have been established following the next rules [3]:

- The range of the marks must be the same for each criterion (from 0 to 10);
- The marks must be as higher as the solution is closer to optimum.

The objective function is determined with relation

$$N_i = \sum_{j=1}^{m} a_j N_{ji} \ , \tag{1}$$

where a_j represents the *j* criterion importance coefficient; *m* – the total number of criteria; N_{ji} – the mark for the *j* criterion of the *i* solution. The a_j criterion importance coefficient has to be as big as the importance of the criterion. Establishing the reference order for the k solutions is done by descendent ordering the objective function (1).

The main imposed restrictions for external cylindrical gearings are [6, 9, 10]:

– Avoiding the teeth profile interference;
– Achieving a minimum transverse contact ratio of $\varepsilon_{\alpha min}$=1.3;
– Avoiding the sharpening of the gearing wheels teeth $s_{a1} \geq s_{amin}$ and $s_{a2} \geq s_{amin}$;
– Choosing the minimal normal module according to the applied treatment. (m_{nmin} ≥1.5 mm, for improvement treatment, respectively $m_{nmin} \geq$ 2.0 mm, for carburation, nitrating and superficial hardening).
– The superficial stresses for the two main stress types should not be bigger then the corresponding permissible stresses, $\sigma_H \leq \sigma_{HP}$; $\sigma_{F1} \leq \sigma_{FP1}$, respectively $\sigma_{F2} \leq \sigma_{FP2}$.

The imposed optimizing criteria and their mathematical models have been presented in [3] and Table 1 is presenting their synthesis.

Table 1

Mathematical model of the optimizing criteria

Optimizing criteria	Mathematical model
Technological, constructive and functional criteria	
Volume of material of the wheels criterion C_V	$N_{CVi} = 10\dfrac{V_{\max} - V_i}{V_{\max} - V_{\min}}$; $0 \leq N_{CVi} \leq 10$ $V_{\max} = \max(V_i, i = 1...k)$; $V_{\min} = \min(V_i, i = 1...k)$ $V = \pi a_{wi}^2 b_i \dfrac{u_i^2 + 1}{(u_i + 1)^2}$
Gear centre distance criterion C_{aw}	$N_{Cawi} = 10\dfrac{a_{w\max} - a_{wi}}{a_{w\max} - a_{w\min}}$; $0 \leq N_{Cawi} \leq 10$ $a_{w\max} = \max(a_{wi}, i = 1...k)$; $a_{w\min} = \min(a_{wi}, i = 1...k)$
Gear width criterion C_b	$N_{Cbi} = 10\dfrac{b_{2\max} - b_{2i}}{b_{2\max} - b_{2\min}}$; $0 \leq N_{Cbi} \leq 10$ $b_{\max} = \max(b_i, i = 1...k)$; $b_{\min} = \min(b_i, i = 1...k)$
Technological accuracy criterion C_{prt}	$N_{Cprti} = 10\dfrac{Trpri_i - Trpr_{\min}}{Trpr_{\max} - Trpr_{\min}}$; $0 \leq N_{Cprti} \leq 10$ $Tr_{pr\max} = \max(Tr_{pri}, i = 1...k)$; $Tr_{pr\min} = \min(Tr_{pri}, i = 1...k)$
Gear efficiency criterion C_η	$N_{C\eta i} = 10\dfrac{\eta_i - \eta_{\min}}{\eta_{\max} - \eta_{\min}}$; $0 \leq N_{C\eta i} \leq 10$, $\eta_{\max} = \max(\eta_i, i = 1...k)$; $\eta_{\min} = \min(\eta_i, i = 1...k)$, where [8]: $\eta_i = 1 - \mu\dfrac{\cos^2\beta}{2\cos\alpha_n}\dfrac{\upsilon_{ali1}^2 + \upsilon_{ali2}^2}{\upsilon_{ali1} + \upsilon_{ali2}}$ $\upsilon_{ali1} = \dfrac{u_i + 1}{u_i}\cdot$ $\cdot\left[\sqrt{\left(\dfrac{z_{1i}/\cos\beta + 2(h_{an}^* + x_{n1i} - k_i)}{z_{1i}/\cos\beta + 2(h_{an}^* + x_{n1i})}\right)^2 - \cos^2\alpha_n} - \sin\alpha_n\right]$

Table 1 (continues)

	$\upsilon_{ali2} = (u_i + 1)\cdot$ $\left[\sqrt{\left(\frac{z_{2i}/\cos\beta + 2(h_{an}^* + x_{n2i} - k_i)}{z_{2i}/\cos\beta + 2(h_{an}^* + x_{n2i})}\right)^2 - \cos^2\alpha_n} - \sin\alpha_n\right]$ $k_i = (x_{n1i} + x_{n2i}) - \left(\frac{z_{1i} + z_{2i}}{2\cos\beta}\right)\left(\frac{\cos\alpha_{ti}}{\cos\alpha_{wti}} - 1\right)$ [2]
Geometric and kinematics criteria	
Gear ratio precision criterion $C_{\Delta u}$	$N_{C\Delta ui} = 10\frac{\Delta u_{max} - \Delta u_i}{\Delta u_{max} - \Delta u_{min}};\ 0 \le N_{C\Delta ui} \le 10$ $\Delta u_{max} = \max(\Delta u_i, i = 1...k);\ \Delta u_{min} = \min(\Delta u_i, i = 1...k)$ $\Delta u_i = \left\lvert\frac{u_i - u_{dat}}{u_{dat}}\right\rvert$
Frontal contact ratio criterion $C_{\varepsilon\alpha}$	$N_{C\varepsilon\alpha i} = 10\frac{\varepsilon_{\alpha i} - \varepsilon_{\alpha min}}{\varepsilon_{\alpha max} - \varepsilon_{\alpha min}};\ 0 \le N_{C\varepsilon\alpha i} \le 10,$ $\varepsilon_{\alpha max} = \max(\varepsilon_{\alpha i}, i = 1...k);\ \varepsilon_{\alpha min} = \min(\varepsilon_{\alpha i}, i = 1...k)$
Specific addendum sliding criterion C_{ξ}	$N_{C\Delta\xi i} = 10\frac{\Delta\xi_{max} - \Delta\xi_i}{\Delta\xi_{max} - \Delta\xi_{min}};\ 0 \le N_{C\Delta\xi i} \le 10$ $\Delta\xi_{max} = \max(\Delta\xi_i, i = 1...k);\ \Delta\xi_{min} = \min(\Delta\xi_i, i = 1...k)$ $\Delta\xi_i = \lvert\xi_{Ei} - \xi_{Ai}\rvert$, where: $\xi_E = \frac{u_i + 1}{u_i}\left[1 - \frac{\cos\alpha_{ti}tg\alpha_{wti}}{\sqrt{\left(\frac{z_{1i}/\cos\beta + 2(h_{an}^* + x_{n1i} - k_i)}{z_{1i}/\cos\beta + 2(h_{an}^* + x_{n1i})}\right)^2 - \cos^2\alpha_{ti}}}\right]$ $\xi_A = (u_i + 1)\left[1 - \frac{\cos\alpha_{ti}tg\alpha_{wti}}{\sqrt{\left(\frac{z_{2i}/\cos\beta + 2(h_{an}^* + x_{n2i} - k_i)}{z_{2i}/\cos\beta + 2(h_{an}^* + x_{n2i})}\right)^2 - \cos^2\alpha_{ti}}}\right]$
Strength criteria	
Loading to the limit of contact stress criterion C_{σ}	$N_{C\sigma Hi} = 10\frac{\Delta\sigma_{H max} - \Delta\sigma_{Hi}}{\Delta\sigma_{H max} - \Delta\sigma_{H min}};\ 0 \le N_{C\sigma Hi} \le 10$ $\Delta\sigma_{H max} = \max(\Delta\sigma_{Hi}, i = 1...k);\ \Delta\sigma_{H min} = \min(\Delta\sigma_{Hi}, i = 1...k)$ $\Delta\sigma_{Hi} = \sigma_{HPi} - \sigma_{Hi}$

Table 1 (continues)

Bending stress approach between pinion and wheel criterion $C_{\Delta\sigma F}$	$N_{C\Delta\sigma Fi}=10\dfrac{\Delta\sigma_{F\max}-\Delta\sigma_{Fi}}{\Delta\sigma_{F\max}-\Delta\sigma_{F\min}};\ 0\le N_{C\Delta\sigma Fi}\le 10$ $\Delta\sigma_{F\max}=\max(\Delta\sigma_{Fi},\ i=1...k);\ \Delta\sigma_{F\min}=\min(\Delta\sigma_{Fi},\ i=1...k)$ $\Delta\sigma F_i=\left\|\dfrac{Y_{F1i}Y_{S1i}}{b_{1i}\sigma_{FP1i}}-\dfrac{Y_{F2i}Y_{S2i}}{b_{2i}\sigma_{FP2i}}\right\|$

3. SOFTWARE

Gear dimensioning or verification calculus usually takes a large amount of work. Following calculus methodology, imposing optimization criteria and the complex and large calculus amount recommends development of dedicated software for gear calculus. All these and also the purpose of obtaining optimal solutions of external cylindrical gears for any specific conditions have been the base for developing the software presented in this paper.

The software is using the mathematical models of the optimization criteria presented in [3] and also the design methodology, notes and recommendations presented in [6, 10]. For a friendly user interface, the software is using the advantages of the existing Windows objects.

The main menu of the software is presented in fig. 1.

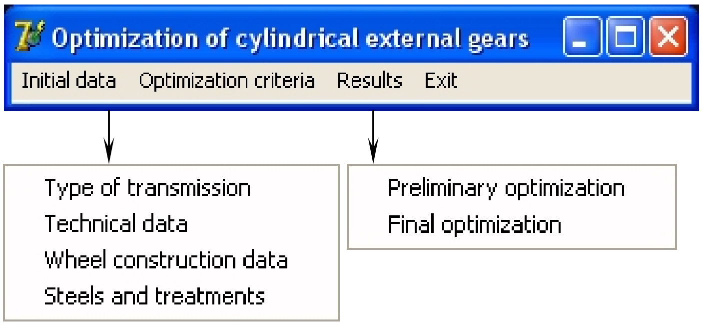

Fig. 1. Main menu

In order to show the stages of running the software, this paper is presenting, using the screen images, the steps followed for a concrete optimal calculus of an external cylindrical gear.

The Input data are introduced in the following steps:

- Screen ***Type of Transmission*** (fig. 2) allows choosing the destination of the gear, with direct effect on the range of the gear precision and also on the standardized or non-standardized values of the centre distance.
- Screen ***Technical Data*** (fig. 3) allows the input of the necessary design data for strength and geometric dimensioning of cylindrical external gears, used for establishing of the parameters of the possible solutions. For certain input data, like transmitted load or imposed durability, there is the possibility to choose between specific known inputs like torque or power, respectively running time or number of running cycles. The application factor is determined by accessing the *Choose application factor* button, which opens the screen ***Application factor*** (fig. 4). By choosing the type of the driving machine and the character of load, the value of the application factor is automatically taken from the recommended values.

The variables of the multi-criteria optimization algorithm are: pinion number of teeth, gear helix angle, gear width coefficient, normal addendum modification coefficient at pinion. For each of them, the *Technical Data* screen gives the possibility to set the range (minimum and maximum value) and iteration step.

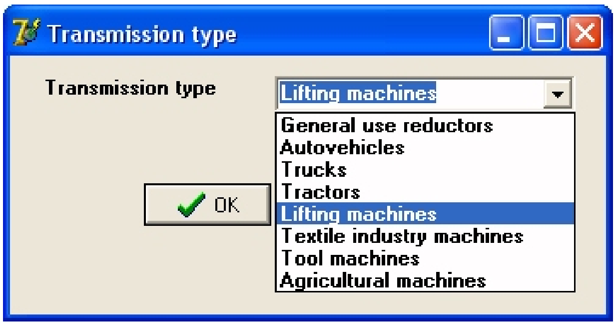

Fig. 2. Type of transmission

• Screen ***Wheels Construction Data*** (fig. 5) allows introduction of the constructive features, separate for pinion and wheel, and also the length of the shafts sustaining the pinion and the wheel (considering the same length). These data have influence on the reduced mass of pinion and wheel and also on the critical input rotation of the gear, which determine the K_{υ} dynamic factor. The length of the shafts is necessary for determining the $K_{H\beta}$ and $K_{F\beta}$ factors.

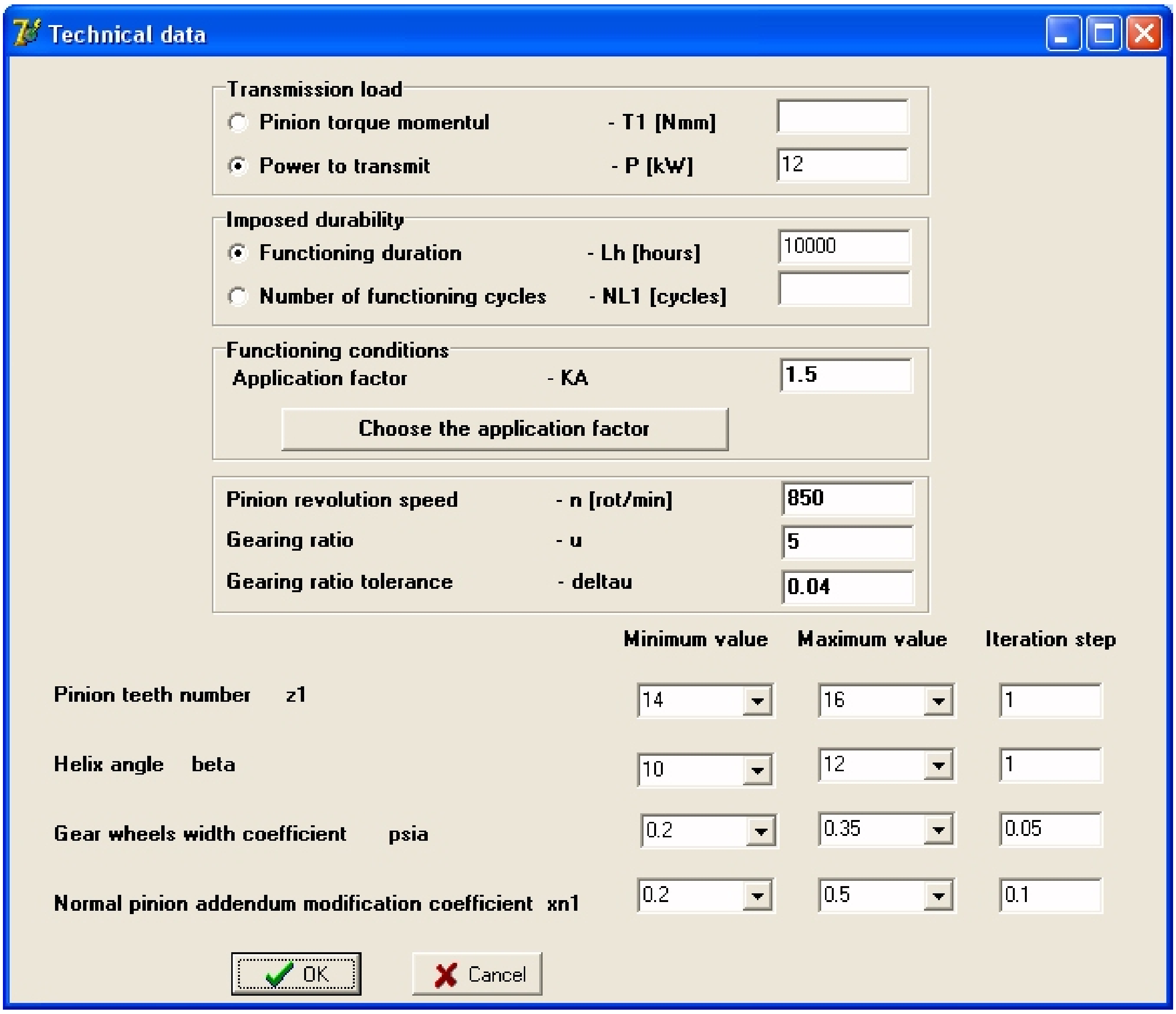

Fig. 3. Technical data

• Screen ***Steels and Treatments***, presented in fig. 6, allows choosing the materials of pinion and wheel and their features. There have to be set the type of steel and specific treatment, and also the steel symbol. The contact and bending limit stresses for pinion and respectively wheel have to be chosen depending on the recommendations presented in the panel.

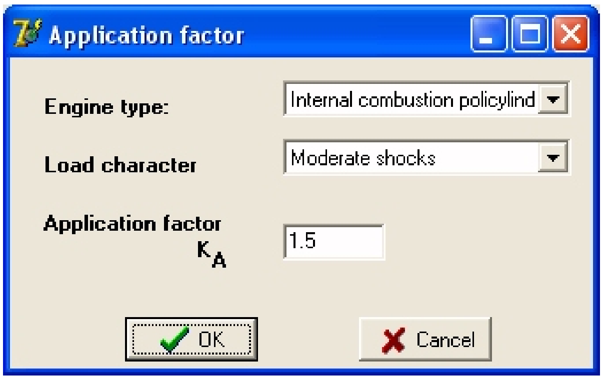

Fig. 4. Application factor

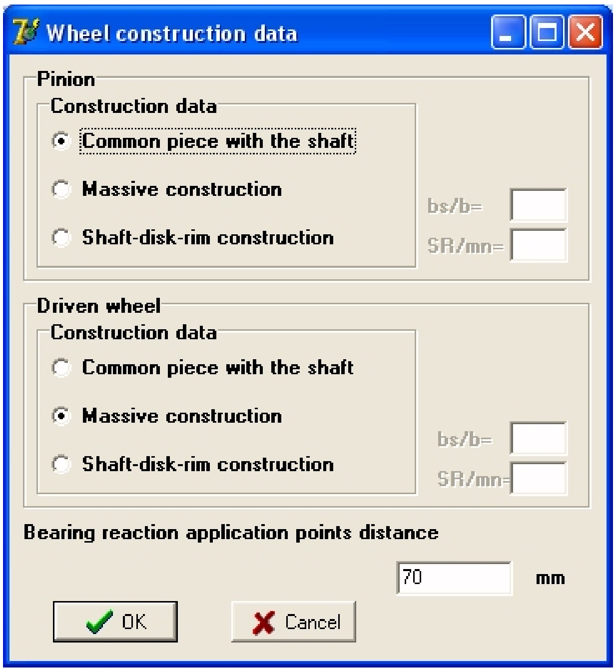

Fig. 5. Wheels construction data

After introducing the technical data, by pushing the *OK* button (see. fig. 3), the screen ***Supplementary data for gear strength calculus*** (fig. 7) opens. In this screen, the following elements must be set: data on gear machining (roughness of the profile and of the foot of the tooth); exploiting specifications on accepting or not any pits on the tooth profile and safety coefficients for the two contact and bending stresses (according to recommendations).

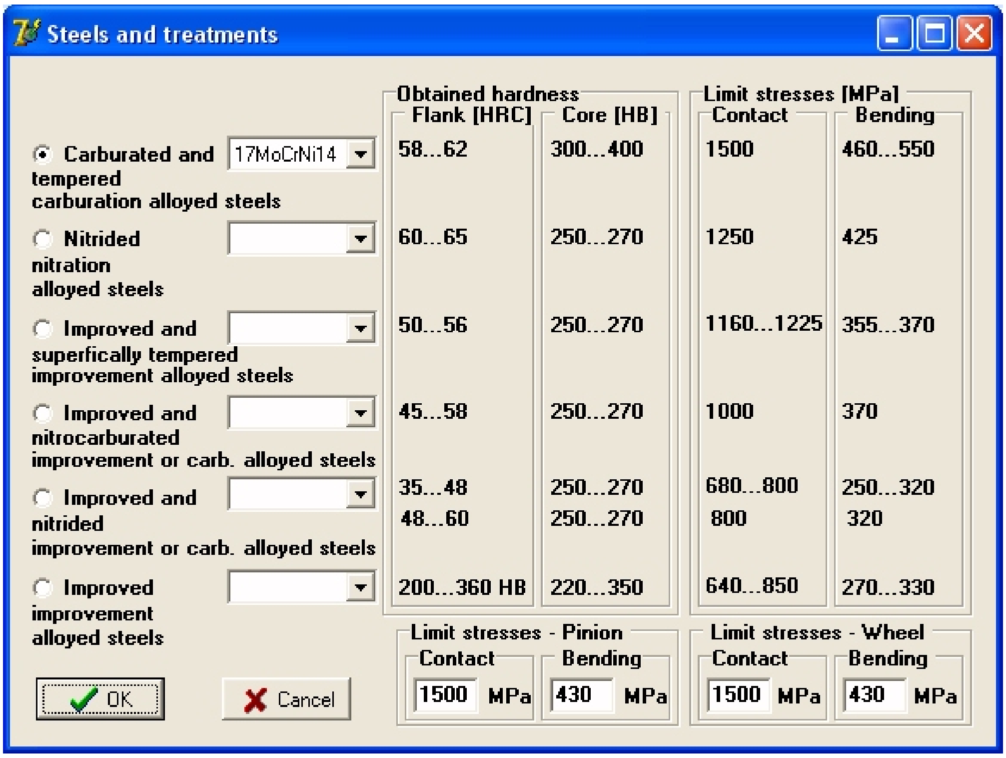

Fig. 6. Steels and Treatments

Screen ***Optimizing criteria***, presented in fig. 8, allows choosing of the optimizing criteria and the importance of each criterion. For the presented example, four criteria have been selected: volume of material criterion; technological accuracy criterion; frontal contact ratio criterion and loading to the limit of contact stress criterion. The importances of the three criteria are (in percents): 100, 50, 50 and 90. The software is following the calculus methodology presented in [6]. The parameters of all possible solutions of the cylindrical gear are determined first. Then, the marks N_{ij} of each solution i are established, for each of the j selected optimizing criteria and, based on relation (1), the objective function (general mark N_i) is determined for each solution. The solutions are then ordered by their approach to optimum. A part of the table containing the solutions from the preliminary optimization is presented in fig. 9. It shows that there are 8969 solutions fulfilling the imposed restrictions, the optimal solution being evaluated with a general mark of 8.679.

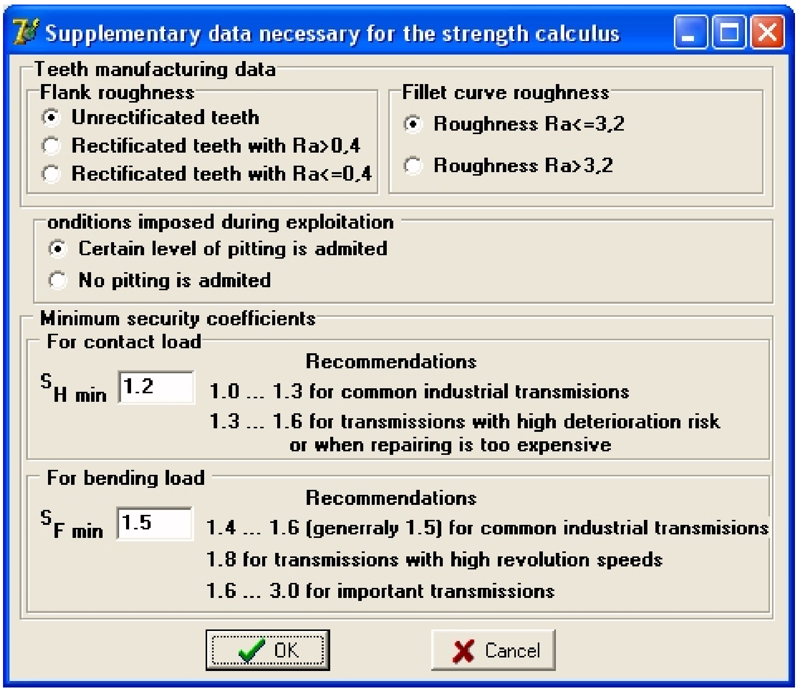

Fig. 7. Supplementary data for the strength calculus

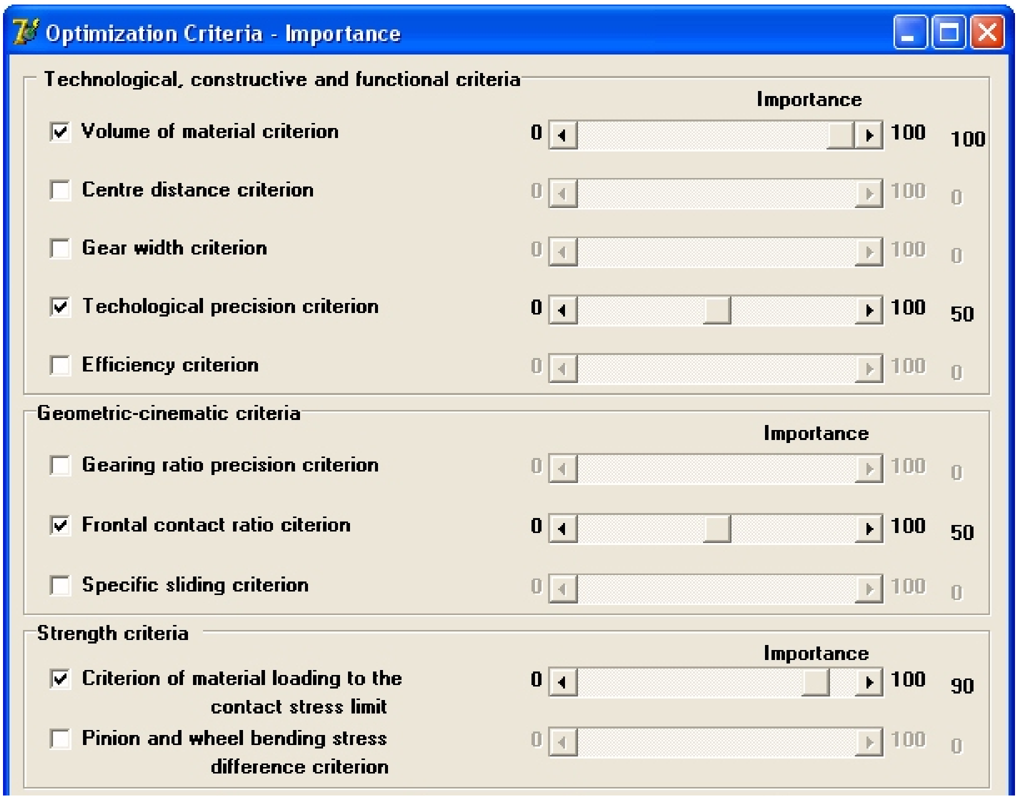

Fig. 8. Optimizing criteria and their importance

Results of preliminary optimization

Nr.var.	z1	z2	beta	aw	mn	xn1	xn2	b2	psia	Trpr	Nota	N Vm	N pr	N eps	N Cont
1	16	77	12.000	142	3.000	0.2000	-0.4022	45.90	0.32	8	8.679	9.095	5.000	9.176	9.985
2	15	72	12.000	155	3.500	0.2000	-0.3832	54.00	0.35	9	8.623	6.716	10.000	8.713	9.926
3	16	77	12.000	142	3.000	0.2000	-0.4022	49.50	0.35	8	8.520	8.629	5.000	9.176	9.989
4	15	72	12.000	155	3.500	0.3000	-0.4832	53.50	0.35	9	8.481	6.793	10.000	7.695	9.950
5	16	77	12.000	143	3.000	0.2000	-0.0709	46.20	0.32	8	8.427	8.972	5.000	7.941	9.996
6	15	72	12.000	156	3.500	0.2000	-0.0996	54.30	0.35	9	8.381	6.561	10.000	7.596	9.941
7	15	72	12.000	155	3.500	0.2000	-0.3832	43.60	0.28	8	8.319	8.317	5.000	8.713	9.947
8	15	73	12.000	157	3.500	0.2000	-0.3246	54.90	0.35	9	8.306	6.326	10.000	8.504	9.453
9	15	72	12.000	155	3.500	0.2000	-0.3832	51.50	0.33	9	8.303	7.101	10.000	8.713	8.467
10	16	78	12.000	144	3.000	0.2000	-0.2498	47.00	0.33	8	8.296	8.759	5.000	8.642	9.420
11	16	77	12.000	143	3.000	0.3000	-0.1709	46.10	0.32	8	8.271	8.985	5.000	7.062	9.968
12	16	77	12.000	143	3.000	0.2000	-0.0709	49.90	0.35	8	8.256	8.486	5.000	7.941	9.984
13	15	72	12.000	156	3.500	0.3000	-0.1996	54.00	0.35	9	8.246	6.608	10.000	6.696	9.951
14	16	79	12.000	145	3.000	0.2000	-0.4239	46.10	0.32	8	8.244	8.773	5.000	9.270	8.888
15	16	78	11.000	143	3.000	0.2000	-0.4095	49.70	0.35	8	8.201	8.491	5.000	9.476	8.948
16	15	72	12.000	155	3.500	0.3000	-0.4832	43.30	0.28	8	8.159	8.363	5.000	7.695	9.944
17	15	73	12.000	157	3.500	0.3000	-0.4246	54.50	0.35	9	8.157	6.390	10.000	7.509	9.457
18	16	78	12.000	144	3.000	0.3000	-0.3498	46.70	0.32	8	8.149	8.799	5.000	7.691	9.431
19	15	72	12.000	155	3.500	0.3000	-0.4832	51.10	0.33	9	8.149	7.162	10.000	7.695	8.470
20	16	77	12.000	144	3.000	0.2000	0.2768	46.30	0.32	8	8.139	8.873	5.000	6.496	9.980
21	16	77	12.000	142	3.000	0.2000	-0.4022	38.90	0.27	7	8.125	10.000	0.000	9.176	9.973
22	16	77	10.000	165	3.500	0.2000	-0.2740	52.20	0.32	9	8.123	5.916	10.000	9.208	8.929
23	14	68	12.000	146	3.500	0.2000	-0.3981	48.60	0.33	8	8.121	8.371	5.000	8.355	9.446
24	16	77	12.000	143	3.000	0.4000	-0.2709	45.80	0.32	8	8.119	9.024	5.000	6.107	9.965

Number of solutions considered for optimization 8969

Number of solutions for final optimization 30

OK Cancel

Fig. 9. Results of the preliminary optimization

Results of final optimization

Geometry of gear | Geometry of pinion and wheel | Control elements | Close

Substitute gear | Calculus factors | Stresses

Nr.var.	z1	z2	beta	aw	mn	xn1	xn2	b2	psia	Trpr	Nota	N Vm	N pr	N eps	N Cont
1	16	77	12.000	142	3.000	0.2000	-0.4022	45.90	0.32	8	8.214	7.783	5.000	9.201	9.929
2	16	77	12.000	142	3.000	0.2000	-0.4022	38.90	0.27	7	8.091	10.000	0.000	9.201	9.849
3	16	77	12.000	142	3.000	0.2000	-0.4022	49.50	0.35	8	7.829	6.643	5.000	9.201	9.957
4	16	77	12.000	143	3.000	0.2000	-0.0709	46.20	0.32	8	7.565	7.482	5.000	5.913	10.000
5	15	72	12.000	155	3.500	0.2000	-0.3832	43.60	0.28	8	7.268	5.879	5.000	7.967	9.683
6	16	77	12.000	143	3.000	0.2000	-0.0709	49.90	0.35	8	7.132	6.293	5.000	5.913	9.925
7	16	77	12.000	143	3.000	0.3000	-0.1709	46.10	0.32	8	7.116	7.514	5.000	3.573	9.817
8	15	72	12.000	155	3.500	0.3000	-0.4832	43.30	0.28	8	6.834	5.992	5.000	5.258	9.664
9	16	77	12.000	144	3.000	0.2000	0.2768	46.30	0.32	8	6.786	7.241	5.000	2.067	9.895
10	16	77	12.000	143	3.000	0.3000	-0.1709	49.70	0.35	8	6.749	6.358	5.000	3.573	9.921
11	15	72	12.000	155	3.500	0.2000	-0.3832	54.00	0.35	9	6.735	1.958	10.000	7.967	9.543

Fig. 10. Results of the final optimization

The final optimization is accomplished by considering a relative small number of the preliminary solutions, closest to optimum. The marks N_{ij} of the first solutions are re-calculated for the selected optimizing criteria. Figure 10 is presenting the re-ordered solutions according to the final optimization and their general an criteria marks. There can be seen that the solution set as optimal is the same with the one from the preliminary optimization phase. The order of the following solutions is slightly changed. For each of the shown solutions the software can perform the geometric calculus (geometry of pinion, wheel and gear, geometry of the substitute gear, control elements) and the strength calculus (calculus factors, real stresses and permissible stresses) by simple selection of the row of any solution. By pushing one of the buttons placed at the top of the screen, a new screen is automatically opening showing the selected outputs of the selected solution of cylindrical gear.

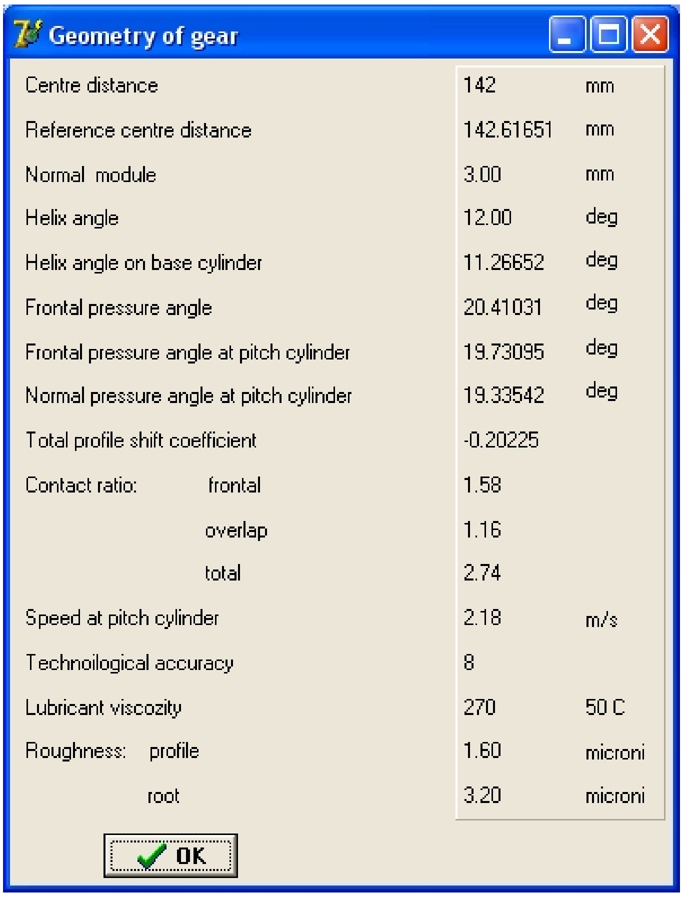

Fig. 11. Gear geometrical elements

As an example, for the first solution, figures 11...13 are showing: *gear geometrical elements* (fig. 11), *pinion and wheel control elements* (fig. 12) and *geometrical elements of pinion and wheel* (fig. 13).

Optimization outputs are mostly depending on the chosen optimizing criteria and their importance. The software can be used by setting only one optimizing criterion or a small group of criteria, giving an image of the significant influences on each optimizing criterion.

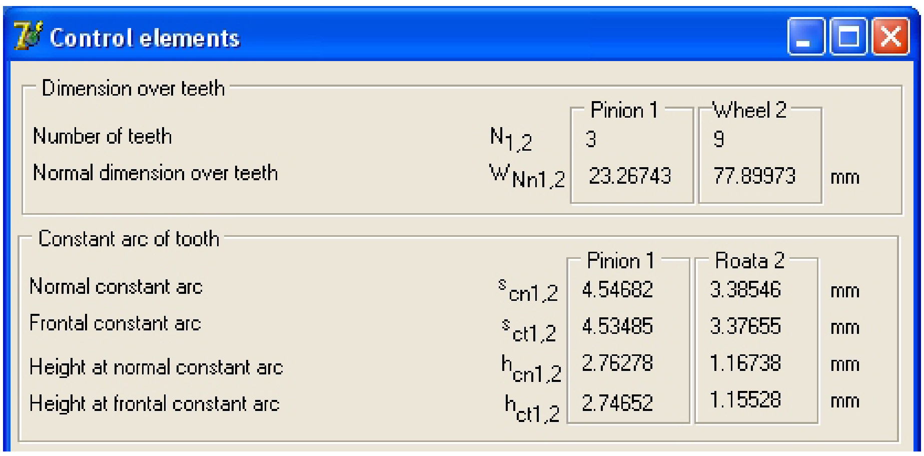

Fig. 12. Pinion and wheel control elements

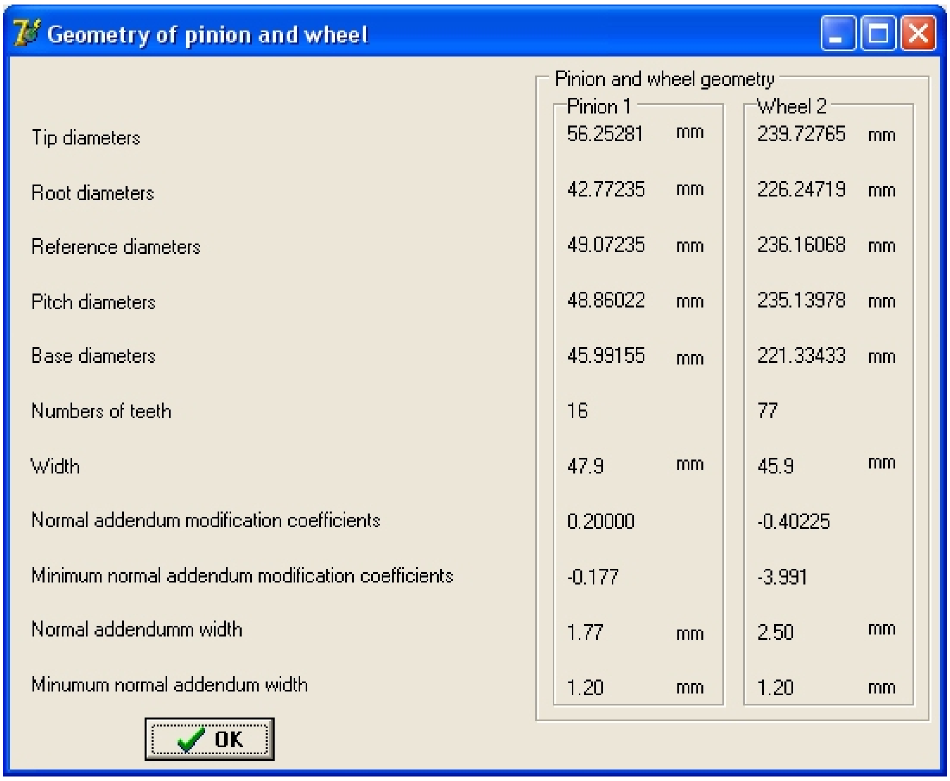

Fig. 13. Pinion and wheel geometrical elements

References

[1]. Bozidar, R. *Multicriterion Optimization of Multistage Gear Train Transmission*, Facta Universitas, University of Belgrad, Series: Mechanical Engineering, Vol.1, No 8, 2001, pp. 1107-1115

[2]. Deaky, B., Gavrilă, C.C. *Influenta deplasarilor de profil asupra scurtarii dintilor angrenajelor cilindrice. (Influence of addendumm modifications on tooth shortening of cylindrical gears)*, PRASIC'06, vol. II, Braşov, Romania, 2006, pp. 25-30

[3]. Deaky, B., Moldovean, Gh., Velicu, R. *Multi-criteria optimization of the external cylindrical gears,* Monography MACHINE DESIGN, University of Novi Sad, 2008

[4] Dudley, D. W. *Gear Handbook*, McGraw-Hill, New York Toronto London

[5] Mirica, R.F., Dobre, G. On the distribution of the profilr shift coefficients between mating gears in the case of cylindrical gear, 12th IFToMM World Congress, Besancon, France, june 2007

[6] Moldovean, Gh., Velicu, D., Velicu, R. and others. *Angrenaje cilindrice şi conice, Calcul şi construcție*. (*Cylindrical and bevel gears. Calculus and constructions*), Lux Libris, Braşov, Romania 2001

[7] Sauer, L. and others. *Angrenaje. Proiectare. Materiale*. (*Gears. Design. Materials*.) Editura Tehnica, Bucuresti Romania 1970

[8] Shigley, J. E., Mischke, C. R. *Mechanical Engineering Design*, McGraw-Hill International Edition, New York, 1989

[9] Su, D. *GearOpt: A Software Package for Evolutionary Optimisation of Gear Design* (www.admec.ntu.ac.uk)

[10] Velicu, R., Moldovean, Gh. *Angrenaje cilindrice. Reductoare cilindrice (Cylindrical Gears. Cylindrical Speed Reducers)*, Transilvania University Publishing Brasov, Romania, 2002

[11] KISSsoft Tutorial: *Lifetime analysis of helical gears* (www.KISSsoft.AG)