ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume VII (XVII), 2008

METHOD FOR CONSTRUCT THE SURFACE OF THE 3D SHAPE USING TRIANGLE MESHES

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Keywords shoe last, cloud data, triangle meshes, surface patches

Abstract Models for surface modeling of free-form surface and massive data points are becoming an important feature in commercial computer aided design/computer-aided manufacturing software. However, there are many problems to be solved in this area, especially for closed free-form surface modeling. This article presents an effective method for cloud data closed surface modeling from asynchronous profile modeling measurement. It includes three steps: first, the cloud data are preprocessed for smoothing; second, a helical line is segmented to form triangle meshes; and third, Bezier surface patches are created over a triangle mesh and trimmed to shape on an entire surface. In the end, an illustrative example of shoe last surface modeling is given to show the availability of this method.

1 INTRODUCTION

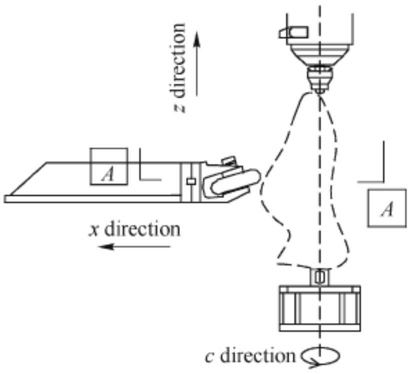
Computer-aided manufacturing (CAM) system is widely used in shoe last manufacturing, which has realized the digitalization of the machining program from the free-form surface detection method to produce numerical control (NC) requirement by the shoe last CAM software system automatically. In this type of free-form surface NC machining, it is common to digitalize the shape of the entity through the threedimensional (3D) measuring system after obtaining the scanning data and the process of the natural pattern; the next step is the rebuilding of the surface model [1], [2], [3]. There are two steps involved in remodeling the discrete data, which has become an entity shape in recent years [1–3]. The first is the sequencing of the measurement data and the second is the generating of the triangle mesh. There are two methods of sequencing the measurement data. One is the automatic region segmentation method, which involves combining "the side and surface" to undertake feature extraction. Here the surface region segmentation is usually divided into one or several rectangle domains, and then a mathematical model is built according to the character boundary. This method can also be used for closed surface modeling; but when the surface is very complicated, region segmentation and feature extraction will be very difficult [4].

To make the mesh surface reflect the structure character of the complicated surface, this paper proposes a triangle mesh fitting method of free-form surface for the cloud data from asynchronous profile measurement. This method has easy calculation and high efficiency, and has already been applied by us.

2 DATA ACQUISITION AND DISPOSAL

Currently, data acquisition can be divided into two categories: the contact measurement and noncontact measurement [4], [5], [6]. Noncontact measurement is widely used in reverse engineering (RE). Although the noncontact measurement method has high efficiency and needs no offset probe radius, it is not accurate and is not suitable for certain measure types, such as shoe last measurement. This paper discusses only the asynchronous profile measurement method, in which, the measurement data are easily changed into NC machining path. The acquisition method of the surface profile measurement data is shown in Fig. 1.

Owing to the fact that shoe last is a complicated closed free-form surface, it was clipped on the *z* axis; the probe moved along the direction of the *x* axis; and the probe head was driven by the cylinder. Using the raster rule record *x* axial coordinate, shoe last was driven by the thumbstall around the *c* axis. The measure rod reciprocated from the top to bottom in the direction of *z* axis; and the probe moved in the *x* axis to-and-fro in the drive of the cylinder, which pressed the probe to contact the shoe last surface while measuring. The measure trajectory is a space spiral curve, which contains uneven, dense, and large number of points. Hence it is difficult to construct the surface with common modeling methods.



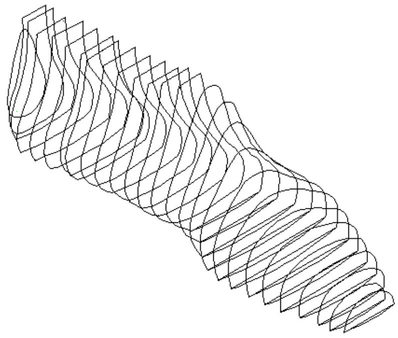


Fig. 1, The principle of shoe last measuring

Before becoming an NC program, shoe last surface measurement data should be reconstructed. It includes several essential technologies, such as the probe radius offset, the shoe last size magnification and shrinkage, the surface tool-path generation, and so on.

3. SURFACE MODELING TECHNOLOGY

Currently, there are three types of surface structure methods in computational geometry [4]: one is based on B-spline or nonuniform rational B-spline (NURBS); the second is the triangle Bezier surface (TBS); and the third is the polyhedra method. The B-spline and the NURBS are the representations of the surface method that are currently adopted to a large extent in the commercialization of the CAD/CAM system. It occupies a strict request to the data point; the data form is distributed by the tensor, and it cannot be changed much. Otherwise the surface smoothness cannot be obtained satisfactorily.

In RE, the TBS modeling method has high speed and can be used to interpolate the arbitrary boundary. It is becoming a general method and getting more attention. On the basis of the shoe last profile measurement data character, this paper suggests a closed free-form surface triangle mesh modeling algorithm as follows.

3.1 SEQUENCE FOR THE DATA

Firstly, let us assume that the data points trace is a space spiral and each point is contextual. After knowing the data loop numbers, the locations of the data point are ensured completely.

Let the data formats be ρ,θ, z, where:

- **ρ** is the pole radius of the point,
- **θ** is the pole angle
- ***z*** is the length.

When adopting a measure method of angle increments, we have,

$$\boldsymbol{\theta = 2n\pi+\varphi},$$

where **φ** is the absolute angle of the point.
By the formula:

$$n = \left[\frac{\theta}{2\pi}\right] \qquad (1)$$

where the simple means getting an integer, we can obtain the turn number where this spot is located.

3.2 TRIANGLE GRID GENERATION

According to the actual measurement data distribution rule, as Fig. 2 shows the data per loop links one by one in order; only adjacent two loop data points can link mutually. For the points in two adjacent loops, the existent condition is of two types:

1) the number of points in each loop is almost equal as shown in Fig. 2(a);

2) the number of points in each loop is different as shown in Fig. 2(b).

The links of all data points are as shown in Fig. 3.

Firstly, points p_0 and q_0 are linked. Only then two methods of linking can be chosen, either p_0 and q_1 or q_0 and p_1. According to the rule the triangle patch should be an isosceles triangle; we can judge which method is more reasonable. If p_0 and q_1 are more reasonable, $\Delta q_1 p_0 q_0$ is the best result. Then choosing $q_1\, p_0$ as one side of the isosceles triangle, we repeat the above-mentioned process to get another result, $\Delta q_1\, q_2\, p_0$. This process will go on till all data points in the spiral curve are linked.

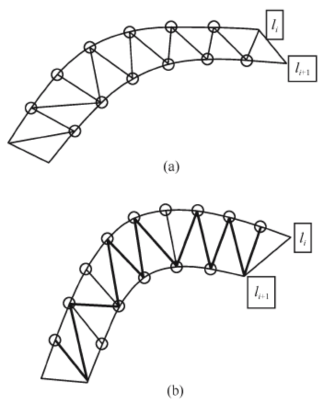


Fig. 2 The link of two adjacent data points
(a) Number of points in each loop is almost equal;
(b) Number of points in each loop is different

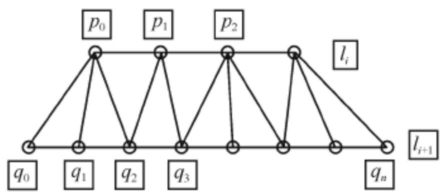


Fig. 3, The principle of shoe last measuring

3.3 VERTEX TABLE AND ADJACENCY TOP RELATIONSHIPS TABLE

To construct the vertex table and the triangle adjoin table, the following steps are involved:

1) Input all points into the vertex table;

2) Establish the original triangle mesh:

(i) choose two vertexes from the vertex table, and put them into the null table;

(ii) select two vertexes V_i *and* V_{i+1} from the next loop;

(iii) if $\Delta V_1 V_2 V_i$ is more reasonable than $\Delta V_1 V_2 V_{i+1}$, then chose $\Delta V_1 V_2 V_i$ and give the number of the angle; input two vertexes v_1 and v_i into the null TBL table; and input *v*2 into the head of the TBL table. Otherwise, choose $\Delta V_1 V_2 V_{i+1}$ and generate the number of the angle; input two vertexes v_1 and v_i into the null TBL table; and input v_2 into the head of the TBL table;

3) Main process of triangulation:

(i) get a point *NN* as the new vertex from the submit table according to two points in the TBL table, and seek the next circle, two neighbor points;

(ii) repeat the process of 2); **(iii)** end the process when the vertex table is null. Otherwise return to (i).

3.4 THE TRIANGLE PATCH PARAMETRIC

To divide each triangle patch into nine equal parts and let each center be barycentric, the algorithm step is as follows: let the parameter of the triangle surface be of the same step length:

$$u = 0, 1/3, 2/3, 1;\ v = 0, 1/3, 2/3, 1;\ w = 0, 1/3, 2/3, 1;$$

and then determine the node normal vector and orientation.

Suppose a surface is ***M***, the point sets $\boldsymbol{P\{P_1,P_2,P_3,\dots,P_n\}}$ are measured from ***M***, each $\boldsymbol{P_i}$ should correspond to a tiny slice plane $\omega(\boldsymbol{P_i})$, geometrically, $\omega\ (\boldsymbol{P_i})$ is the linear approximate of the surface *M* at the point P_i $\omega\ (P_i)$. can be defined by a center O_i and a unit normal vector $\boldsymbol{R_i}$ (corresponding to a point at O_i), i.e.: $\omega\ (P_i)=(O_i, \boldsymbol{R_i})$.

Because the geometric information of the part region of the surface is contained in the point sets on the region, the $\omega\ (P_i)$ of point P_i should be decided with the neighbor point sets around point P_i . if *Pi€P*, the neighbor point sets of a point P_i is combined with *k* points with which the point has the shortest distance, these points belong to *P* and include point P_i . so $K(P_i)=\{P_1,P_2,P_3,\dots,P_n\}$. Commonly, every four neighbor points can be used to construct a tiny slice plane. For p_i in $K(p_i)$, the equation of the plane is:

$$\boldsymbol{z = ax+by+c} \tag{2}$$

where, *a*, *b*, and *c* are constant. To make this plane equation, it is necessary to make sure that the distance square sum from the plane to all points in $K(p_i)$ are a minimum.

Therefore, we can consider choosing a proper constant *a*, *b*, and *c* to make sure the formula (3) has a minimum:

$$M := \left[\sum_{i=1}^{k} \left[z_i - \left(a\cdot x_i + b\cdot y_i + c\cdot z_i\right)\right]\right] \tag{3}$$

With M as a three-component function of the independent variable a, b, and c, the problem can be brought down to seek the minimum value in a point in:

$$M = M(a,b,c)$$

The solution is to use M to get a partial derivative to a, b, and c, respectively. That is: Considering various components inside the brackets that undertake settle amalgamations, and separate unknown a, b, c, we have:

$$\begin{cases} \dfrac{\partial M}{\partial a} = -2\sum_{i=1}^{k} x_i[z_i - (ax_i + by_i + c)] = 0 \\ \dfrac{\partial M}{\partial b} = -2\sum_{i=1}^{k} y_i[z_i - (ax_i + by_i + c)] = 0 \\ \dfrac{\partial M}{\partial c} = -2\sum_{i=1}^{k} [z_i - (ax_i + by_i + c)] = 0 \end{cases} \tag{4}$$

$$\begin{cases} a\sum_{i=1}^{k} x_i^2 + b\sum_{i=1}^{k} x_i y_i + c\sum_{i=1}^{k} x_i = \sum_{i=1}^{k} x_i z_i \\ a\sum_{i=1}^{k} x_i y_i + b\sum_{i=1}^{k} y_i^2 + c\sum_{i=1}^{k} y_i = \sum_{i=1}^{k} y_i z_i \\ a\sum_{i=1}^{k} x_i + b\sum_{i=1}^{k} y_i + ck = \sum_{i=1}^{k} z_i \end{cases} \tag{5}$$

According to the kth data points in the point sets $K(p_i)$ that belong to the point pi neighbor domain, the following:

$$\sum_{i=1}^{k} x_i^2, \sum_{i=1}^{k} y_i^2, \sum_{i=1}^{k} x_i y_i, \sum_{i=1}^{k} x_i z_i, \sum_{i=1}^{k} y_i z_i, \sum_{i=1}^{k} x_i, \sum_{i=1}^{k} y_i, \sum_{i=1}^{k} z_i$$

Replacing these items into Eq. (5), we have a three component simple equation group. The Gauss method is then used to get the solution of the equation group. By finding the minimum of two multiplication plane from the neighbor domain $K(p_i)$ of p_i, the normal vector $\boldsymbol{r}_i$ in the tiny slice plane $u(p_i)$ can be calculated.

4 APPLICATION EXAMPLES

To confirm the validity of the algorithm in the article, we have carried out the triangle grid division to the shoe last model data, made the primitive data point to triangularize, and saved it as the standard Simon Torrance Limited (STL) document format[2], [3]. The file is 2.14 MB, the shoe last model size is 275 mmx85 mmx125 mm. When the output precision of the STL format is 0.01 mm, there are 2 678 triangles on the surface model as shown in Fig. 4.

Fig. 4 Shoe last triangle model

5 CONCLUSIONS

In view of this kind of screw "cloud" data, this paper proposes a fast and effective closed surface modeling technology. It can effectively carry on the data processing, restructure surface to the profile scanning survey data, and enhance the product modeling efficiency. However, the technology such as surface structure based on the Bézier surface, needs to be studied further.

Acknowledgements: The paper is within the research activities of the SIMSANO project. The project is funded by Romanian Ministry of Education and Research within the framework of the 2nd National Plan, under the CNM Program.

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