ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume VII (XVII), 2008

CONSIDERATIONS ON THE GRINDER-GEAR GEARING THEORY

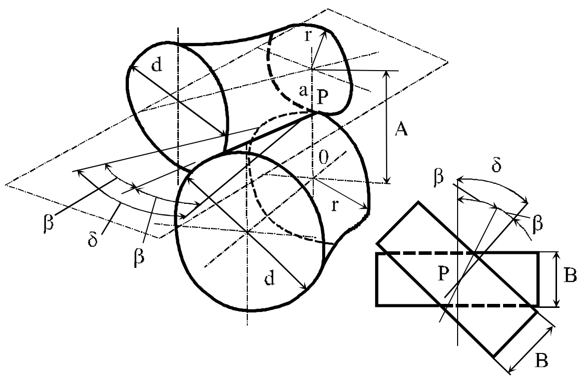
Dan GOMBOŞ, Ovidiu GAVRILESCU

University of Oradea,
dgombos@uoradea.ro, ogavrilescu@uoradea.ro

Keywords: rotation, gearing, generatrix, surfaces.

ABSTRACT: The gearing formed by the gear-grinder and the processing gear represents a cross-axed gearing, which has specific properties – the sliding between profiles – which allows, in fact, the processing.

A cross-axed gearing transmits the movement of rotation between two shafts whose axes are neither concurrent, nor parallel. In order to transmit the rotation between the two shafts, the existence of two teethed surfaces with a common generatrix is necessary. In the case of cross-axed transmission it is demonstrated that these surfaces are hyperboloidal, these being the only surfaces that allow a common generatrix. Thus, the cross-axed gearings have two hyperboloidal surfaces as rolling surfaces, tangent to one another by a line. Figure 1 shows the transmission between crossed axes in an exterior gearing, the main elements of this transmission being shown, whereas figure 2 shows an interior gearing transmission.



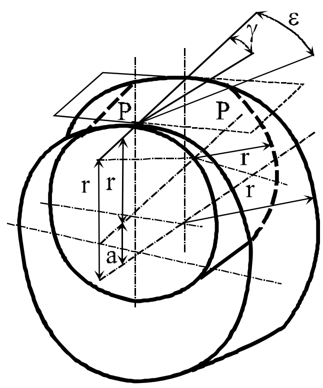


Figure 1. Cross-axed transmission Figure 2. Cross-axed transmission (exterior gearing) (interior gearing)

The execution of a teething on the two hyperboloids in contact leads to the forming of a hyperboloidal gearing. Since the execution of teething on hyperboloidal surfaces implies outstanding technological difficulties, this gearing is replaced with a crossed cylindrical gearing, made out of two cross-axed gears with inclined teeth. Through this rolling surface switch – the hyperboloidal surfaces being replaced with cylindrical surfaces – there arises a fundamental modification of the gearing, as to the fact that the rolling is no longer realized in accordance to a common generatrix, but around a point instead. The contact between the flanks of the teeth is done, therefore, in a single point, which greatly reduces the possibility of transmitting torsion moments.

Since in this type of gearing the peripheric velocities in the contact point are differently faced – because of the crossed axes – the outcome is that between the two surfaces there is no pure rolling movement. In this gearing there arises a relative sliding of the flanks' profiles of the teeth, which contributes to their accentuated usage.

Given these inconveniences, the cylindrical gearing is only utilized in cinematic purposes. The disadvantages of this gearing morph into advantages for the grinding process, since the relative sliding helps with lifting splinters, whereas the punctiform contact determines a large contact pressure.
The cylindrical cross-axed gearing is composed, as previously shown, of two cross-axed gears with inclined teeth. It is known from the theory of gearing that each of the two gears can gear with an inclined-tooth cremailliere; the rolling cylinders of the two gears roll without sliding over the planes of rolling of the cremaillieres. Furthermore, each inclined-tooth cremailliere can be attached to a straight-tooth cremailliere, whose profile is identical to that of the normal profile of the cremailliere it is being attached to. The gearing of the two inclined-tooth gears takes place if the associated straight-tooth cremaillieres overlap, condition which determines the calculus of the angle between the axes:

$$\delta_A = \beta_{r1} \pm \beta_{r2} \tag{1}$$

where:βr1; βr2 – the angles of inclination of the rolling cylinders for the two gears.
The crossing angle of the two gears depends on the angles of inclination of the rolling cylinders' teeth. Through the modification of the rolling cylinders' diameters – done by displacing the gears' profiles – a modification in the angle of crossing δA is obtained. The calculus of the angle δA being laboriously modified, the use of zero gearings or zero displaced gearings is recommended, for which:

- the angle between the axes of the two gearing gears is equal to the angle between the two inclined-tooth cremaillieres, measured in all the directions of their displacement
- the gearing's transmission report is that of the angular speeds of the two gears:

$$i = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{z_2}{z_1} \tag{2}$$

In the normal plane of the gearing, the peripheric speeds are equal, therefore:

$$\omega_1 \cdot R_{r1} \cdot \cos\beta_{r1} = \omega_2 \cdot R_{r2} \cdot \cos\beta_{r2} \quad deoarece: \quad v_1 \cos\beta_{r1} = v_2 \cos\beta_{r2};$$
$$i = \frac{R_{r2} \cdot \cos\beta_{r2}}{R_{r1} \cdot \cos\beta_{r1}}; \tag{3}$$

or, for the zero or zero-displaced gearing:

$$i = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{R_{d2} \cdot \cos\beta_2}{R_{d1} \cdot \cos\beta_1}; \tag{4}$$

- the distance between the gearing's axes is the sum of the rolling radiuses:

$$A = R_{r1} + R_{r2} = (R_{d1} + R_{d2}) \cdot \frac{\cos\alpha_n}{\cos\alpha_w} = \frac{m_n}{2} \cdot \left(\frac{z_1}{\cos\beta_1} + \frac{z_2}{\cos\beta_2} \right) \cdot \frac{\cos\alpha_n}{\cos\alpha_w}, \tag{5}$$

relation which demonstrates that for the same number of teeth, the distance between the axes varies dependant to β1, β2,αw.
The diameter of the beginning of the active profile Da1 which, in concordance with figure 2, is deducted from the formula:

$$D_{a1}^2 = d_{b1}^2 + K_1 \cdot S_2^2 \tag{6}$$

where K1·S2: - represents the minimum curving radius of the active profile in frontal plane, and is noted with ρmin1. This radius is deducted as following:

$$\rho_{\min 1} = K_1S_2 = S_1K_1 - S_1S_2; \qquad S_1K_1 = \rho_{\max 1} = 0{,}5 \cdot \sqrt{d_{a1}^2 - d_{b1}^2}; \tag{7}$$

where K1·S1: - represents the minimum curving radius of the active profile in frontal plane, given by the preceding relation,

S1·S2: - is the length of the frontal plane gearing segment

$$S_1S_2 = l_a = S_1P + S_2P; \qquad S_1P = S_1K_1 - PK_1 = \sqrt{O_1S_1^2 - O_1K_1^2} - O_1P \cdot \sin\alpha_w \tag{8}$$

because:

$$O_1S_1 = 0{,}5 \cdot d_{a1} \quad ºi \quad O_1K_1 = 0{,}5 \cdot d_{b1}, \ \ se\ obtine$$

$$S_1P = 0{,}5 \cdot \sqrt{d_{a1}^2 - d_{b1}^2} - O_1P \cdot \sin\alpha_w \quad similar \quad S_2P = 0{,}5 \cdot \sqrt{d_{a2}^2 - d_{b2}^2} - O_2P \cdot \sin\alpha_w; \tag{9}$$

$$S_1S_2 = S_1P + S_2P = 0{,}5 \cdot \left(\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2}\right) - \left(O_1P + O_2P\right) \cdot \sin\alpha_w;$$

but:

$$O_1P + O_2P = O_1O_2 = a_w \tag{10}$$

and finally, the outcome is:

$$l_a = 0{,}5 \cdot \left(\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2}\right) - a_w \cdot \sin\alpha_w; \tag{11}$$

the angle αw represents the gearing angle of the two gears, and is calculated by the formulas:

$$\cos\alpha_w = \frac{a}{a_w} \cdot \cos\alpha_t; \quad sau \quad inv\alpha_w = 2 \cdot \frac{x_1 + x_2}{z_1 + z_2} \cdot tg\alpha_n + inv\alpha_t. \tag{12}$$

The minimum range of the active profile's curving becomes:

$$\rho_{\min 1} = 0{,}5 \cdot \sqrt{d_{a1}^2 - d_{b1}^2} - l_a = a_w \cdot \sin\alpha_w - 0{,}5 \cdot \sqrt{d_{a2}^2 - d_{b2}^2}. \tag{13}$$

Another essential element of the processed gear, the importance of which will be noticed in the grinder-gear gearage, is represented by the angle between the gearing line and the generator of the base cylinder (σ1), and is calculated from the condition that the pitch of the tooth's helix on the base cylinder be equal to the pitch of the helix on the division cylinder. This angle is equal to the angle of ascent of the teeth's helix on the base cylinder. It is of importance, as it determines the direction of the gearing line (figure 3). The pitch of the grinder's teeth's helix on the division cylinder is equal to that of the base cylinder.

$$\eta_1 = 2 \cdot \pi \cdot r_1 \cdot ctg\beta_1 = 2 \cdot \pi \cdot r_{b1} \cdot ctg\beta_{01} = \pi \cdot d_1 \cdot ctg\beta_1 = \pi \cdot d_{b1} \cdot ctg\beta_{01} \tag{14}$$

where from the calculus of the base cylinder's radius results:

$$r_{b1} = r_1 \cdot \frac{ctg\beta_1}{ctg\beta_{01}}; \quad (*) \tag{15}$$

Since between the division diameter and the base diameter, for a gear with exterior teething, there exists the relation:

$$d_{b1} = d_1 \cdot \cos\alpha_{t1}; \tag{16}$$

where αt1is the angle of the reference profile in frontal plane:

$$tg\alpha_{t1} = \frac{tg\alpha_n}{\cos\beta_1}; \quad \cos\alpha_{t1} = \frac{1}{\sqrt{tg^2\alpha_{t1} + 1}} = \frac{\cos\beta_1}{\sqrt{tg^2\alpha_n + \cos^2\beta_1}}; \tag{17}$$

The relation of the base cylinder's radius results:

$$r_{b1} = r_1 \cdot \frac{\cos\beta_1}{\sqrt{tg^2\alpha_n + \cos^2\beta_1}}; \tag{18}$$

Taking into account the relation (*), the result is:

$$\frac{ctg\beta_1}{ctg\beta_{01}} = \frac{\cos\beta_1}{\sqrt{tg^2\alpha_n + \cos^2\beta_1}}; \qquad (19)$$

or for the inclination angle of the base cylinder's teeth:

$$ctg\beta_{01} = \frac{\sqrt{tg^2\alpha_n + \cos^2\beta_1}}{\sin\beta_1}. \qquad (20)$$

Through the expansion of this relation, by expressing the cotangent taking into account the sinus and cosinus, the outcome is:

$$\frac{1}{\sqrt{1+ctg^2\beta_{01}}} = \frac{\sin\beta_1}{\sqrt{1+tg^2\alpha_n}} = \cos\alpha_n \cdot \sin\beta_1; \qquad (21)$$

Since the angles β01 and σ1 are complementary, for the angle σ1, there results:

$$\cos\sigma_1 = \sin\beta_{01} = \cos\alpha_n \cdot \sin\beta_1. \qquad (22)$$

For the avoidance of contact between the grinder and the big toothed anullus, the distance between the axes of the grinded gear and the grinder must be greater that the sum of distances from the point A to the axes of the grinder and the gear in vertical plane:

$$A_{1s} = 0{,}5 \cdot \left(D_{sr} + D_{1r}\right) > y + y'. \qquad (23)$$

For the design calculus of a new grinder, in correlation with figure 3, the accomplishment of the following essential points of the design calculus must be taken into account. The value of the inclination angle between the grinder's ax and that of the gear is recommended to be between 10÷15°, reaching up to values of 5° for grinding gears having their teething in the vicinity of a shoulder.

For the design of a new grinder, the accomplishment of a crossed zero-cylinder is taken into account, in which the distance between the axes is equal to or close to the reference distance, and the gearing angle is situated in proximity to the angle of the reference profile. Thus, for the new grinder there is expected an increase in angle of 1°÷1°30`, and the calculus of its elements is done starting from this angle. For the used grinder, the gearing angle is smaller by 1°÷1°30` than that of the reference profile's angle. The inclination angle of the grinder's teeth on the division cylinder is calculated by the formula:

$$\beta_{ds} = \beta_1 \pm \Sigma, \qquad (24)$$

in which the sign (+) is used for the inclination of the teeth in the same direction, while the sign (-) is used for the inclination in different directions.

The angle of the reference profile of the grinder's tooth in frontal plane is calculated with the relation:

$$tg\alpha_f = \frac{tg\alpha_n}{\cos\beta_{ds}}, \qquad (25)$$

whereas the angle between the gearing line and the generator of the base cylinder is calculated with the relation:

$$\cos\sigma_s = \cos\alpha_n \cdot \sin\beta_{ds}. \qquad (26)$$

The main elements of the grinder-to be processed gear are presented in figures 4 and 5.

The calculus of the grinder is done in three stages:

- *stage one* – it is made out of the calculus of the new grinder, for which there have not been made any resharpenings;
- *stage two* – it is made out of the calculus of the half-used grinder;
- *stage three* – it is made out of the calculus for the used grinder.

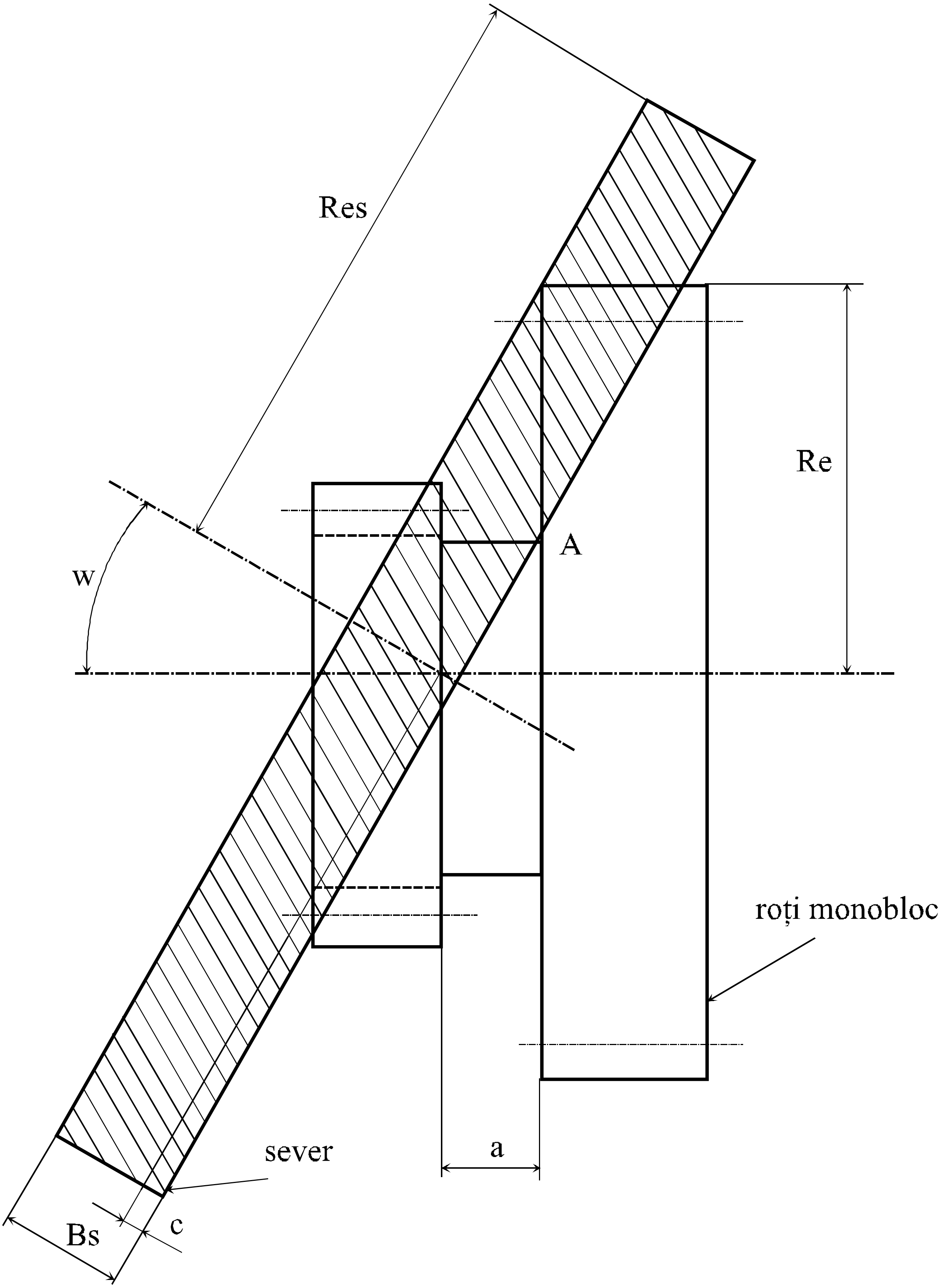


Figure 3. The grinding of monoblock gears.

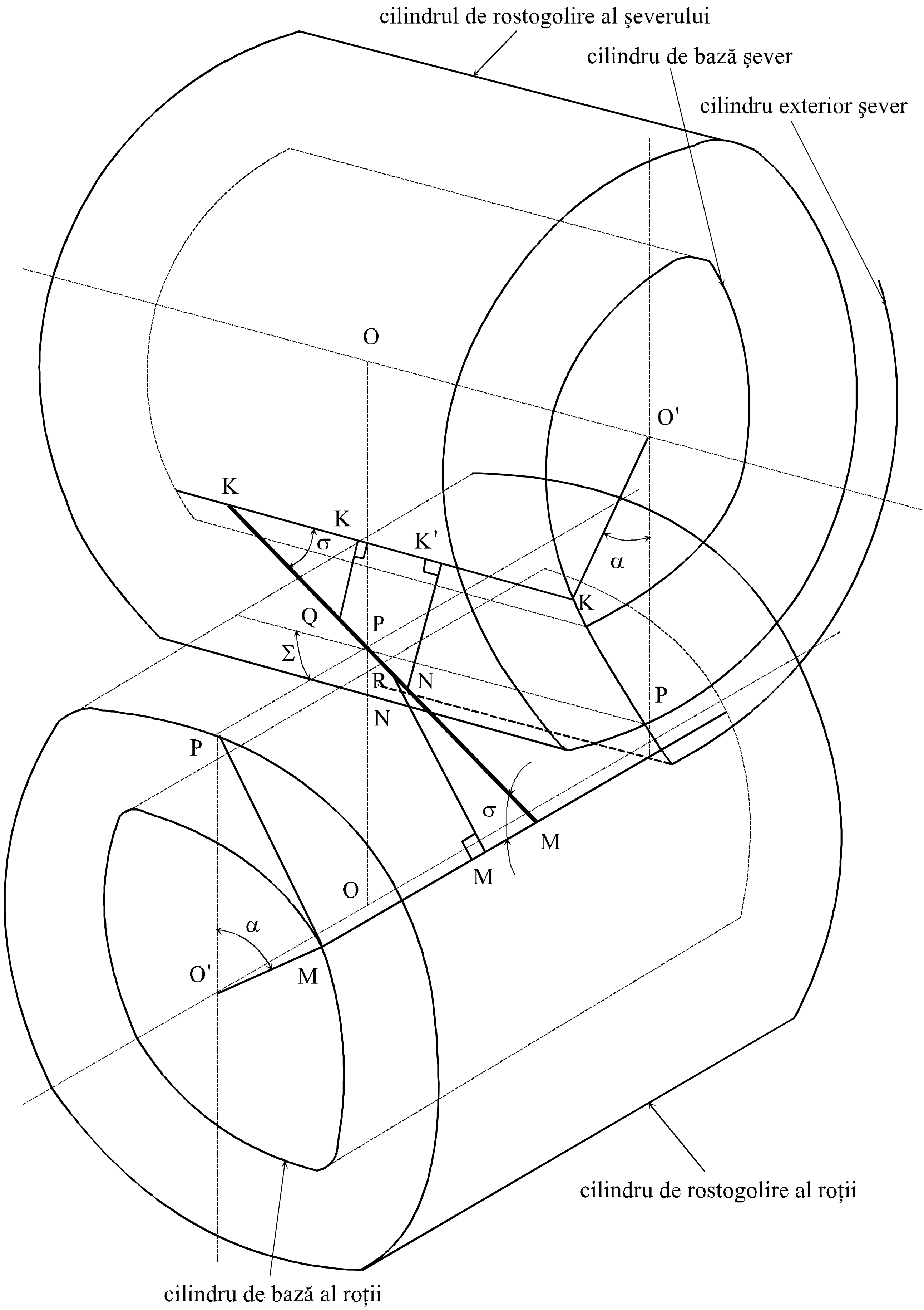


Figure 4. The elements of the grinder-gear gearing.

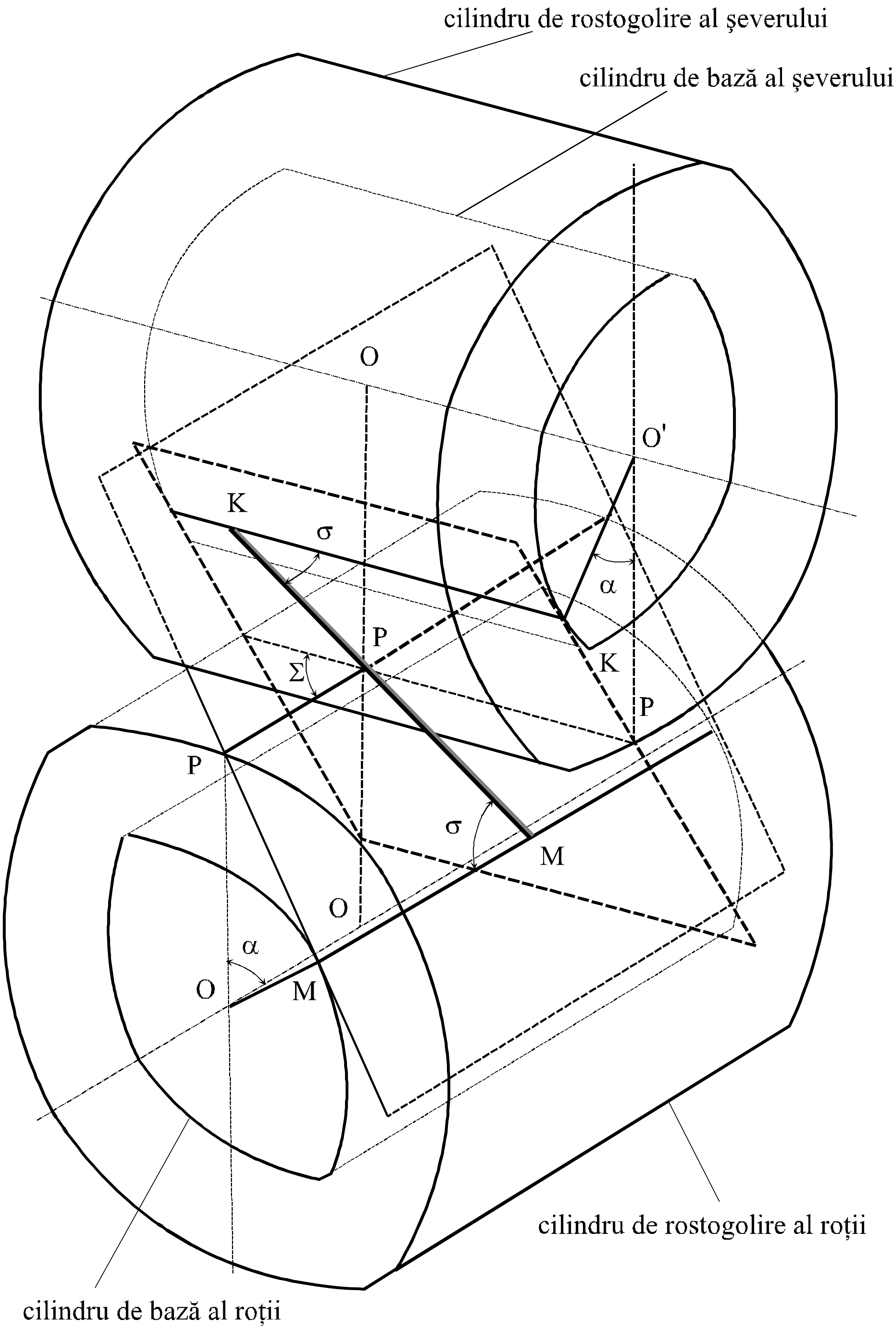


Figure 5. The gearing line for the grinder-gear gearing.

In all of these three stages of calculi, the starting point must be the gearing angle, which for the second stage is equal to the angle of the reference point. This determines the gearing's parameters, and imposes the profile displacements of the grinder's teeth. It is noticeable that, from the very start, the calculus of the grinder differs from that of the stock grinders, because the starting point is the zero or zero-displaced gearing, obtaining the displaced gear-type grinder, the displacement of which depends on the displacement of the gear to be processed. It is reminded that the design of stock grinders is done for the purpose of obtaining an undisplaced profile for the half-used grinder. The inconvenience of this method has been previously mentioned, and will be noticed in the verification calculi for the utilization of these types of grinders for processing displaced-teeth gears.
Another important element of this gearing is the length of the gearing line length, the importance of which resides in assuring a proper gearing throughout the entire length of the profile. According to the notations in figures 4 and 5, the following is deduced:

$$KM = KP + PM = \frac{\sqrt{O_s^{'}P_s^2 - O_s^{'}K_s^2}}{\sin\sigma_s} + \frac{\sqrt{O_1^{'}P_1^2 - O_1^{'}M_1^2}}{\sin\sigma_1}, \qquad (27)$$

where:

$KM = L_{1s}$ – *the length of the grinding line*;

$O_1^{'}P_1 = \frac{1}{2}\cdot D_{1r}$ – *the length of the gear's rolling cylinder*;

$O_1^{'}M_1 = \frac{1}{2}\cdot d_{b1}$ – *the radius of the gear's base cylinder*;

$O_s^{'}P_s = \frac{1}{2}\cdot D_{sr}$ – *the radius of the grinder's rolling cylinder*;

$O_s^{'}K_s = \frac{1}{2}\cdot D_{bs}$ – *the radius of the grinder's base cylinder*.

From these notations results:

$$L_{1s} = \frac{\sqrt{D_{1r}^2 - d_{b1}^2}}{2\cdot\sin\sigma_1} + \frac{\sqrt{D_{sr}^2 - D_{bs}^2}}{2\cdot\sin\sigma_s}. \qquad (28)$$

For the calculus of the exterior diameter of the grinder, the biggest curving radius of the grinder's tooth in frontal plane must be taken into account, necessary for the processing of the entire active flank of the gear. This curving radius is calculated depending on the minimum curving radius of the gear's active profile in frontal plane, which is decreased by a value Δl, in order to allow the processing of the gear's profile on a length greater than that of the active profile. The value Δl must be chosen so that the head of the grinder's tooth not process the unevolventic area from the tooth's foot of the gear to be processed. It is recommended that this value be calculated by the formula:

$$\Delta l = \frac{0{,}15\cdot m_t}{\sin\alpha_t}. \qquad (29)$$

The largest curving radius of the grinder's tooth's profile results form figure 4, and is deduced thus:

$$L_{1s} = KM = KN_s + RM - RN_s; \quad \textit{with the following notations}$$

$$KN_s = \frac{\rho_{s\max}}{\sin\sigma_s};\ RM = \frac{\rho_{\min 1}}{\sin\sigma_1};\ RN_s = \frac{\Delta l}{\sin\sigma_1}\cdot \quad \textit{results} \qquad (30)$$

$$L_{1s} = \frac{\rho_{s\max}}{\sin\sigma_s} + \frac{\rho_{\min 1} - \Delta l}{\sin\sigma_1}; \quad sau \quad \rho_{s\max} = \left[L_{1s} - \frac{\rho_{\min 1} - \Delta l}{\sin\sigma_1}\right]\cdot\sin\sigma_s\cdot$$

An important element in the design of the grinder is the variation of the fixture during the grinding, of the exterior cylinder of the grinder and the interior cylinder of the gear, which must be between (0.1÷0.15)·mn.
The thickness of the grinder's tooth on the division cylinder is calculated related to the thickness of the gear's tooth on the division cylinder. Thus, the thickness of the gear's tooth on the division cylinder is calculated, from which results the thickness of the gear's tooth on the rolling cylinder. The thickness of the grinder's tooth on the rolling cylinder results as the difference between the normal pitch of the rolling cylinder and the thickness of the gear's tooth on the rolling cylinder. Thereafter, the thickness of the grinder's tooth on the division cylinder is calculated.
The thickness of the gear's tooth on the division cylinder, in normal plane, is given by the relation:

$$s_{d1} = m_n \cdot \left(\frac{\pi}{2} + 2 \cdot x_1 \cdot tg\alpha_n \right); \tag{31}$$

On the rolling cylinder, the thickness of the tooth is equal to:

$$s_{r1} = D_{1r} \cdot \cos\beta_{r1} \cdot \left(\frac{s_{d1}}{d_1 \cdot \cos\beta_1} + inv\alpha_t - inv\alpha_{1f} \right); \tag{32}$$

and the pitch on the rolling cylinder, in normal plane, is:

$$t = \frac{\pi \cdot D_{r1}}{z_1} \cdot \cos\beta_{r1}, \tag{33}$$

where βr1 is the inclination angle of the gear's teeth on the rolling cylinder.
Thus results the thickness of the grinder's tooth on the rolling cylinder, in normal plane, having the value:

$$s_{rs} = t - s_{r1}, \tag{34}$$

whilst the thickness of the grinder's tooth on the division cylinder, in normal plane, will be:

$$s_{ds} = \left[\frac{D_{ds}}{D_{sr}} \cdot \frac{s_{rs}}{\cos\beta_{rs}} - D_{ds} \cdot \left(inv\alpha_f - inv\alpha_{sf} \right) \right] \cdot \cos\beta_{ds}, \tag{35}$$

where:

- Dds – the division diameter of the grinder;
- Dsr – the rolling diameter of the grinder;
- Brs – the inclination angle of the grinder's teeth on the rolling cylinder.

REFERECNCES :

1. Belous, V. Sinteza sculelor aşchietoare. Iaşi, Editura Junimea 1980.
2. Gavrilescu, O., Bococi, D., ş.a. Prelucrarea roţilor dinţate prin şeveruire. Proiectarea şeverelor–disc, Editura universităţii din Oradea, ORADEA 2001.
3. Kleticov, V.D. Sevingprocess, Moskva, Masgiz, 1996.