

APPROACH TO SENSITIVITY OPTIMIZATION

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Abstract: In this paperwork are presented a series of theoretical notions regarding elements of sensitivity in optimization analysis of spatial structures. First are presented briefly the basic notions of design optimization, such as Design variables, Objective function, and Behavior constraints, and what they can be in COSMOS/M. Then is presented the notion of Sensitivity in relation with optimization analysis and the sensitivity types in COSMOS/M: global sensitivity, offset sensitivity, local sensitivity, or optimization sensitivity results, and finally conclusions regarding using sensitivity analysis prior optimization analysis.

1. DESIGN OPTIMIZATION

Design optimization refers to the automated redesign process that attempts to minimize or maximize a specific quantity (objective function) subject to limits or constraints on the response by using a rational mathematical approach to yield improved designs.

Figure 1 shows minimum weight design of a structure.

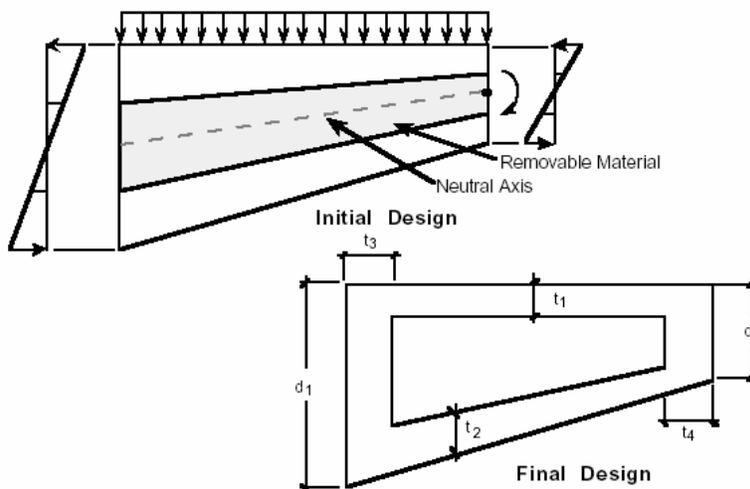


Fig. 1. Minimum weight design of a structure

A *feasible design* is a design that satisfies all of the constraints. A feasible design may not be optimal. An *optimum design* is defined as a point in the design space for which the objective function is minimized or maximized and the design is feasible. If relative minima exist in the design space, other optimal designs can exist.

Basic terminology in design optimization are: **design variables**, **objective function**, and **behavior constraints**. They are explained in the following sections.

1.1 Design Variables

Design variables are the parameters (independent quantities) that users seek to find their values for an optimum design.

Figure 2 shows a structure having four geometry dimensions defined as design variables.

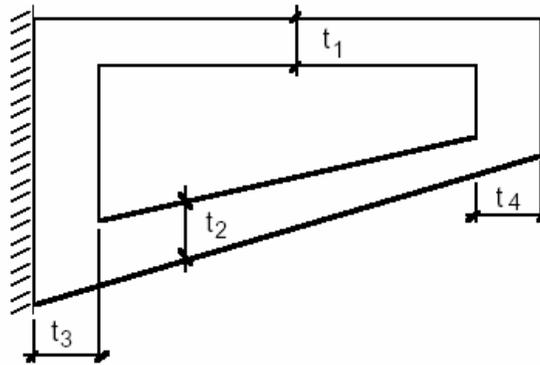


Fig. 2. Structure with 4 design variables

Upper and lower bounds are specified for each design variable. Lower and upper bounds are also referred to as *side constraints*.

$$\text{Lower Bound} \quad 10 \leq t_1 \leq 25 \quad \text{Upper Bound}$$

Depending on design variables, there are two types of optimization applications: *sizing* optimization and *shape* optimization.

Sizing optimization refers to the class of problems where a change in design variables does *not* change the problem's geometry or mesh.

Shape optimization refers to the class of problems where any change in design variables causes changes in the problem's geometry or mesh.

Besides purely sizing optimization and shape optimization mentioned above, there is a class of problems where both sizing and shape parameters are defined as design variables.

1.2 Objective Function

Objective function is a single quantity that the optimizer seeks to minimize or maximize. The objective function must be a continuous function of the design variables.

The **weight** (or volume) of a structure is an example of the commonly used objective functions.

Other possible quantities are:

- Stress;
- Reaction Force;
- Natural Frequency;
- Temperature Gradient;
- User-Defined Functions.
- Strain;
- Velocity;
- Linearized Buckling Load Factor;
- Heat Flux;
- Displacement;
- Acceleration;
- Temperature;
- Fatigue Usage Factor;

The objective function can be composed of different sets of the same type, and can reflect different weight (importance) factors for different portions of the model.

1.3 Behavior Constraints

A behavior constraint is defined as an inequality that must be satisfied in order to have a feasible design. The behavior constraints are typically response quantities that are functions of the design variables. Von Mises stress is a typical example in structural problems.

Ex: von Mises stress \leq allowed stress

Other possible quantities are:

- Volume;
- Strain;
- Velocity;
- Buckling Load Factor;
- Heat Flux;
- User-Defined Functions.
- Weight;
- Displacement;
- Acceleration;
- Temperature;
- Fatigue Usage Factor;
- Stress;
- Reaction Force;
- Natural Frequency;
- Temperature Gradient;

Multiple constraint sets of different types can also be specified.

In COSMOS/M, users have to specify lower and upper limits for constraints.

For example: $0 \leq$ von Mises stress \leq allowed stress

2. SENSITIVITY STUDY

A sensitivity study is the procedure that determines the changes in a *response quantity* for a change in a *design variable*. Figure 3 shows a sensitivity study of a control arm bracket and Figure 4 shows its result.

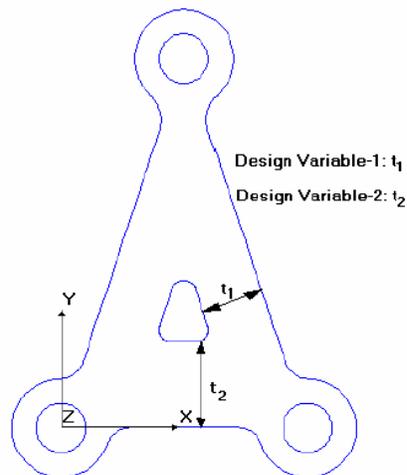


Fig. 3. Sensitivity study of a control arm bracket in Frequency analysis

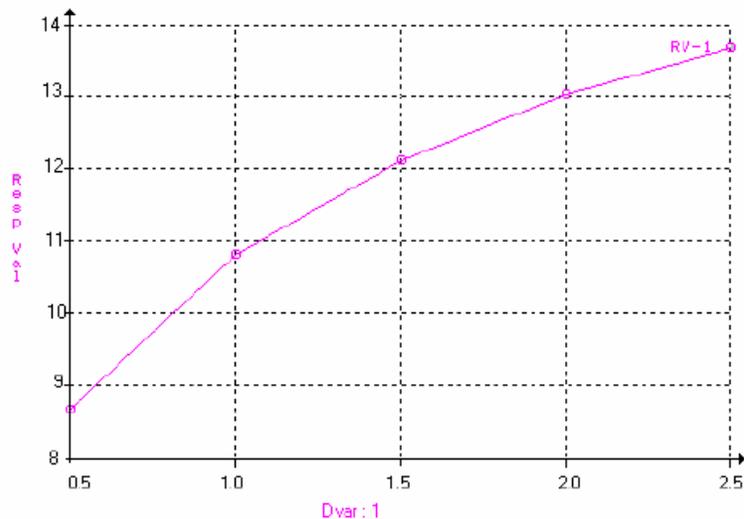


Fig. 4. Fundamental frequency versus Design variable-1, t_1

Basic terminology in sensitivity study are: *Design variables* and *response quantities*. The definition of design variables is the *same* as that in design optimization. Response quantities are functions of the design variables. All the post processing quantities which are suitable for the *objective function* and *behavior constraints* are also suitable for the sensitivity response quantities.

3. SENSITIVITY TYPES

In COSMOS/M there are four types of sensitivity study, namely, global sensitivity, offset sensitivity, local sensitivity, and optimization sensitivity results. They are explained in the following paragraphs.

3.1 Global sensitivity

Is where design variables are changed between their lower and upper bounds in a user-specified number of *steps*. The number of steps is the *same* for all the design variables. Under this type of sensitivity, the user can change all the design variables *simultaneously* or *one at a time*. Have the frequency analysis of a control arm bracket as an example where:

$$0.5 \leq \text{design variable-1} \leq 2.5 \text{ and}$$

$$1.5 \leq \text{design variable-2} \leq 3.5$$

The plots of response quantity versus design variable are shown in Figure 5, 6 and 7.

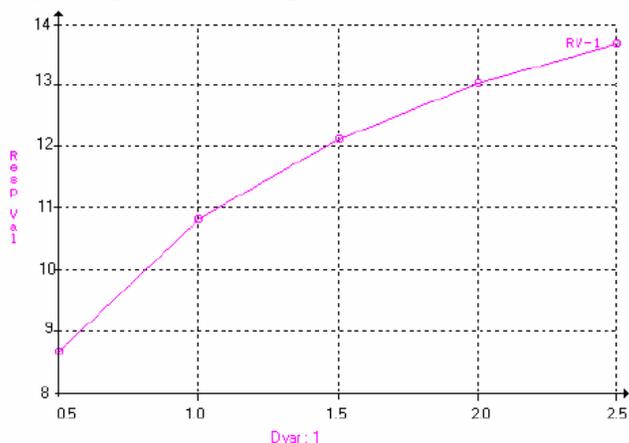


Fig. 5. Global sensitivity - One at a time: fundamental frequency versus Design variable-1, t_1

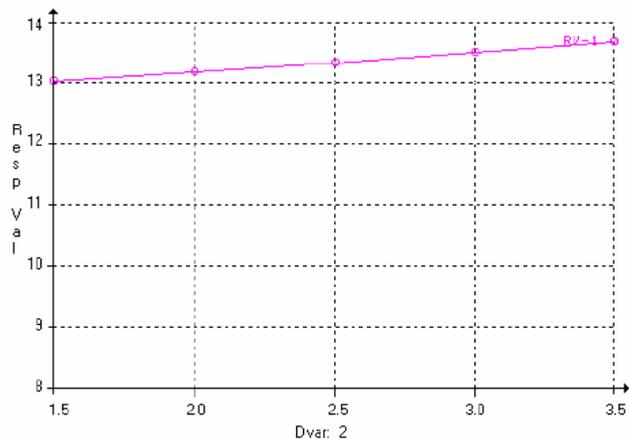


Fig. 6. Global sensitivity - One at a time: fundamental frequency versus Design variable-2, t_2

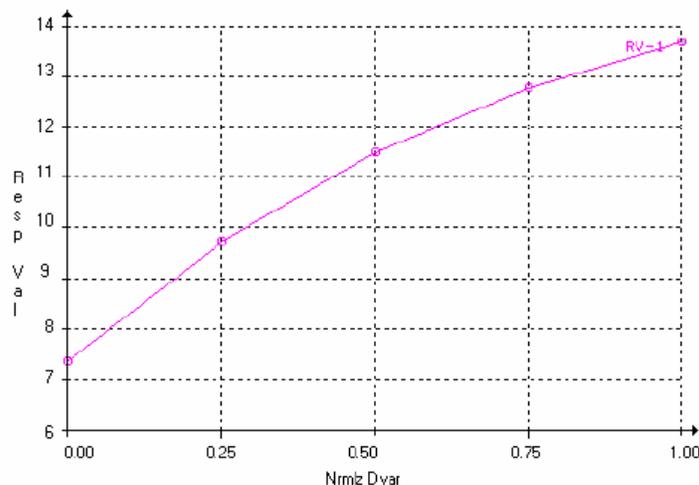


Fig. 7. Global sensitivity - Simultaneously: fundamental frequency versus normalized Design variable-1 and -2

3.2 Offset sensitivity

Is where users specify the values of a series of design variables in a user-defined sets. The design variables are defined either by the *actual values* or by a *perturbation ratio* with respect to the initial value.

Have the frequency analysis of a control arm bracket as an example where the series of design variables are:

Sensitivity Set Number	Design Variable-1	Design Variable-2
1	0.5	3.5
2	1.0	3.0
3	1.5	2.5
4	2.0	2.0
5	2.5	1.5

The plot of response quantity versus sensitivity set is shown in Figure 8.

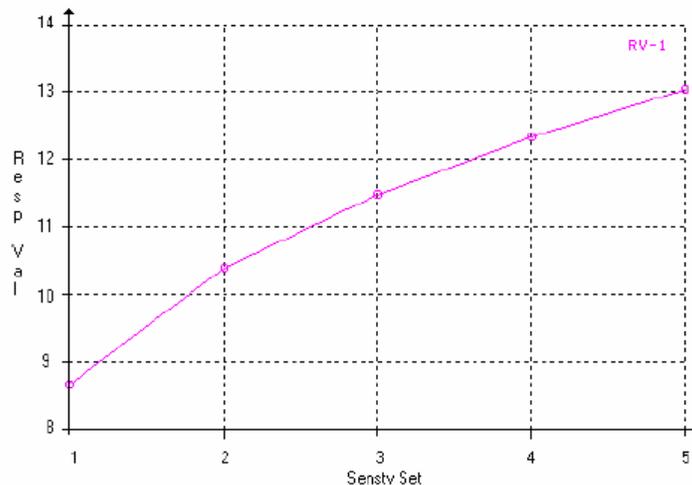


Fig. 8. Offset sensitivity: fundamental frequency versus Sensitivity set number

3.3 Local sensitivity

Is where a design variable is *perturbed* at a time by a user specified value while the rest of the design variables are kept unchanged. The perturbed design variables are defined either by the *actual values* or by a *perturbation ratio* with respect to the initial value. The *gradients* of the response quantities with respect to the design variables are computed based on the finite difference method.

Have the frequency analysis of a control arm bracket as an example where:

Initial Value: design variable-1=2.5,
design variable-2=3.5

Perturbation Ratio: design variable-1=+0.1,
design variable-2=+0.1

The plot of gradient of the response quantity versus design variable set is shown in fig.9.

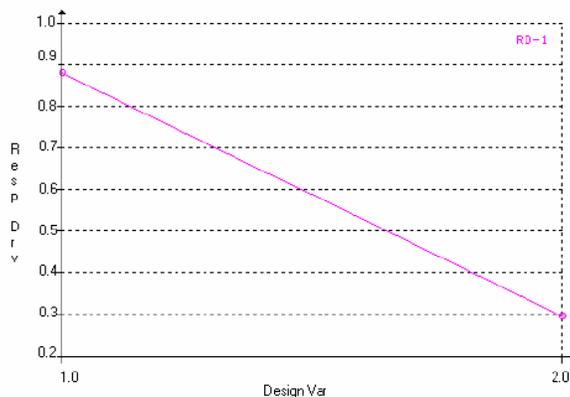


Fig. 9. Local sensitivity: Gradient of fundamental frequency versus Design variable number

3.4 Optimization sensitivity results

Is where both gradients of behavior constraints and objective function are computed during the optimization process. The gradients are obtained by taking the derivatives of the *approximation* functions with respect to the design variables. This type of sensitivity study is available *only* when the design optimization is to be performed.

4. CONCLUSIONS

By performing a sensitivity study before optimization analysis, using one of the methods described previously (global sensitivity, offset sensitivity, local sensitivity, or optimization sensitivity results), can be drawn conclusions about response quantity in objective function versus design variable perturbation, and therefore can be retained for subsequent optimization analysis only those design variables that produce a significant response in objective function, and by that reducing the time and difficulty of analysis.

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