A GENERALIZED TRANSPORTATION MODEL IN SOCIAL ASSISTANCE MANAGEMENT

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Abstract: The most common decision methods in choosing the method of supply in social assistance management is that of auction. Just one supplier is usually the winner, but ones offer is sometimes not the optimum version. The optimum should be considered not only in financial terms but also in time terms. We solve the problem of optimally managing the food and medical care supply by means of multi-index transportation model of tri-axial type. We use the algorithm of K. B. Haley [2] for problem solving, with a model of computation elaborated in Excel. The example we give refers to the optimization of the food supply of the network of Care Centers for Elderly in Arad County.

1. INTRODUCTION

The transportation problem solves one product supply problems. Generally in social assistance, the supply problems refer to more products as food, drugs, clothes, etc. Therefore the classical transportation problem is not enough. A multi-index transportation model is suitable in this framewotk. We deal with tri-axial problem [3], which contains three types of linear constraints.

The standard form of the model is, according to [1]:

$$f = \sum_i \sum_j \sum_k c_{ijk} x_{ijk} \to \min, \tag{1}$$

submitted to:

$$\sum_{k=1}^{p} x_{ijk} = a_{ij}, \quad i \in I, \quad j \in J, \tag{2}$$

$$\sum_{i=1}^{m} x_{ijk} = b_{jk}, \quad j \in J, \quad k \in K, \tag{3}$$

$$\sum_{j=1}^{n} x_{ijk} = c_{ik}, \quad i \in I, \quad k \in K, \tag{4}$$

$$x_{ijk} \geq 0, \quad i \in I, \quad j \in J, \quad k \in K \tag{5}$$

The terms on the right side should satisfy the conditions:

$$\sum_j a_{ij} = \sum_k c_{ik} = a_i, \quad i \in I, \tag{6}$$

$$\sum_{k=1}^{p} x_{ijk} = a_{ij}, \quad i \in I, \quad j \in J, \tag{7}$$

$$\sum_j b_{jk} = \sum_i c_{ik} = c_k, \quad k \in K, \tag{8}$$

$$\sum_i a_i = \sum_j b_j = \sum_k c_k = S, \tag{9}$$

$$c_{ijk} \geq 0, \quad a_{ij} > 0, \quad b_{jk} > 0, \quad c_{ik} > 0, \quad i \in I, \quad j \in J, \quad k \in K.$$

The following existence and optimality results have been proven in [2].

Theorem 1. *If m, n, p* > 1*, then the number of the linear independent equations from the constraints of the model does never exceed the number of unknowns.*

Theorem 2. *The tri-axial problem has an infinity of admissible solutions.*

Theorem 3. *The tri-axial problem has at least an optimal solution.*

Remark 4. The tri-axial transportation problem is non-degenerated if any program contains exactly $M = mnp - (m-1)(n-1)(p-1)$ values $x_{ijk} > 0$, the other ones being 0. If a program contains $M - r$ values $x_{ijk} > 0 \quad (r \in N \geq 1)$ the problem is degenerated, having a multi-plane degeneration of order r .

In order to determine an optimal program of a tri-axial transportation problem we use the Haley [2] procedure:

Step a. Determine a scale repartition using the recurence formula

$$u_{ij} + v_{jk} + w_{ik} = c_{ijk}, \tag{10}$$

which is admissible if the non-negativity conditions:

$$x^{\circ}_{ijk} = \min(a_{ij} - \sum_{k=1}^{k-1} x^{\circ}_{ijk}, b_{jk} - \sum_{i=1}^{i-1} x^{\circ}_{ijk}, c_{ik} - \sum_{j=1}^{j-1} x^{\circ}_{ijk}) \geq 0, \quad i \in I, \ j \in J, \ k \in K.$$

are fulfilled.

Step b. Verify the optimality of the solution extending the modified distributive method (see [1]) to this type of problems. To do that one solves the system (10), c_{ijk} being the costs corresponding to the values $x_{ijk} > 0$. As in case of a non-degenerate solution there are exactly

$$M = mn + np + mp - (m + n + p - 1)$$

values $x_{ijk} > 0$, and the system (10) has $mn + np + mp$ unknowns, one consider arbitrary $m + n + p - 1$ unknowns equal to 0. So, the system (10) will ce compatible, with an unique solution. Then, one determines the costs

$$\bar{c}_{ijk} = u_{ij} + v_{jk} + w_{ik} \tag{11}$$

corresponding to values $x_{ijk} = 0$.

The following situations are possible:

- If: $c_{ijk} > \bar{c}_{ijk}$, for $x_{ijk} = 0$,
  $c_{ijk} = \bar{c}_{ijk}$, for $x_{ijk} > 0$,
  then the solution is optimal and unique.
- If $c_{ijk} > \bar{c}_{ijk}$, for $x_{ijk} = 0$,
  $c_{ijk} = \bar{c}_{ijk}$, for some $x_{ijk} = 0$,
  then the problem has an inhinity of optimal solutions.
- If $c_{ijk} < \bar{c}_{ijk}$, for some $x_{ijk} = 0$
  then the solution is not optimal and may be improved.

Step c. Improve the solution for each cycle corresponding to the cells in which

$$c_{ijk} < \bar{c}_{ijk}.$$

The optimality is verified after improving each cycle, which leads to huge calculus if there are numerous cells in which $c_{ijk} < \bar{c}_{ijk}$.

2. APPLICATION IN SOCIAL ASSISTANCE MANAGEMENT

We use the above described multi-index transportation problem for solving the problem of optimizing the process of food supply in case of the Care Centers for Elderly on Arad County area. In the framework of the Social Asistance Direction of Arad county, there are three subordinate budgetary centers: Center 1, Center 2 and Center 3. The Direction has to ensure the delivery of the necesary goods (foods, medicines, materials) to the subordinate centers.

The prolem of delivery eficiency has three parts: ensuring the need of goods, reducing the delivery time and reducing the delivery costs. This article will solve the problem of ensuring the need of goods and reducing the delivery costs suggesting a shaping as a multidimensional transport issue. The suggested algorithm takes into consideration the upgrade of delivery regularity, minimizing the cost for each delivery trance fulfilled.

Next we are going to use the following notations:

$i \in I$ - denomination given to the center of production / providers

$j \in J$ - denomination given to the product (or to the assortment which is about to be sent)

$k \in K$ - denomination given to the consumer center which has to get the product (or the assortment)

a_{ij} - denomination given to the quantity of product *j*, available in the production center *i* ;

b_{jk} - denomination given to the quantity of product *j*, necessary to the consumer center *k* ;

c_{ik} - denomination given to the quantity of products which are to be sent from the producing center *i* to the consumer center *k ;*

a_i - denomination given to the whole quantity of those *n* products available in the production center *i ;*

b_j - designation given to the quantity of product *j* available in the production centers *m ;*

c_k - designation given to the whole quantity of that *n* product necessary to the consumer center *k ;*

c_{ijk} - designation given to the delivery cost for one piece of the product *j*, produced in the production center *I* and sent to the consumer center *k ;*

x°_{ijk} - designation given to the quantity of product *j* which is to be sent from the production center *i* to the comsumer center *k*.

We are going to make an algorithm for each class of products mainly for food. For food supply have the following data:

- Three caterers:

Name for the production center	Denomination
Caterer A	i=1
Caterer B	i=2
Caterer C	i=3

- Ten categories of products:

Product name	Denomination
Dairy products	j=1
Poultry	j=2
Pork	j=3
Beef	j=4
Breading products and pasta	j=5
Vegetables	j=6
Fruit	j=7
Soft drinks	j=8
Sweets and deserts	j=9
Cans	j=10

- Three consumer centers, Care Centers for Elderly of Arad County:

Consumer center	Denomination
Center 1	k=1
Center 2	k=2
Center 3	k=3

To determine a proper transport schedule we use the data from Table 1.

Step a. The determination of the innitial solution:

Using the reccurence formula

$$x^{\circ}_{ijk} = \min(a_{ij} - \sum_{k=1}^{k-1} x^{\circ}_{ijk}; b_{jk} - \sum_{i=1}^{i-1} x^{\circ}_{ijk}; c_{ik} - \sum_{j=1}^{j-1} x^{\circ}_{ijk}) \geq 0$$

we succesively get:

$$x^{\circ}_{111} = \min(a_{11}; b_{11}; c_{11}) = \min(69, 23, 159) = 23$$

....

$$x^{\circ}_{3103} = \min(a_{310} - \sum_{k=1}^{k-1} x^{\circ}_{310k}; b_{103} - \sum_{i=1}^{i-1} x^{\circ}_{i103}; c_{33} - \sum_{j=1}^{j-1} x^{\circ}_{3j3}) = \min(12, 12, 10) = 10$$ The initial solution thus obtained in Table 2 leads us to the admissible solution.

10				5				13				7				6				11				15				17				6				7				169			
	15				17				15				8				3				8				6				9				10				9				169		
		11				20				16				9				2				7				9				8				9				11				151	
			69				59				57				52				47				45				47				61				38				14				498
12				19				20				19				23				19				5				18				21				18				169			
	11				12				17				11				17				8				20				9				15				12				159		
		9				16				27				9				15				5				13				13				17				6				167	
			45				59				57				43				39				25				43				47				67				70				495
6				6				15				20				29				17				19				9				21				14				178			
	4				3				7				5				17				15				12				17				13				9				150		
		14				10				19				13				15				13				10				5				9				24				142	
			10				18				12				18				74				69				70				77				69				53				470
23				53				42				50				55				69				53				67			185	63				41				516			
	66				43				41				26				63				19				59				59				67				35				478		
		35				40				43				37				42				51				48				59				44				61				460	
			124				136				126				113				160				139				160				185				174				137				1454

Table 1. Initial data

This solution is not degenerated because it exactly contains:

$$M = mnp - (m-1)(n-1)(p-1) = 90 - 36 = 54$$

values $x^{\circ}_{ijk} > 0$, the other *mnp* – *M* = 90 – 54 = 36 values equal to 0 .

Step b. Checking optimality:

We rewrite the costs c_{ijk} that correspond to the values $x^{\circ}_{ijk} > 0$ from the Table 1 into the Table 3 and solve the system:

$$u_{ij} + v_{jk} + w_{ik} = c_{ijk}\ ; \quad i = \overline{1,3}\,; \quad j = \overline{1,10}\,; \ k = \overline{1,3}\,.$$

As this system contains *mnp* = 90 unknowns and only *M* = 54 equations, as we considered in an arbitrary way:

$$u_{12} = 0, u_{13} = 0, u_{14} = 0, u_{15} = 0, u_{16} = 0\ u_{17} = 0,\ u_{18} = 0, u_{19} = 0, u_{110} = 0\,;$$

$$u_{21} = 0, u_{22} = 0, u_{23} = 0, u_{24} = 0, u_{25} = 0,$$

$$u_{26} = 0\ u_{27} = 0,\ u_{28} = 0, u_{29} = 0, u_{210} = 0\,;$$

$$w_{11} = 0,\ w_{13} = 0,\ w_{21} = 0,\ w_{22} = 0, w_{23} = 0,\ w_{31} = 0,\ w_{33} = 0\ .$$

The other values u_{ij}, v_{jk}, w_{ik} are directly obtained in Table 3. Using the relation

$$\bar{c}_{ijk} = u_{ij} + v_{jk} + w_{ik}$$

we can also determine the costs $\bar{c}_{ijk}$ that correspond to the 36 values $x^{\circ}_{ijk} = 0$.

23				53				42				50				1				0				0				0				0				0				169
	46				6				15				2				46				19				35				0				0				0			169
		0				0				0				0				0				28				12				59				38				14		151
			69				59				57				52				47				45				47				61				38				14	489
0				0				0				0				39				25				8				47				50				0				169
	20				37				28				24				0				0				0				0				17				35			159
		25				22				31				19				27				0				13				0				0				35		167
			45				59				57				43				39				25				43				47				67				70	495
0				0				0				0				15				44				45				29				13				41				178
	0				0				0				0				17				0				24				57				50				0			150
		10				18				12				18				42				25				1				0				6				10		142
			10				18				12				18				74				69				70				77				69				53	470
23	66	35	124	53	43	40	136	42	41	43	126	50	26	37	113	55	63	42	160	69	19	51	139	53	59	48	160	67	59	59	185	63	67	44	174	41	35	61	137	1454

Table 2.

The numbers highlighted in green in Table 4 represent the costs $\bar{c}_{ijk}$, and the others the corresponding costs c_{ijk}.

Having $c_{113} < \bar{c}_{113}$ (10<9) for $x^{\circ}_{113} = 0$, the innitial solution in Table 2 is not optimal.

10				5				13				7				6				0				0				0				0				0				w11
	15				17				15				8				3				8				6				0				0				0			w12
		0				0				0				0				0				7				9				8				9				11		w13
			u11				u12				u13				u14				u15				u16				u17				u18				u19				u110	
0				0				0				0				23				19				5				18				21				0				w21
	11				12				17				11				0				0				0				0				15				12			w22
		9				16				27				9				15				0				13				0				0				6		w23
			u21				u22				u23				u24				u25				u26				u27				u28				u29				u210	
0				0				0				0				29				17				19				9				21				14				w31
	0				0				0				0				17				0				12				17				13				0			w32
		14				10				19				13				15				13				0				5				9				24		w33
			u31				u32				u33				u34				u35				u36				u37				u38				u39				u310	
v11	v12	v13		v21	v22	v23		v31	v32	v33		v41	v42	v43		v51	v52	v53		v61	v62	v63		v71	v72	v73		v81	v82	v83		v91	v92	v93		v101	v102	v103	0	

Table 3

Step c. Improving the initial solution:

We determine the minimum $x^{\circ}_{ijk} > 0$ written in the even corners that correspond to cell c_{113}:

min(25, 35, 46) = 25

10				5				13				7				6				11	0			15	0			17	0			6	0			7	0			0
	15				17				15				8				3				8				6				9	-6			10	9			9	6		-6
		10	19			20	0			16	0			9	0			2	0			7				9				8				9				11		0
			10				0				0				0				0				0				0				0				0				0	
12	0			19	5			20	13			19	7			23				19				5				18				21				18	0			0
	11				12				17				11				17	0			7	0			20	0			9	0			15				12			0
		9				16				27				9				15				8	5			13				13	8			17	0			6		0
			0				0				0				0				0				0				0				0				0				0	
6	0			6	5			15	13			20	7			29				17				19				9				21				14				0
	4	0			3	1			7	6			5	0			17				15	-11			12				17				13				35	1		-11
		14				10				19				13				15				13				10	9			5				9				24		0
			0				0				0				0				0				0				0				0				0				0	
0	11	9		5	12	0		13	17	0		7	11	0		6	0	0		0	0	7		0	0	9		0	0	8		0	15	0		0	12	0		

Table 4

Modifying this cycle we obtain a better solution in Table 5 .

10				5				13				7				6				11	0			15	0			17	0			6	0			7	0			0
	15				17				15				8				3				8				6				9	-6			10	9			9	6		-6
		25	19			20	0			16	0			9	0			2	0			7				9				8				9				11		0
			10				0				0				0				0				0				0				0				0				0	
12	0			19	5			20	13			19	7			23				19				5				18				21				18	0			0
	11				12				17				11				17	0			7	0			20	0			9	0			15				12			0
		9				16				27				9				15				8	5			13				13	8			17	0			6		0
			0				0				0				0				0				0				0				0				0				0	
6	0			6	5			15	13			20	7			29				17				19				9				21				14				0
	4	0			3	1			7	6			5	0			17				15	-11			12				17				13				35	1		-11
		14				10				19				13				15				13				10	9			5				9				24		0
			0				0				0				0				0				0				0				0				0				0	
0	11	9		5	12	0		13	17	0		7	11	0		6	0	0		0	0	7		0	0	9		0	0	8		0	15	0		0	12	0		

Table 5.

Checking the optimality of this solution se observe that it is proper because $c_{ijk} > \bar{c}_{ijk}$ for $x_{ijk} = 0$.

Problem T-3A admits an infinity of proper programs (minimums), but all these programs lead to the same minimum value of a: min *f* = 1737 .

The interpretation of the results from Tabel 5 is done like: $x^{\circ}_{111} = 10$ represents the quantity of diary products which is to be sent from Caterer A to Center 1, etc. The same procedure is applied for optimizing the delivery time.

The use of multi-index transportation model is more suitable than the linear programming, together with the related tool pack from Excel. The multi-index transportation problem is equivalent to a linear programming problem of enormous dimension. It is difficult to approach due to the large simplex table, which should be written, the most values being zero. Also, the volume of calculus involved considerably reduces when multi-index transportation model is used.

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