

## COMPARATIVE STUDY BETWEEN ANALYTICAL AND EXPERIMENTAL RESULTS FOR RSRC MECHANISM KINEMATICS

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**Abstract:** *The paper presents a comparative study between analytical, numerical and experimental results regarding the kinematics of the RSRC mechanism. The experimental results were obtained using a device designed and constructed by the authors. The last element of the mechanism has a complex motion (screw motion) which is not met in planar mechanisms. One of the purposes of the paper is to identify the main error sources in experimental works. These sources should be considered in analysis of more complex systems such as multi-contour linkages or robots, where an analytical model, for validation, does not exist.*

### 1. INTRODUCTION

The authors obtained the analytical relations for the RSRC mechanism, specifically the relative displacements from the mechanism pairs, as shown in figure 1, using the homogenous operators method which was first proposed by Hartenberg and Denavit, [1], [2], and afterwards improved by McCharty, [3].

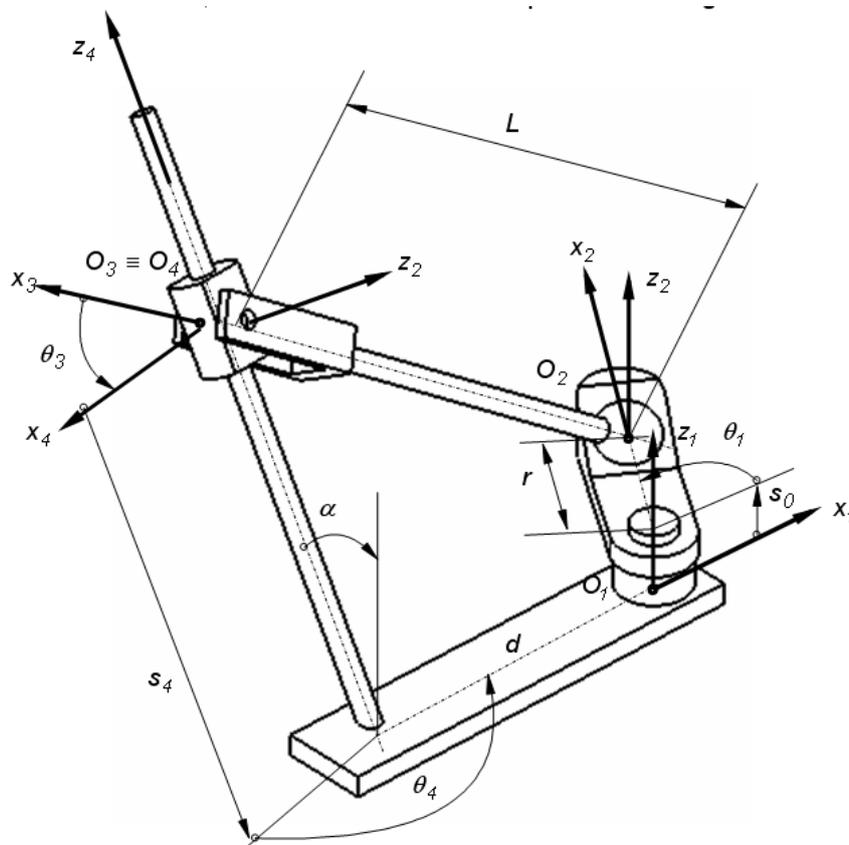


Fig. 1. The RSRC mechanism with Hartenberg-Denavit attached reference systems and the geometrical and kinematics parameters

Among spatial mechanisms, only a very few ones have analytical relations describing

the motion equations. For the other mechanisms, the position of the elements is obtained via numerical methods. Generally, the experimental value of a parameter, e.g., a distance, can be obtained using various methods and instruments. The present work illustrates some aspects regarding the proper alternative for the sensor device choice and, using the analytical model, a comparative analysis between experimental and theoretical results is performed. Despite the employment of an accurate measuring method, some discrepancies between theoretical and experimental results occur. The causes of these aspects and the surmounting approaches are identified.

## 2. THEORETICAL ASPECTS

The Hartenberg-Denavit method was first assumed to solve the position of linkages with cylindrical pairs, which in particular are cylindrical or prismatic. A distinctiveness of the position analysis of the mechanism is due to the presence of the spherical joint "S" in the structure of the mechanism. For the present mechanism, the method was applied by replacing the relative instantaneous motion from spherical joint by three rotations with respect to three axes, specifically:  $Oz$ ,  $Oy$ , and  $Ox$ , reciprocally normal and concurrent in the centre of the sphere of parameters  $\zeta$ ,  $\eta$  și, respectively  $\xi$ , following the idea of Yang, [4], [5].

The closure matrix equation for kinematics linkage of the mechanism is:

$$Z(\theta_1, s_0)X(0, r)T_{sph}(\zeta, \eta, \xi)X(0, L)Z(\theta_3, 0)X(\pi/2, 0)Z(\theta_4, s)X(\alpha, d_0) = I_4 \quad (1)$$

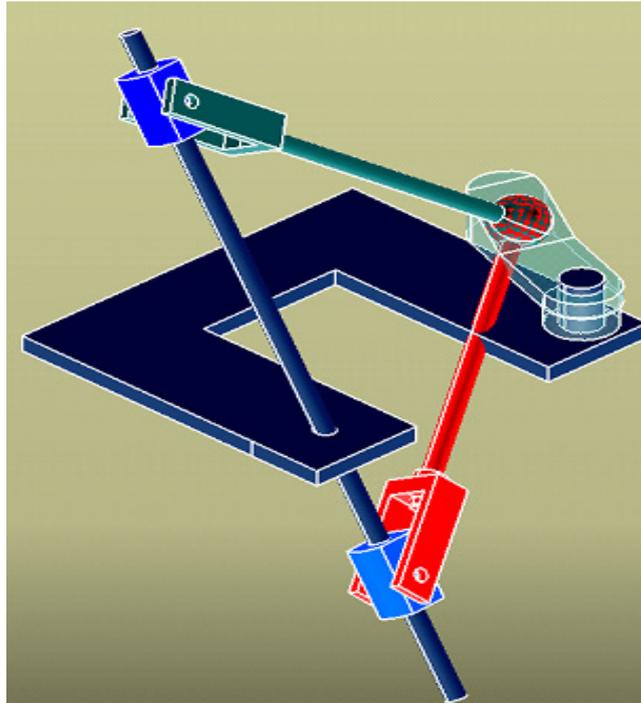
where  $T_{sph}(\zeta, \eta, \xi)$  represents the matrix for the relative motion from the spherical joint and is given by the relation:

$$T_{sph} = \begin{bmatrix} \cos \zeta & \sin \zeta & 0 & 0 \\ -\sin \zeta & \cos \zeta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \eta & 0 & \sin \eta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \eta & 0 & \cos \eta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \xi & -\sin \xi & 0 \\ 0 & \sin \xi & \cos \xi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

In equation (1),  $I_4$  represents the unit matrix of order four. The unknowns from the matrix equation are the rotations  $\zeta, \eta, \xi$  from the spherical joint, the rotation  $\theta_3$  from the intermediary rotation pair and the rotation and, from the cylindrical pair, the rotation  $\theta_4$  and the translation  $s_4 \equiv s$ .

The expressions of these parameters, with respect to the geometrical parameters and the position  $\theta_1$  of the crank, are, [6]:

$$\begin{aligned} s &= -(s_0 \cos \alpha + r \sin \alpha \sin \theta_1) \pm \\ &\sqrt{(s_0 \cos \alpha + r \sin \alpha \sin \theta_1)^2 - 2rd_0 \cos \theta_1 - s_0^2 - d_0^2 - r^2 + L^2} \\ \theta_3 &= -\text{asin}[(s + s_0 \cos \alpha + r \sin \alpha \cos \theta_1) / L] \\ \theta_4 &= \text{atan2}[-(r \cos \theta_1 + d_0) / L \cos \theta_3, (r \cos \alpha \sin \theta_1 - s_0 \sin \alpha) / L \cos \theta_3] \\ \eta &= -\text{asin}[\sin \alpha \cos \theta_3 \sin \theta_4 + \cos \alpha \sin \theta_3] \\ \zeta &= \text{atan2} \left[ \frac{(\cos \theta_1 \cos \theta_3 \cos \theta_4 - \cos \alpha \sin \theta_1 \cos \theta_3 \sin \theta_4 + \sin \alpha \sin \theta_1 \sin \theta_3) / \cos \eta,}{(-\sin \theta_1 \cos \theta_3 \cos \theta_4 - \cos \alpha \cos \theta_1 \cos \theta_3 \sin \theta_4 + \sin \alpha \cos \theta_1 \sin \theta_3) / \cos \eta} \right] \\ \xi &= \text{atan2} [(-\sin \alpha \cos \theta_4) / \cos \eta, (\sin \alpha \sin \theta_3 \sin \theta_4 - \cos \alpha \cos \theta_3) / \cos \eta] \end{aligned} \quad (3)$$



*Fig. 2. The RSRC mechanism and the two assembling positions*

The aim of the present paper is a comparative assessment of analytical results and experimental data regarding the motion of the led element. The effectiveness of the comparison is shown as it proves some aspects that cannot be illustrated for a planar linkage, since the parts of these mechanisms cannot simultaneously perform rotation and translation motions with respect to the same axis.

### **3. EXPERIMENTAL WORK**

#### **3.1 Sensor selection**

The experimental results were obtained using a device designed and constructed by the authors.

The linear displacement was recorded using the displacement sensor from the MultiLogPRO System. The sensor measures the time between the moment of emitting an acoustic signal and the moment of returning in the emitting point, after being reflected by the studied element. In order to use the mentioned sensor, the rig was adapted, that is a plane element was attached and fixed onto the led element of the mechanism, as seen in figure 3b. The role of this plate is of reflecting wave element and also as an obstructive element, preventing the reception of signals from other reflective parts of the mechanism. The employment of the sensor is straightforward since the only constraint implies that the signal emitter-receiver membrane should be parallel to the reflecting surface.

Another advantage of the mounting consists in the fact that the sensor mustn't be placed on the moving element or in direct contact with it, and therefore, the effect of vibrations are removed. Assuming that a direct contacting displacement transducer was used, aspects concerning assembling and linearity or nonlinearity of input-output dependence should occur, as Sinclair shows, [7]. Another solution considered was an acceleration sensor.

Another solution considered was an acceleration sensor. In this case, in order to obtain a proper input, high values for rotative speed are necessary and this would lead to

vibrations and variable rotation speed of the motor drive due to increased friction between the joints' surfaces of the mechanism.



Fig. 3. Initial RSRC mechanism, (a) and actual adapted mechanism, (b)

### 3. 2. Data acquisition

The *MultiLab Software* allows saving output data as *Excel* files. From the *Excel* files the data were transferred and saved as *PRN* files in order to use the *Mathcad* application. The data were plotted and the dependence displacement versus rotating angle was obtained, as seen in figure 4.

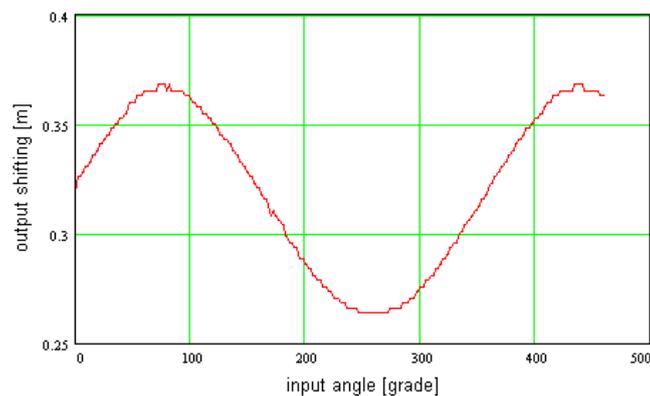
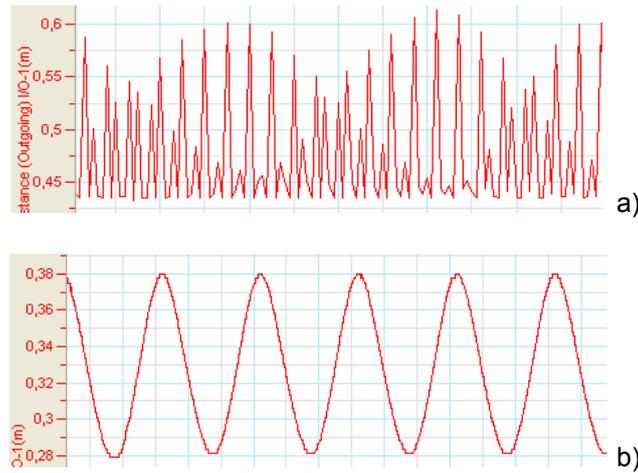


Fig.4. Experimental plot of output shifting versus input angle

In figure 4 it can be observed that the smoothness of the plot is affected by the coarse digitization of the input parameter. The results were not better despite trying to decrease the digitization, due to the fact that the sensor collects the superior harmonics of the vibrations of the last element, as can be observed from figure 5.



**Fig.5. Plots of experimental data for: 500 frames/s and b) 100 frames/s**

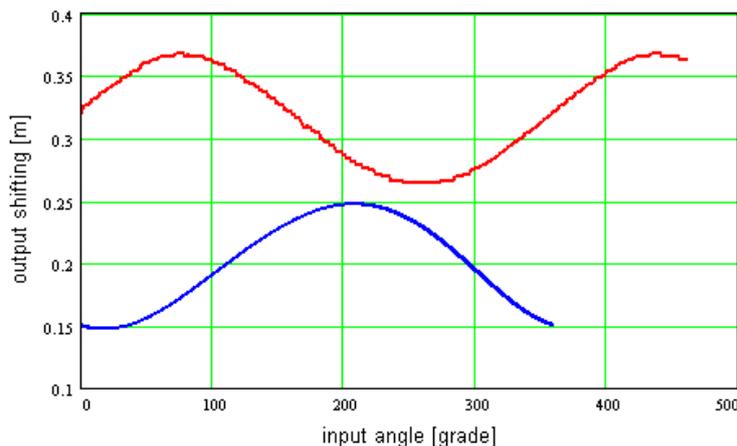
#### 4. DATA ANALYSIS AND INTERPRETATION

The experimental results can be compared to the analytical results if the plot from figure 5 is interpolated by the analytical plot for "s" from relation (5) considered for "+" case. The two graphs are plotted in figure 6. As can be seen from figure 6, there is a lag between plots, considering both axes. In order to estimate the concordance between the experimental and theoretical results, the overlapping of the two plots is needed. Using the analytical expression for  $s = s(\theta_1)$ , that is the displacement of the last element versus the angle of the crank, and the angles  $\theta_{1j}$  and displacements  $s_j$ , experimentally obtained, the purpose-function is chosen, [8], having the form:

$$F(a,b) = \sum_{j=1}^n [s(\theta_{1j} - a) + b - s_j]^2, \quad (4)$$

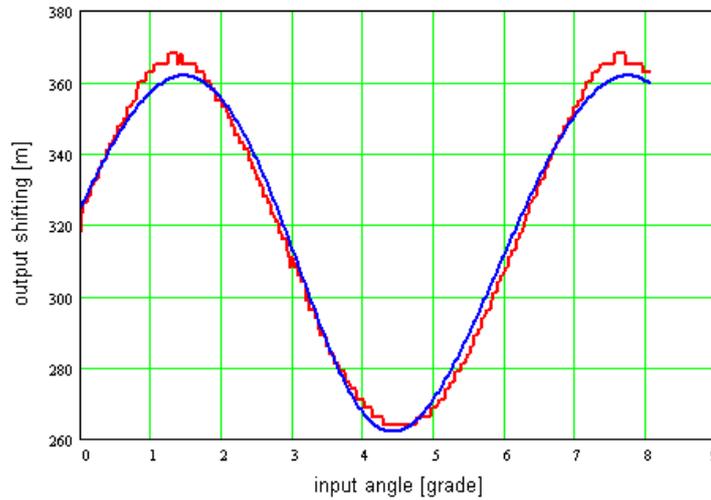
The  $a$  and  $b$  parameters are found from the following conditions:

$$\begin{cases} \frac{\partial}{\partial a} F(a,b) = 0 \\ \frac{\partial}{\partial b} F(a,b) = 0 \end{cases} \quad (5)$$



**Fig. 6. Analytical (blue) and experimental (red) results**

The fulfilment of conditions (5) ensures a minimum square deviation of experimental points versus analytical points. The  $a$  and  $b$  parameters, as can be easily seen, represent the displacement of coordinates of origin point of analytical plot for the optimum overlapping of the two plots. The system was numerically solved, using a Mathcad subroutine. Figure 7 reveals a good agreement between theoretical and experimental results. The geometry of the mechanism was:  $r = 70mm$ ,  $L = 260mm$ ,  $d_0 = 135$ ,  $\alpha_{41} = atan(55/180)$ ,  $s_0 = 10mm$ .



**Fig.7. Analytical and experimental results - overlapping of plots**

One must mention that the MultiLogPRO System acquires data in real time. The correspondence time-angle was set considering a constant crank angular speed, with a period  $T = 2.34s$ , as measured from experimental data. It was observed that for high displacement values, when the plate is far remote from sensor, the two plots are lagged. One of the possible causes of this aspect could be the non-parallelism between the reflecting surface of the plate and the emitting-receiving membrane. By denoting the measure of this non-parallelism by the angle  $\psi$  there is a relationship between the actual value of the displacement,  $s_{real}$  and experimental one  $s_{exp}$ :

$$s_{real} = s_{exp} \cos \psi \tag{6}$$

$$F(a, b, \psi) = \sum_{j=1}^n [s(\theta_{1_j} - a) + b - s_j \cos \psi]^2 \tag{7}$$

and by imposing the minimum conditions.

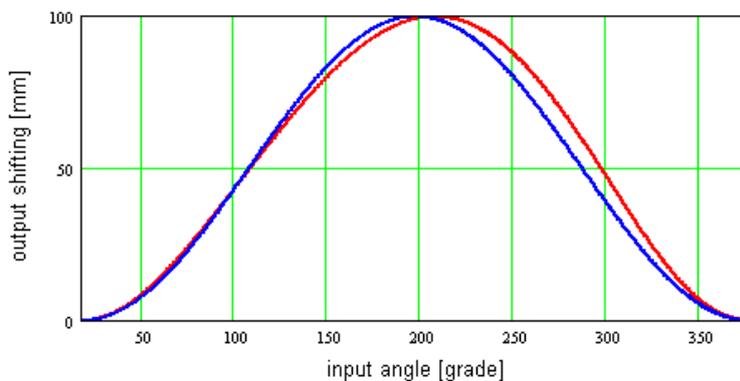
The aim to build a purpose-function having the following form:

$$\begin{cases} \frac{\partial}{\partial a} F(a, b, \psi) = 0, \\ \frac{\partial}{\partial b} F(a, b, \psi) = 0, \\ \frac{\partial}{\partial \psi} F(a, b, \psi) = 0, \end{cases} \tag{8}$$

has as result a system that is impossible to be solved numerically due to strong dependence of guess values and instability of solutions.

Another error source referred to experimental and analytical results is the assumption that the crank has a constant angular speed. Since it is difficult to test this supposition, the constant angular speed it is however *a priori* considered.

Another aspect that must be emphasised shows that the motion law is not harmonic. To this end, the points of extremum for the function  $s(\theta_1)$  were found and one can see that the distance between two successive point of the same type (maximum or minimum) is  $2\pi$ , but, as example, a point of maximum is not located at the middle point between two successive minimum points. Figure 8 presents, comparatively, the displacement function  $s(\theta_1)$  (red plot) and the harmonic function,  $A_0 + A\cos(\theta_1 - \theta_0)$ , (blue plot), where  $A_0, A, \theta_0$  constants were properly found.



**Fig. 8. Plots of displacement function  $s(\theta_1)$  (red) and harmonic function (blue)**

## 5. CONCLUSIONS

The present paper studies the concordance between analytical and experimental results for positional analysis of RSRC mechanism. For this mechanism the authors have found, in a previous work, the analytical relations for relative displacements from its joints, and so, this mechanism was chosen for comparative reasons.

In addition, the last element of the mechanism has a complex motion (screw motion) which is not met in planar mechanisms. One of the purposes of the paper is to identify the main error sources in experimental works. These sources should be considered in future analysis of more complex systems such as multi-contour linkages or robots, where an analytical model, for validation, does not exist.

Considering the experimental work, a special attention was paid to the displacement sensor selection. For the present case, a non-contacting sensor from *MultiLogPRO System* was used. Compared to other types of sensors, this sensor presents the advantage that doesn't influence the dynamics of the analysed mechanism. The only imposed constraint is to ensure parallelism between the active surface of sensor and reflecting surface of analysed object. The *MultiLab Software* allows storage of experimental data regarding displacements and moments of data acquisition, including Excel files. From Excel sheets, the data were transferred in matrix form in *Mathcad* files for further analysis. The plot of experimental displacement-time dependence shows that the digitization of the input parameter is relatively coarse, due to un-smoothness of the graph. The results were not better despite trying to decrease the digitization, due to the fact that the sensor collects the superior harmonics of the vibrations of the last element.

Using the least square method, experimental plots were overlapped on analytical plots. A very good agreement was obtained between experimental and analytical data. There are some deviations observed for the extreme position, when the distance between the sensor and target is maximum, and the plots do not concord well. The cause of this miss-concordance could be the fact that for this position, the pressure angle reaches the maximum value from the whole cycle and this leads to a significant increase of supporting reactions in joints and increase of moment of reaction from the crank.

The consequence is the alteration of the constant nature of the crank angular velocity. The augmented pressure angle value generates vibrations and the stick-slip phenomenon. An improved analysis is proposed by the authors using two methods. The first method assumes the analysis of the angular speed of the crank in order to verify the hypothesis of constant rotation. The second method for comparative study implies adapting the device for using the angular displacement sensor from *MultiLogPRO System Kit*, for crank angular displacement measurements, simultaneously with the use of linear displacement sensor (as in the present work); the data acquisition and plot of crank rotation versus last element's linear displacement eliminates the time variable and therefore leaving the assumption of constant crank angular speed. Supplementary, this last suggestion can be applied for transitory regimes, when starting and stopping the mechanism. The obtained curve is to be compared to the similar graph from analytical study.

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