DAMPED VIBRATION MODELING THE GEARS PERFECTLY

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Abstract: This paper aims to determine the conditions for the asymptotic motion of gears by a regular motion and corresponding resonance areas. Also determine those cases for which there is regular movement.

In a first approximation is considered singular engaging "fiction" with variable stiffness, but without taking into account the deviations of step (which influences the process of collision of teeth), and any errors of profile (which influences the vibration of the wheels).
Gear is initially charged with the task F_n teeth which deforms under the action of elastic. Position of equilibrium will be considered the position where the elastic forces balance the static load (Fig. 1).

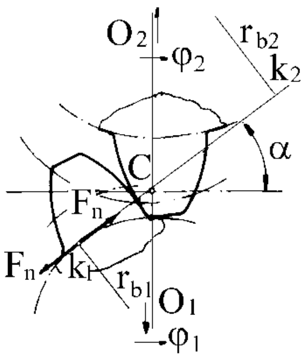

Fig.1 Deflection of gear teeth in the direction of the line

Teeth deformations measured in the direction of contact line will be:

$$x = r_{b1}\varphi_1 - r_{b2}\varphi_2 \tag{1}$$

Given the Lagrange equations, differential equations will get the movement

$$\begin{cases} J_1\ddot{\varphi}_1 + k_z r_{b1} x = 0 \\ J_2\ddot{\varphi}_2 - k_z r_{b2} x = 0 \end{cases} \tag{2}$$

This system leads to a differential equation as:

Depending on how the gear (double, singular) differential equation will have one of the forms:

$$\ddot{x}_D + \frac{k_D}{m_{red}} x_D = 0 \qquad 0\langle t \langle T_2 \text{ (Engaging double)} \tag{3}$$

and the stiffness of the meshing will be singular form:

$$\ddot{x}_S + \frac{k_S}{m_{red}} x_S = 0 \qquad T_2\langle t \langle T_z \qquad \text{(Engaging singular)} \tag{4}$$

$$x_D = r_{b1}\varphi_{1D} - r_{b2}\varphi_{2D} \tag{5}$$

$$x_S = r_{b1}\varphi_{1S} - r_{b2}\varphi_{2S} \tag{6}$$

K_S k_D and is considered constant during the second period. Position of equilibrium of the system is considered for meshing double, as denoted by D, and corresponding static strain F_n / k_D. (Fig.2)

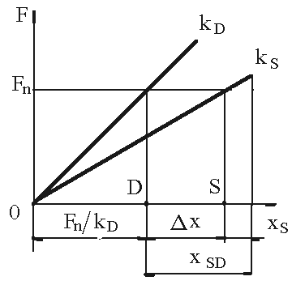

Fig.2 Equilibrium position according to stiffness [1]

Equilibrium positions to which were written equations (3) differ because the rigidities of the two types of gear are not the same.
The distance between the position of equilibrium in engaging double and singular is:

$$\Delta x = F_n\left(\frac{1}{k_S} - \frac{1}{k_D}\right) \tag{7}$$

For both types of gear movements x_D and $x_{SD} = x_S + \Delta x$ is measured from the same origin and therefore can write:

$$\ddot{x} + \frac{k_z}{m_{red}} x = F(t) \tag{8}$$

$$k_z(t) = k_D \qquad F(t) = 0 \qquad \text{for } 0\langle t \langle T_2 \tag{9}$$

$$k_z(t) = k_S \qquad F(t) = \frac{F_n}{m_{red}}\left(1 - \frac{k_S}{k_D}\right) \text{ for } T_2 \langle t \langle T_z \tag{10}$$

When considered and damping (proportional to the velocity of deformation) Gear vibration equation becomes:

$$\ddot{x} + 2\xi x + \frac{k_z(t)}{m_{red}} x = F(t) \tag{11}$$

Where ξ is defined by where k = k_m = 0.5 (k_D + k_S).

The solution equation is of the form

$$\ddot{y} + \frac{k_z(t)}{m_{red}} y + \ddot{z} + \frac{k_z(t)}{m_{red}} z + 2\xi\dot{y} + 2\xi\dot{z} = F(t) \tag{12}$$

Requires that:

$$\ddot{z} + 2\xi\dot{z} + \frac{k_z(t)}{m_{red}} z = F(t) \tag{13}$$

Be satisfied by a periodic function with period gear T_z, which together with its first order derivative must be continuous in the interval.
For the two types of gear (13) is written

$$\ddot{z}_1 + 2\xi.\dot{z}_1 + \frac{k_D}{m_{red}} z_1 = 0 \qquad \text{for } 0\langle t \langle T_2 \tag{14}$$

(15)

$$\ddot{z}_2 + 2\xi.\dot{z}_2 + \frac{k_z(t)}{m_{red}} z_2 = \frac{F_n}{m_{red}}\left(1 - \frac{k_S}{k_D}\right) \quad \text{for } T_2 \langle t \langle T_z$$

Solutions to these differential equations are:

$$z_1 = e^{-\xi t}\left(A_1 \sin p_1 t + B_1 \cos p_1 t\right) \tag{16}$$

$$z_2 = e^{-\xi t}\left(A_2 \sin p_2 t + B_2 \cos p_2 t\right) + \frac{F_n}{k_S}\left(1 - \frac{k_S}{k_D}\right) \tag{17}$$

The constants $A_{1,\ 2}$ and $B_{1,\ 2}$ are determined in terms of frequency and continuity of movement and speed:

$$\begin{cases} z_1(0) = z_2(T_1) \ldots\ldots \dot{z}_1(0) = z_2(T_1) \\ z_1(T_2) = z_2(0) \ldots\ldots \dot{z}_1(T_2) = \dot{z}_2(0) \end{cases} \tag{18}$$

Written explicitly, the conditions (18) are:

$$\begin{cases} B_1 = e^{-\xi T_1}\left(A_2 \sin p_2 T_1 + B_2 \cos p_2 T_1\right) + \frac{F_n}{k_S}\left(1 - \frac{k_S}{k_D}\right) \\ p_1 A_1 - \xi B_1 = e^{-\xi T_1}\begin{bmatrix} p_2\left(A_2 \cos p_2 T_1 - B_2 \sin p_2 T_1\right) + \\ -\xi\left(A_2 \sin p_2 T_1 + B_2 \cos p_2 T_1\right) \end{bmatrix} \\ e^{-\xi T_2}\left(A_1 \sin p_1 T_2 + B_1 \cos p_1 T_2\right) = B_2 + \frac{F_n}{k_S}\left(1 - \frac{k_S}{k_D}\right) \\ p_2 A_2 - \xi B_2 = e^{-\xi T_2}\begin{bmatrix} p_1\left(A_1 \cos p_1 T_2 - B_1 \sin p_1 T_2\right) + \\ -\xi\left(A_1 \sin p_1 T_2 + B_1 \cos p_1 T_2\right) \end{bmatrix} \end{cases} \tag{19}$$

Equations (19) have solutions in all cases except those where the system determinant is zero, ie when:

$$\frac{1}{2}\left\|\begin{bmatrix} \left(2 + \frac{p_1}{p_2} + \frac{p_2}{p_1}\right)\cos(\alpha_1 + \alpha_2) + \\ -\left(\frac{p_1}{p_2} + \frac{p_2}{p_1} - 2\right)\cos(\alpha_1 - \alpha_2) \end{bmatrix}\right\| = e^{\xi T_z} + e^{-\xi T_z} \tag{20}$$

Since z (t) is regular and bounded (contains factor), then the stability of the solution x(t) occurs when the stability function y(t) and vice versa. Therefore, to study stability Gear Drive can start from equation (11), which differs from (13) with no right member, which means that its presence does not affect the stability of vibration. [2]
By substitution:

$$y(t) = u(t)e^{-\xi t} \tag{21}$$

ecuation (11) becomes:

$$\ddot{u} + \theta(t).u = 0 \tag{22}$$

$$\theta(t) = \frac{k_z(t)}{m_{red}} - \xi^2 \tag{23}$$

Since (23) is regular, equation (22) is of the Hill, having the general solution of form

$$u(t) = A_1 u_1(t) + A_2 u_2(t) \tag{24}$$

If we accept that

$$u(t+T_z)=s.u(t) \tag{25}$$

$$(A_1\alpha_1 + A_2\beta_1 - sA_1)u_1(t) + (A_1\alpha_2 + A_2\beta_2 - sA_2)u_2(t) = 0 \tag{26}$$

Fundamental solutions are linearly independent, their coefficients of (26) must vanish, ie:

$$\begin{cases}(\alpha_1 - s)A_1 + \beta_1 A_2 = 0\\ \alpha_2 A_1 + (\beta_2 - s)A_2 = 0\end{cases} \tag{27}$$

The system will have different solutions to zero if

$$\begin{vmatrix}(\alpha_1 - s) & \beta_1\\ \alpha_2 & (\beta_2 - s)\end{vmatrix} = 0 \tag{28}$$

$$s^2-(\alpha_1+\beta_2)s+\alpha_1\beta_2-\alpha_2\beta_1=0 \tag{29}$$

Admit that the two solutions $u_1(t)$, $u_2(t)$, form a normal key, ie satisfying the following conditions for t = 0:

$$\begin{cases}u_1(0)=1 \quad \dot{u}_1(0)=0\\ u_2(0)=0 \quad \dot{u}_2(0)=1\end{cases} \tag{30}$$

$\alpha_{1.2}$ and $\beta_{1.2}$ parameters are determined using the conditions (30). Follows:

$$\begin{matrix}\alpha_1 = u_1(T_z) & \alpha_2 = \dot{u}_1(T_z)\\ \beta_1 = u_2(T_z) & \beta_2 = \dot{u}_2(T_z)\end{matrix} \tag{31}$$

Wronskianul a normal fundamental solution system of equation (22) is constant. As for t=0:

$$W(0)=\begin{vmatrix}u_1(0) & u_2(0)\\ \dot{u}_1(0) & \dot{u}_2(0)\end{vmatrix} = \begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} = 1$$

$$W(0)=\begin{vmatrix}u_1(0) & u_2(0)\\ \dot{u}_1(0) & \dot{u}_2(0)\end{vmatrix} = \begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} = 1 \tag{32}$$

Equation (29) becomes:

$$s^2 - \lambda s + 1 = 0 \tag{33}$$

$$\lambda = u_1(T_z) + \dot{u}_2(T_z) \tag{34}$$

Stability or instability of the vibratory movement of the transmission gear depends on the values of s, derived from (33):

$$s_{1,2} = \frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} - 1} \tag{35}$$

If the term under the radical:

$$\frac{\lambda^2}{4} - 1.\langle.0\text{, adică:} \quad -2.\langle.\lambda.\langle.2 \tag{36}$$

Then the two roots are complex conjugate:

$$s_{1,2} = \frac{\lambda}{2} \pm i\sqrt{1 - \frac{\lambda^2}{4}} \tag{37}$$

Having the module: $|s_1| = |s_2| = 1$

In this case equation solution will be stable, whatever the initial conditions and the function y (t), defined by the relation (21) satisfies the condition:

$$y(t+T_z)=u(t+T_z).e^{-\xi(t)}=s.y(t).e^{-\xi T_z} \quad (38)$$

If (38), it is the root of the largest and satisfies the condition:

$$|s|_{max}\langle .e^{\xi T_z} \quad (39)$$

Then the solution will be asymptotically stable, ie for t→∞, we have y(t)→0.
If you:

$$|s|_{max}=e^{\xi T_z} \quad (40)$$

Function y(t) will be stable without being asymptotically stable. The condition (37), if the term under the radical:

$$\frac{\lambda^2}{4}-1.\rangle.0 \text{ , adică: } |\lambda|.\rangle.2 \quad (41)$$

Then the roots equation (33), are real and distinct. You can determine the value of λ for maintaining the condition (39). Follows:

$$|\lambda|=|.u_1(T_z)+\dot{u}_2(T_z)|.\langle .e^{\xi T_z}+e^{-\xi T_z} \quad (42)$$

which is very asymmetric stability condition equation (11). If this condition is satisfied, after a certain time, the function y(t) becomes negligible, so that:

$$x(t)=r_{b1}\varphi_1-r_{b2}\varphi_2=z(t) \quad (43)$$

It follows that the motion will be periodic toothed wheel, ie after the time T_z angular velocity variation will be zero.
For dual meshing, fundamental solutions are:

$$u_1(t)=cop_1t$$
$$u_2(t)=\frac{1}{p_1}\sin p_1t \quad (44)$$

For engaging single, fundamental solutions are:

$$u_1(t)=a_1\cos p_2t+b_1\sin p_2t$$
$$u_2(t)=a_2\cos p_2t+b_2\sin p_2t \quad (45)$$

Replacing the constants $a_{1,2}$ and $b_{1,2}$, derived from the continuity conditions (44), we obtain the relations:

$$\begin{cases} u_1(T_z)=\cos\alpha_1\cos\alpha_2-\frac{p_1}{p_2}\sin\alpha_1\sin\alpha_2; \\ u_2(T_z)=\frac{1}{p_1}\sin\alpha_1\cos\alpha_2+\frac{1}{p_2}\cos\alpha_1\sin\alpha_2.; \\ \dot{u}_1(T_z)=-p_2\cos\alpha_1\sin\alpha_2-p_1\sin\alpha_1\cos\alpha_2.; \\ \dot{u}_2(T_z)=-\frac{p_2}{p_1}\sin\alpha_1\sin\alpha_2+\cos\alpha_1\cos\alpha_2 \end{cases} \quad (46)$$

Substituting u_1 (T_z) and (46) in (34), we get:

$$\lambda = \begin{bmatrix} \left(2+\frac{p_1}{p_2}+\frac{p_2}{p_1}\right)\cos(\alpha_1+\alpha_2)+ \\ -\left(\frac{p_1}{p_2}+\frac{p_2}{p_1}-2\right)\cos(\alpha_1-\alpha_2) \end{bmatrix} \tag{47}$$

Asymptotic stability condition (42) becomes:

$$\frac{1}{2}\cdot\left\| \begin{bmatrix} \left(2+\frac{p_1}{p_2}+\frac{p_2}{p_1}\right)\cos(\alpha_1+\alpha_2)+ \\ -\left(\frac{p_1}{p_2}+\frac{p_2}{p_1}-2\right)\cos(\alpha_1-\alpha_2) \end{bmatrix} \right\| .\langle .e^{\xi T_z}+e^{-\xi T_z} \tag{48}$$

Comparing conditions (18) with (42) that those cases for which there is regular movement, will be to limit the zone of stability [3]. If the condition (42) is satisfied there will be a periodic motion, toward which the asymptotic motion gears. When the condition (42) is not satisfied, then the transmission gear will be in a state of resonance parameters. Inequality (42) allows finding those areas of the corresponding resonance rotating toothed wheel.

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