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AN EFFICIENT HEURISTIC METHOD OF BALANCING FOR RANDOM TIMES AND UNIQUE MODEL

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Abstract: In this item, it is presented the problem of a technical line balancing for a single model with random working times and a given operating rhythm of the line. In theoretical studies, there is often the supposition of fixed working times of the phases of technical process. This supposition is only a hypothesis for simplicity, because in fact, working time for each phase is a random variable. This is evidently, especially for phases that are executed by human workers. Study of undetermined aspect of operating of technical line is important in practice and need reexamination of the given restrictions and purposed objectives.

1. INTRODUCTION

The development of cybernetic and system theory allowed approaching of enterprise as industrial cybernetic system wherein inputs are turned into outputs for satisfy some social necessities. The transformation must be enough qualitative and quantitative, that is, to run having a maximum productivity and a minimum resource consume.

Within industrial cybernetic system, the transformation is produced in technical lines, in manufacturing points rightly technical equipped, when they are able to operate a programmed action over work objects. Its efficiency depends on the putting on the same time between machine and object availability [3].

Systemic approaching of technical lines gives a high level of generalization of classical concept of line. The feature of a fabrication system as technical line, supposes finding the elements and relations that to make possible operation acting. The structure has fabrication task as starting point. In a technical line fabrication system, the fluxes of worked and basic materials, energy and information are directly inter-correlated for getting half-finished materials and parts having information put in at a higher and higher level. System competitiveness will be established by this inside information [4, 5].

Main problem in the management of a production line (of processing or assembling) in flux is its balancing. The aim of line balancing is costs minimization for a desired working rhythm or during line operation, with a more and more uniform and complete charge of workplaces and men, considering work system restrictions.

Balancing models follow in turn a better and better charge either of work power or of equipments under the conditions of mechanization and automatization extension, or simultaneously of both objectives.

In the most cases, there is accepted that phase operating times must be considered as being established. For technical lines wherein human operators' activities impose, phase operating times are random variables with known repartition functions [3, 10].

In the followings, we'll study balancing problem for stochastic times and unique model. We'll also give some ideas concerning the solving of balancing problem with stochastic times exposing an efficient heuristic like that from determinist case wherein the condition of rhythm fitting of operation times from every workstation is replaced by lower limitation of the likelihood of rhythm fitting.

2. PROBLEM FORMULATION

Let be $F = \{1,2,...,n\}$ the set of technical process' phases and $G = (F,U)$ acyclic digraph of precedence restrictions. In the followings, we'll suppose that phase operating times t_i, $i = 1,2,...,n$, are independent random variables having real values bigger than zero. Let be f_i repartition function associated to $t_i, i \in F$. Line operating rhythm R means now a desired rhythm, not an imposed one. Let be $\alpha \in [0,1]$ and let be $P(E)$ the likelihood to realize event *E* [8].

A partition $Q = \{Q_1,...,Q_m\}$ of F is a solution of balancing problem with stochastic times if it satisfies the conditions:

$$(x,y) \in U, x \in Q \text{ and } y \in Q \text{ implies } r \le s, \text{ for every } (x,y) \in U \tag{1}$$

$$P(T(Q_j) \le R) \ge \alpha,\ 1 \le j \le m, \tag{2}$$

where $T(Q_j) = \sum_{k \in Q_j} t_k$ is random variable representing operating time in station $Q_j, 1 \le j \le m$.

We mark δ_S the set of these solutions.

Pause time in station Q_j is random variable $R - T(Q_j)$, $j = 1,...,m$. Total pause time associated to solution Q is random variable:

$$TN(Q) = \sum_{j=1}^{m} (R - T(Q_j)) = mR - \sum_{i=1}^{n} t_i \tag{3}$$

Minimization of total pause time being without sense, a reasonable way to define the objective of balancing in this case, consists of average pause time minimization. Marking $\overline{X}$ the average of random variable X, it results that for every $Q = \{Q_1,...,Q_m\} \in \delta_S$ total average pause time is:

$$\overline{TN}(Q) = mR - \sum_{i=1}^{n} \bar{t}_1 \tag{4}$$

Minimization of $\overline{TN}$ returns to getting a solution $Q^* \in \delta_S$ so that:

$$|Q^*| = \min\{|Q| \mid Q \in \delta_S\} \tag{5}$$

that is, a solution having a minimum number of workstations.

It is clearly that if G is acyclic and $f_i(R) \ge \alpha$, for $i = 1,2,...,n$, then $Q = \{Q_1,...,Q_n\}$ with $Q_j = \{x_j\}, 1 \le j \le n$, where $(x_1,...,x_n)$ is an acceptable permutation of F relative to digraph G, is a solution of balancing problem with stochastic times [11]. In the followings, we'll work under these hypotheses which warrant that $\delta_S \ne \varnothing$.

3. AN EFFICIENT HEURISTIC

We mark $\pi(x)$ the set of direct predecessors of phase x in digraph $G = (F,U)$. We'll also note φ_A repartition function of random variable $X = \sum_{x \in A} t_x$, where $A \subset F$ and $A \ne \varnothing$.

The method showed after, for establish a solution of balancing problem with stochastic times is like that of determinist case wherein the condition of rhythm fitting of operation times from every workstation is replaced by lower limitation of the likelihood of rhythm fitting, given by (2).

Algorithm 1.
Step 1. It is established the set of phases without direct predecessors:

$$F_{fp} := \{x / x \in F, \pi(x) = \varnothing\},$$

it is made $m := 1$ (start number of workstations) and $k := 1$ (start number of assigned phases from the *n* phases of F), there is calculated $x_1 \in F_{fp}$ so that

$$f_{x_1}(R) = \max\{f_x(R) / x \in F_{fp}\},$$

we take $Q_1 = \{x_1\}$ (setting up of first workstation), there is initialized the set of assigned phases $C = \{x_1\}$ and we pass to next step.

Step 2. If $k = n$, then stop.

Contrarily, $k := k + 1$, there is established the set of phases having assigned predecessors,

$$F_{pa} := \{x / x \in F \setminus C, \pi(x) \subseteq C\},$$

for every $x \in F_{pa}$ it computes $p(x) = \varphi_{Q_m \cup \{x\}}(R)$, there is established the set of phases which can fill in last workstation (considering condition (2)).

$D := \{x / x \in F_{pa},\ p(x) \geq \alpha\}$ and it passes to next step.

Step 3. If $D \neq \varnothing$ then there is set $x_k \in D$ so that $p(x_k) = \max\{p(x) / x \in D\}$, last station is filled in, $Q_m := Q_m \cup \{x_k\}$, it updates $C := C \cup \{x_k\}$ and it passes to step 2; contrarily (if $D = \varnothing$), it is found $x_k \in F_{pa}$ so that $f_{x_k}(R) = \max\{f_x(R) / x \in F_{pa}\}$,
we make $m := m + 1$,
we put $Q_m := \{x_k\}, C := C \cup \{x_k\}$ and go to step 2.
About algorithm 1, next theorem can be formulated:
Theorem 1. If $G = (F, U)$ is acyclic and $f_i(R) \geq \alpha$ for every $i \in F$ and if $Q = \{Q_1, ..., Q_m\}$ is get according to algorithm 1, then $Q \in \delta_S$ [6].

4. A LOWER EDGE

In the followings, there is analyzed a particular case wherein optimum solution of balancing problem with stochastic operating times can be found in polynomial time. This case corresponds to the situation wherein the digraph $G = (F, U)$ has a Hamiltonian way, that means that $U = \{(x_i, x_{i+1}) / i = 1, ..., n-1\}$, $(x_1, ..., x_n)$ is the only admissible permutation of the set F. In this case, using next algorithm allows finding an optimum solution of balancing problem with stochastic times [8].

Algorithm 2.
Step 1: We put $u := 1,\ i := 1,\ Q_1 := \varnothing$ and go to step 2.
Step 2: If $i = n$ then stop,
contrarily $i := i + 1$ and go to step 3.
Step 3: If $\varphi_{Q_m \cup \{x_i\}}(R) \geq \alpha$ then $Q_m := Q_m \cup \{x_i\}$ and go to step 2,
contrarily $m := m + 1,\ Q_m := \{x_i\}$ and go to step 2.

Theorem 2. If $f_i(R) \geq \alpha$, $1 \leq i \leq n$ and $Q = \{Q_1,...,Q_m\}$ is get according to algorithm 2, then Q is an optimum solution.
It is outstanding that if G is not reduced to a Hamiltonian way but has such a way, than we can prove that $|F_U| = 1$ and an optimum solution can be get applying the algorithm 2 over the only admissible permutation of F (we marked F_U the set of admissible permutations associated to the digraph $G = (F,U)$) [7] .
An important consequence of theorem 2 is the possibility to get a lower margin of the number of workstations. This lower limit offer the possibility of conformation, through right changes, of some heuristic methods used for solving balancing problem in determinist case to stochastic one [9].

CONCLUSIONS

It is outstanding that however stochastic model is nearer reality than that determinist, the results gained for the former are much less then those which refer to the latter. This is explained on a side through bigger complexity of balancing problem with stochastic times respective to that with determinist times and as a consequence, through smaller performances of solving methods.
On other side, adopting of stochastic model implies additional sophistication of standardization activity because for every phase it must be established the kind of distribution wherein its operating time fits and the parameters which characterize that distribution.
For technical lines wherein human operators' activities are dominated, phase operating times are random variables with known repartition functions. When the dispersions of these random variables are small, for simplifying mathematical models, phase operating times are considered as determinist and having as values, the averages of associated random variables. In this case, balancing aims minimization of the number of workstations under the condition of satisfying precedence restrictions and lower limitation of the likelihood to have operating time of every station to be smaller than established rhythm.

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