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# CREATING THE SPATIAL FORM OF THE SHOE LAST WITH 3D BI-CUBIC SURFACES LIKE B-SPLINE FUNCTION

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**Abstract** Models for surface modeling of free-form surface and massive data points are becoming an important feature in commercial computer aided design/computer-aided manufacturing software. However, there are many problems to be solved in this area, especially for closed free-form surface modeling. This article presents an effective method for cloud data closed surface modeling from asynchronous profile modeling measurement. It includes three steps: first, 3D Bi-cubic Surface's Definition; second, The creation of the bi-cubic surfaces like B-spline function; and third, creating the spatial form of the shoe last with 3D bi-cubic surfaces In the end, an illustrative example of shoe last surface modeling is given to show the availability of this method.

## 1 INTRODUCTION

Computer-aided manufacturing (CAM) system is widely used in shoe last manufacturing, which has realized the digitalization of the machining program from the free-form surface detection method to produce numerical control (NC) requirement by the shoe last CAM software system automatically. In this type of free-form surface NC machining, it is common to digitalize the shape of the entity through the three dimensional (3D) measuring system after obtaining the scanning data and the process of the natural pattern; the next step is the rebuilding of the surface model [1], [2], [3]. There are two steps involved in remodeling the discrete data, which has become an entity shape in recent years [1–3]. Here the surface region segmentation is usually divided into one or several rectangle domains, and then a mathematical model is built according to the character boundary. This method can also be used for closed surface modeling; but when the surface is very complicated, region segmentation and feature extraction will be very difficult [4].

To make the mesh surface reflect the structure character of the complicated surface, this paper proposes a method for creating the spatial form of the shoe last with 3D bi-cubic surfaces like B-spline function. The article presents an effective method for cloud data closed surface modeling from asynchronous profile modeling measurement. It includes three steps: first, 3D Bi-cubic Surface's Definition second, the creation of the bi-cubic surfaces like B-spline form; and third, creating the spatial form of the shoe last with 3D bi-cubic surfaces. In the end, an illustrative example of shoe last surface modeling is given to show the availability of this method.

## 2. 3D BI-CUBIC SURFACE'S DEFINITION

For designing a 3D shape it has to be chosen, suitable, an amount of points situated on it's surface or in the neighborhood, that is memorized in a list. These points are named also knot or vortex and the list that contains their coordinates is named vortex list.

The points can be joined through right or curbed lines accordingly chosen, obtaining simplifying of the object. For a compete presentation, through the network's points are assigned parts of 3D curbed surfaces of parametric form. This method is used currently in the projecting process of some irregular tree-dimensional forms.

### 2.1 Obtaining the equation of a 3D bi-cubic surface

For presenting this method there is considered two 3D cubic curbs families: family C and family D of whose equations are presented in the tab.1:

***Table 1.*** *Parametric equations of the two families of curves.*

| Curbs family | Curbs family's equation | | |
|---|---|---|---|
| (C) | x=x(t) | y=y(t) | z=z(t) |
| (D) | x=x(s) | y=y(s) | z=z(s) |

whose variables s and t vary in [0.1] domain. The two curb families define bi-cubic surface in space.

In the fig 1 are presented two curb families {C} and {D} that form a bi-cubic surface in space. Curb family {C} is formed from {C1, C2, C3, C4 } curbs and family {D} is formed from {D1, D2, D3, D4 }.

{C: 3D bi-cubic curb family is obtained by introducing in the parametric equations of these: x=x(t), y=y(t), z=z(t), for t={0,0.33, 0.66, 1} another parameter s that vary in the [0,1] domain. So the curb's equation C1, C2, C3, C4 for t=0, t=0,33, t=0,66, t=1, are written viewing the s parameter. Their form is presented in the tab 2.

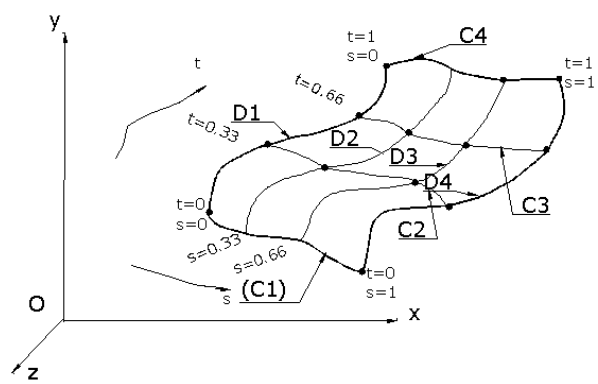


***Fig. 1, Two curb families {C} and {D}. That form a bi-cubic surface in space***

***Table 2.*** **The curbs equation for {D} family**

| Curb | Curb's equation | | |
|---|---|---|---|
| C1: | x=x(0,s) | y=y(0,s) | z=z(0,s) |
| C2 | X=x(0.33,s) | y=y(0.33,s) | z=z(0.33,s) |
| C3 | x=x(0.66,s) | y=y(0.66,s) | z=z(0.66,s) |
| C4 | x=x(1,s) | y=y(1,s) | z=z(1,s) |

for s varying in the [0,1] domain.

Similar to the curbs from {D}: {$D_1$, $D_2$, $D_3$, $D_4$} family are obtained introducing in the equations: x=x(s), y=y(s), z=z(s) with s=0, 0.33, 0.66, 1 another parameter t that vary in the [0,1] domain. These curbs equation's are found in the tab 3:

***Table3. The curbs equation for {D} family***

| Curb | Curb's equation | | |
|---|---|---|---|
| D1: | x=x(t,0) | y=y(t,0) | z=z(t,0) |
| D2 | X=x(t,0.33) | y=y(t,0.33) | z=z(t,0.33) |
| D3 | x=x(t,0.66) | y=y(t,0.66) | z=z(t,0.66,) |
| D4 | x=x(t,1) | y=y(t,1) | z=z(t,1) |

So, the definition of a surface using the two curbs families will have this form:

$$x=x(t,s)$$
$$y=y(t,s)$$
$$z=z(t,s)$$

relations that define a 3D surface's equations, named bi-cubic surface.
The general form of a coordinate over a surface viewing the 2 parameters s and t, in matrix is:

$$[K(s,t)]= [S] \cdot[C] \cdot[T]^T$$

with:

$$S := \begin{bmatrix} s^3 & s^2 & s & 1 \end{bmatrix} \quad C := \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad T := \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

and analytic form is:

$$\begin{aligned} K(s,t)=&a_{11}s^3t^3+a_{12}s^3t^2+ a_{13}s^3t+ a_{14}s^3+ \\ &a_{21}s^2t^3+ a_{22}s^2t^2+ a_{23}s^2t+ a_{24}s^2+ \\ &a_{31}st^3+ a_{32}st^2+ a_{33}st+ a_{34}s+ \\ &a_{41}t^3+ a_{42}t^2+ a_{43}t+ a_{44} \end{aligned}$$

According to this methodology are defined more 3D curbed surface types similar to the 3D curbs, this way: Hermite, Bezier. B-Spline.
In the paper it is presented the mathematic designing of the shoe last using ***B-spline 3D bi-cubic surfaces***.

## 3. THE CREATION OF THE BI-CUBIC SURFACES LIKE B-SPLINE FORM

A bi-cubic surface like B-spline form is defined of 16 control points, similar to the 3D B-spline curbs. The relations for finding the coordinates correspondent to the (s, t) parameters have the general form:

$$K(s,t) = [S]\cdot[M_s]\cdot[Q_{i,j+1,i+1;\, i+1,j+1}]\cdot[M_s]^T\cdot[T^T]$$

with:

$$Q := \begin{bmatrix} P_{i-1,j-1} & P_{i-1,j} & P_{i-1,j+1} & P_{i-1,j+2} \\ P_{i,j-1} & P_{i,j} & P_{i,j+1} & P_{i,j+2} \\ P_{i+1,j-1} & P_{i+1,j} & P_{i+1,j+1} & P_{i+1,j+2} \\ P_{i+2,j-1} & P_{i+2,j} & P_{i+2,j+1} & P_{i+2,j+2} \end{bmatrix}$$

Where the indices specify that the intermediate points s, t counts in the perimeter delimited by the control points:

$$P_{i,j},\ P_{i+1,j},\ P_{i,j+1},\ P_{i+1,j+1}$$

where i, j belong to the [2,n-2] domain, supposing that for the description of the entire surface using a nxn point network.
[M] is **B-Spline** matrix:

$$M := \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

and [S], [T] are the parameters s, t.

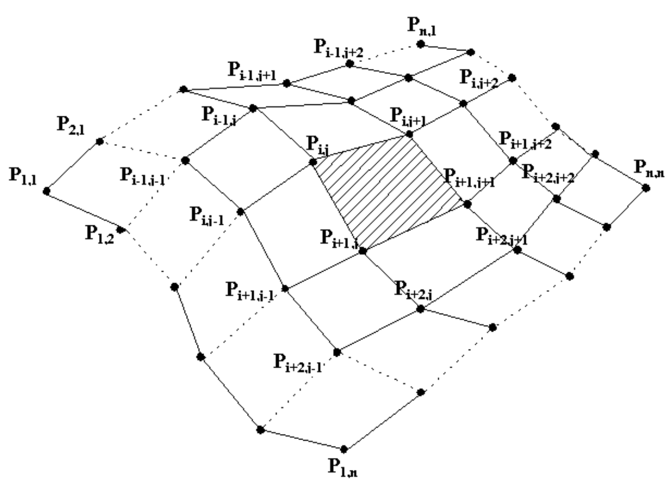


*Fig. 2, A bi-cubic surface like B-spline form is defined of 16 control points*

## 4. CREATING THE SPATIAL FORM OF THE SHOE LAST USING 3D BI-CUBIC SURFACES

For creating the spatial form of the shoe last with 3D bi-cubic surfaces it has been proceeded this way:

1. On the surface of the shoe last are marked sectional and longitudinal sections, and are determined the knot's coordinates through manual methods.

2. It is established a step for browsing the designed surface after s, t between control points.

3. For each s value:

are browsed t values between 0 and 1 for each t value:

- are calculated the point's coordinates:

$$x=x(s,t),\ y(s,t),\ z(s,t)$$

- is calculated the projection(q(s,t), w(s,t)) of the point's (x,y,z) [2];

are calculated the u, v coordinates passing to the next t passing to the next s

4. There are marked the curbs from C and D families through the calculated points, as polygonal lines.

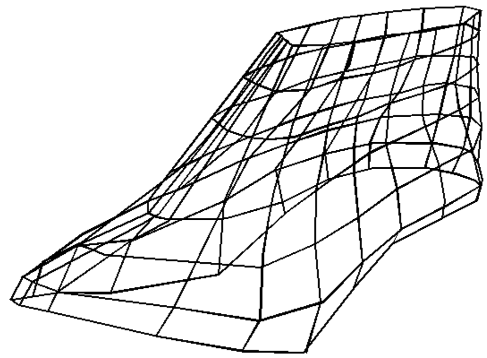

*Fig. 3, Discredited the shoe last's for more surface*

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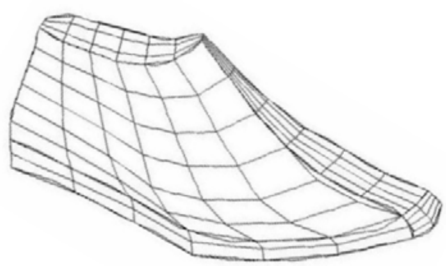

*Fig. 4, The shoe last's surface obtained by bicubic surface*

## 5. CONCLUSIONS.

There has been created a point network that defines the shoe last's spatial surface with B-spline curves. In the fig 3 is presented the shoe last's surface obtained by marking the 2 curbs families C, D, that the transversal and longitudinal sections where through it was defined. In the fig 4 it has proceeded to the varying of the parameters s, t with 0.1 step for visualizing the wire-chassis network of the shoe last's spatial surface.

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