KINEMATICS EFFECTS DUE TO REPLACEMENT OF SPHERICAL JOINT BY REVOLUTE JOINTS IN RSRC MECHANISM

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Abstract

The spherical pair from RSRC mechanism structure determines bounded amplitude for the motion of the last element. A first solution regarding the increase of the amplitude proposed by the authors in a recent paper consists of an optimal assembly for the outer ring of the ball and socket joint and in the present paper the authors propose a solution for increasing the amplitude. Among the replacing solutions, two are simulated and analyzed comparatively to the initial mechanism. For the solution without the interference phenomenon, a numerical simulation is presented in order to highlight the kinematical effects due to replacement. The RSRC mechanism was selected for the study of replacement effects because the availability of the analytical solutions for relative displacements from kinematics pairs.

1. INTRODUCTION

In a series of recent papers, [1], [2], the authors found the analytical expressions for relative displacements from RSRC mechanism pairs, Fig. 1. The method used in finding the relative displacements was the method of homogenous operators proposed by Hartenberg and Denavit, [3]; the closure matrix equation was finally obtained. From these equations the position of the mechanism for imposed values of input parameters can be obtained. This solution is difficult to perform analytically due to redundant equations presence.

2. THEORETICAL REMARKS

The reference frames, were chosen shows according to Hartenberg and Denavit, [3], convention, as seen in Fig. 2. As principle, the method assumes attaching reference frames with Oz axes along axes of cylindrical pairs and Ox axes pointed along common normal directions of two successive Oz axes.

The relative position for two reference frames attached to two neighboring elements is described by means of four parameters:

- kinematical parameters for relative motion from cylindrical pair, namely:
  - rotation θ and translation s about Oz axis.
- geometrical parameters:
  - length of element, *a,* and twisting angle α, about Ox axis.

The signs of the parameters are adopted according to the coincidence to positive direction of the respective axes.

In order to model the relative motion from the spherical joint, where a concrete rotation axis does not exist, the methodology proposed by Yang, [4], [5], was used, which proposes that the motion from the spherical joint should be considered:

- either as a rotation motion around the instantaneous rotation axis,
- or as the result of three successive rotations about Oz, Oy, and Oz axes of a Cartesian reference frame having the origin in the centre of spherical joint.

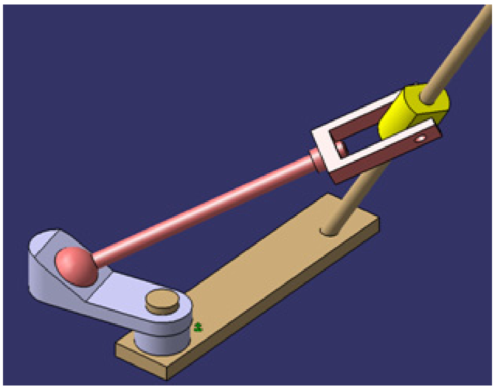

Fig. 1. The revolute-spherical-revolute-cylindrical (RSRC) mechanism

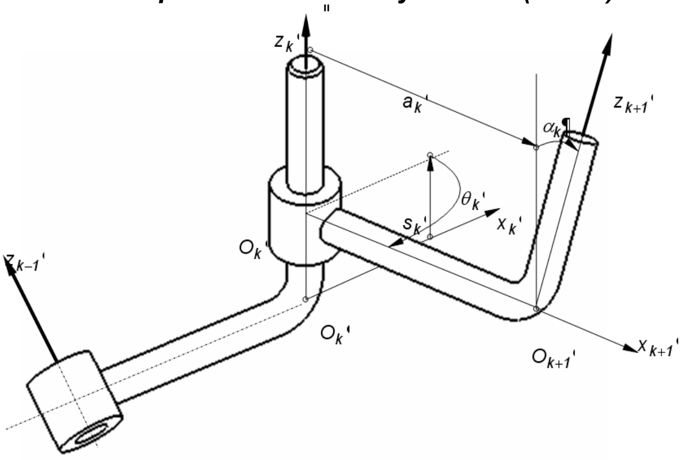

Fig. 2. Hartenberg-Denavit parameters

The homogenous operator is represented by a 4×4 matrix that allows to obtain the coordinates of a point from frame $" k + 1"$ into frame $" k"$. The transformation relation is, [6]:

$$\begin{bmatrix} 1 \\ x_k \\ y_k \\ z_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_k \cos\theta_k & \cos\theta_k & -\cos\alpha_k \sin\theta_k & \sin\alpha_k \sin\theta_k \\ a_k \sin\theta_k & \sin\theta_k & \cos\alpha_k \cos\theta_k & -\sin\alpha_k \cos\theta_k \\ s_k & 0 & \sin\alpha_1 & \cos\alpha_k \end{bmatrix} \begin{bmatrix} 1 \\ x_{k+1} \\ y_{k+1} \\ z_{k+1} \end{bmatrix} \tag{1}$$

or, contracted:

$$\mathrm{x}_k = \mathrm{T}_{k,k+1}\mathrm{x}_{k+1} \tag{2}$$

One can notice that the elements of matrix $\mathrm{T}_{k,k+1}$ depend on the parameters mentioned above, the operator can be formally written as a column matrix containing the parameters:

$$\mathrm{T}_{k,k+1} = \begin{bmatrix} s_k \\ \theta_k \\ a_k \\ \alpha_k \end{bmatrix} \tag{3}$$

Writing the coordinate transformation for a point from frame $"n+1" \equiv "1"$, the matrix closure equation of the linkage is obtained under the form:

$$\mathrm{T}_{1,2}\mathrm{T}_{2,3}\ldots\mathrm{T}_{n-1,n}\mathrm{T}_{n,1} = \mathrm{I}_4 \tag{4}$$

or, in symbolical form, as Hartenberg and Denavit proposed, [6]:

$$\begin{bmatrix} s_1 \\ \theta_1 \\ a_1 \\ \alpha_1 \end{bmatrix} \times \begin{bmatrix} s_2 \\ \theta_2 \\ a_2 \\ \alpha_2 \end{bmatrix} \times \ldots . \times \begin{bmatrix} s_n \\ \theta_n \\ a_n \\ \alpha_n \end{bmatrix} = \mathrm{I}_4 \qquad (5)$$

where I_4 represents the identity matrix of order four.

In the case when only the displacement of the last element was envisaged, a more simple geometrical method could be used instead of homogenous operators method. For instance, for an imposed position of the crank, the position of the internal revolute joint can be obtained directly as intersection of the sphere having the centre in the ball socket joint and the radius equal to the length of the floating element, with the axis of the cylindrical joint, as seen from Fig. 3.

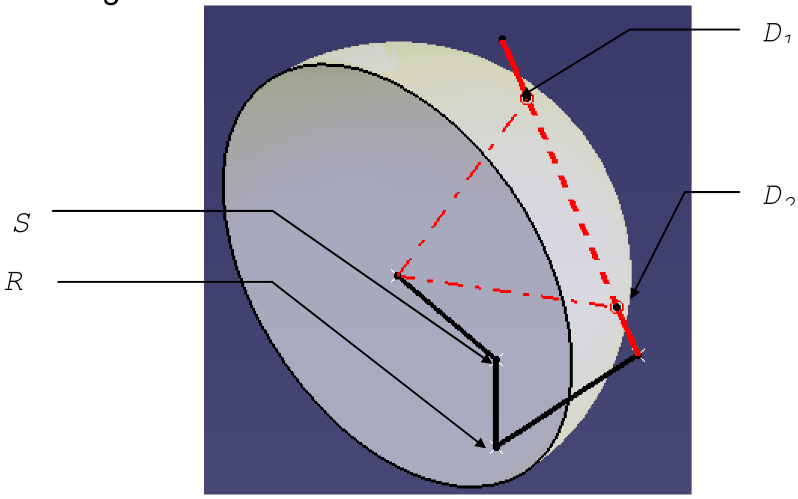

Fig.3. Graphical result for the position of the last element

The mechanism can be assembled in two ways, and the points D_1 and D_2 correspond to these two solutions and their position allows finding the displacement. During running the mechanism, it could be observed that although the amplitude of spherical motion in the spherical joint was large enough, the setting of external ring significantly bounded the motion from the joint. In order to surmount this aspect, the authors proposed an optimization method concerning the outer ring assembly of the joint, [2], to allow the largest amplitude of the spherical motion.

The paper presents a detailed optimization method. In Fig. 4 is presented merely the assembling solution, consisting in fixing the outer ring onto an intermediate wobble plate.

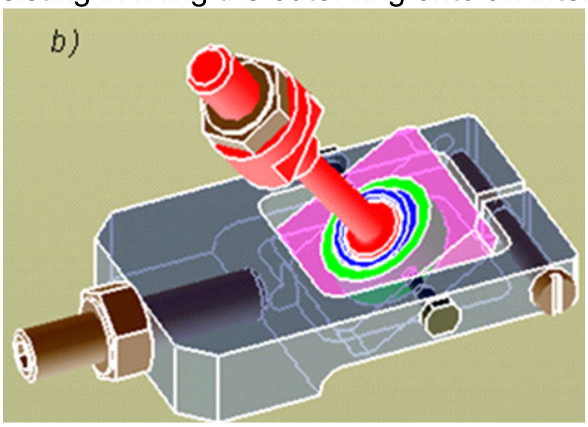

Fig. 4. Optimized spherical joint assembly

3. EQUIVALENT MECHANISM SOLUTION

Despite the presented solution, for certain geometrical parameters of the mechanism, the required amplitude for spherical motion wasn't suitable for a proper functioning. The imposed solution was replacement of spherical joint with three revolute joints with orthogonal but not intersecting axes. For this solution, the phenomenon that should be avoided is the mechanical interference. The study of this phenomenon was performed via some equivalent mechanisms that are simultaneously acted by the same crank, together with the initial mechanism, as shown in Fig. 5.

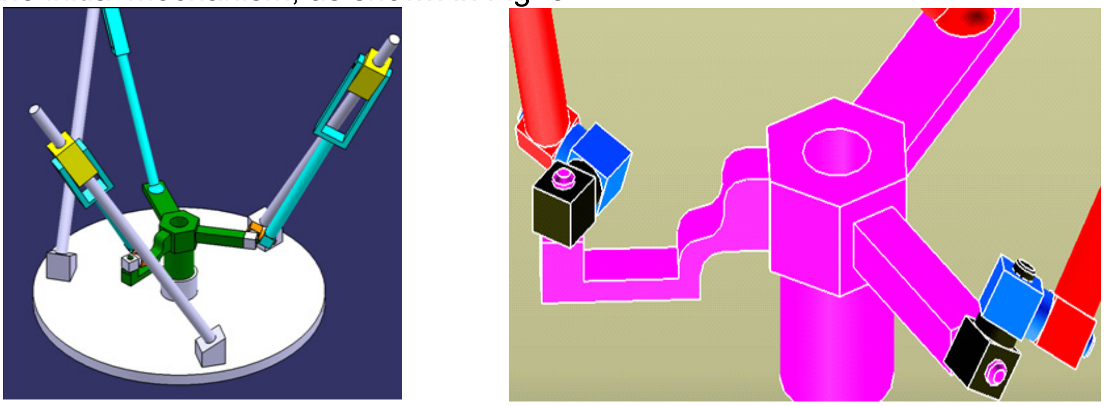

Fig. 5. The initial mechanism and two equivalent mechanisms acted by the same crank

From the two equivalent solutions, the one that eliminates the interference is the mechanism having the first replacing revolute joint with the axis parallel to the crank axis, Fig. 6. The equivalent mechanism and the attached Hartenberg-Denavit frames, [6] are presented in Fig. 7.

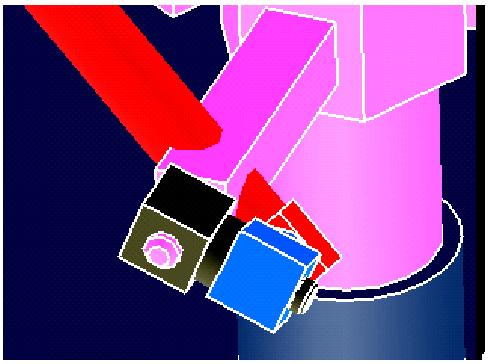

Fig. 6. The interference phenomenon for a replacing mechanism

The concrete symbolic closure equation for the kinematical linkage of the mechanism is:

$$\begin{bmatrix} s_1 \\ \theta_1 \\ a_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} s_2 \\ \theta_2 \\ 0 \\ \pi/2 \end{bmatrix} \times \begin{bmatrix} s_3 \\ \theta_3 \\ 0 \\ -\pi/2 \end{bmatrix} \times \begin{bmatrix} s_4 \\ \theta_4 \\ a_4 \\ \pi \end{bmatrix} \times \begin{bmatrix} 0 \\ \theta_5 \\ 0 \\ \pi/2 \end{bmatrix} \times \begin{bmatrix} s_6 \\ \theta_6 \\ a_6 \\ \alpha_6 \end{bmatrix} = \mathrm{I}_4 \qquad (6)$$

In the above equation, the unknown parameters are represented by the revolute angles from revolute joints and from the cylindrical joint, namely $\theta_2, \theta_3, \theta_4, \theta_5$ and θ_6 respectively, and s_6, the linear displacement from cylindrical joint.

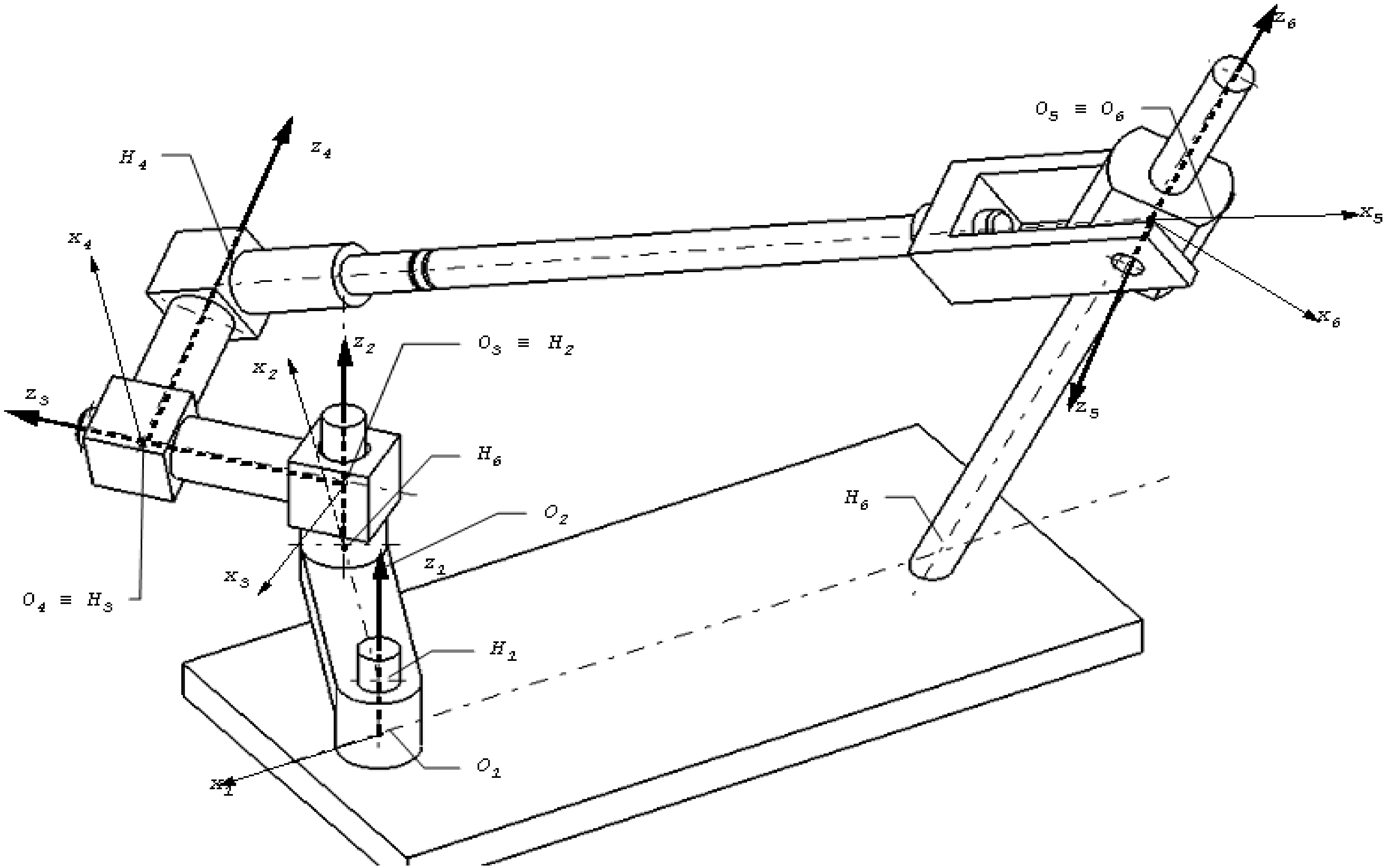

Fig. 7. The equivalent mechanism and the attached Hartenberg-Denavit frames

These parameters are found as functions of position angle of the crank, θ_1, and geometrical parameters occurring in the above equation.

In this situation, is very complicated to look for a geometrical solution instead of homogenous operators method for equation 6, because requires the use of a special surface, named cylindroid, [7], [8], [9], [10], [11], [12] The cylindroid is an algebraic ruled surface of third order, defined as geometrical locus of a linear combination of two screws, Fig. 8. Briefly, by screw, one means a straight line and a positive number named pitch screw, [13].

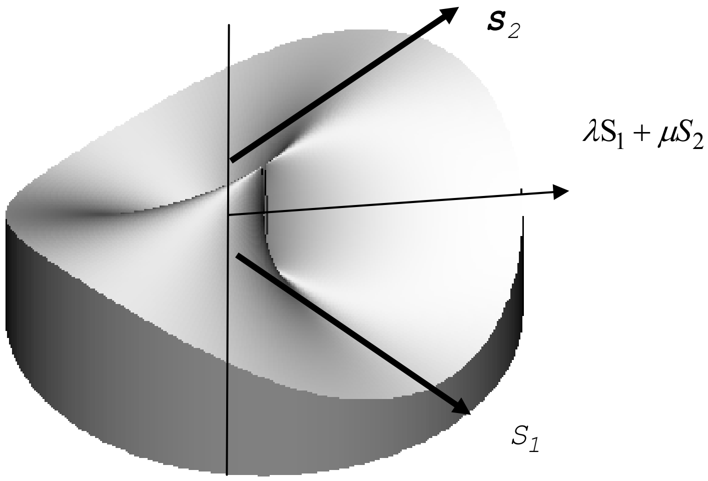

Fig. 8. The cylindroid

Practically, for an imposed crank position, there have to be found those revolute displacements from equivalent pairs, so the axis of internal revolute pair and the axis of cylindrical should be orthogonal.

The difficulty of geometrical solution method also indicates difficulties in analytical positional analysis using matrix closure equation of the equivalent mechanism.

A very convenient method is numerical solving of matrix closure equation. Uicker et al., [14], propose a strong convergent numerical method with the aim of finding the relative displacement of a mono-contour mechanism with cylindrical (particularly revolute or prismatic) pairs.

The most important difficulty of this method consists in choosing the guess value needed to initiate the numerical solving. This guess value determines both the convergence of the numerical process and the convergence point (assembly position). This guess value is commonly obtained from graphical methods, which was already shown to be extremely complicated.

4. NUMERICAL SOLUTION

The recent FEM software based on numerical calculus are a convenient mean for getting the solution of the problem. Using *DMUKinematics* module from CATIA environment, the problem can be solved in a relative simple manner, Zamani [15]. The mechanism being already in the desired assembly position, in *Catia Assembly Design Module*, the problem of mentioning the guess value is avoided.

In Fig. 9, the variation of linear displacement for the last element in initial mechanism and for the replacing mechanism is presented and it can be seen that the behavior is similar, the only difference being the magnitude of amplitude which is greater for the replacing mechanism. This aspect isn't a major inconvenience because there are solutions, such as a convenient change of geometrical parameters, for reducing the above amplitude difference. It was considered to be suggestive a plotting for the dependence of linear displacement for the last element for the replacing mechanism versus linear displacement of the initial mechanism, Fig. 10. Concerning the graph from Fig. 10, the replacement has better results when the area of the enclosed surface by the curve minimizes and the mean slope leans to the bisecting line of the third quadrant.

Similar remarks can be made from Fig. 11, where the variation of revolute displacement for the last element in initial mechanism and for the replacing mechanism are presented and for Fig. 12, presenting the variation of revolute displacement for the last element for the replacing mechanism versus revolute displacement of the initial mechanism. For this case, the angular amplitude difference is smaller compared to the linear displacement case, but the lag angle between the moments when extreme values were reached is larger, and therefore, the area form Fig. 12 is more important.

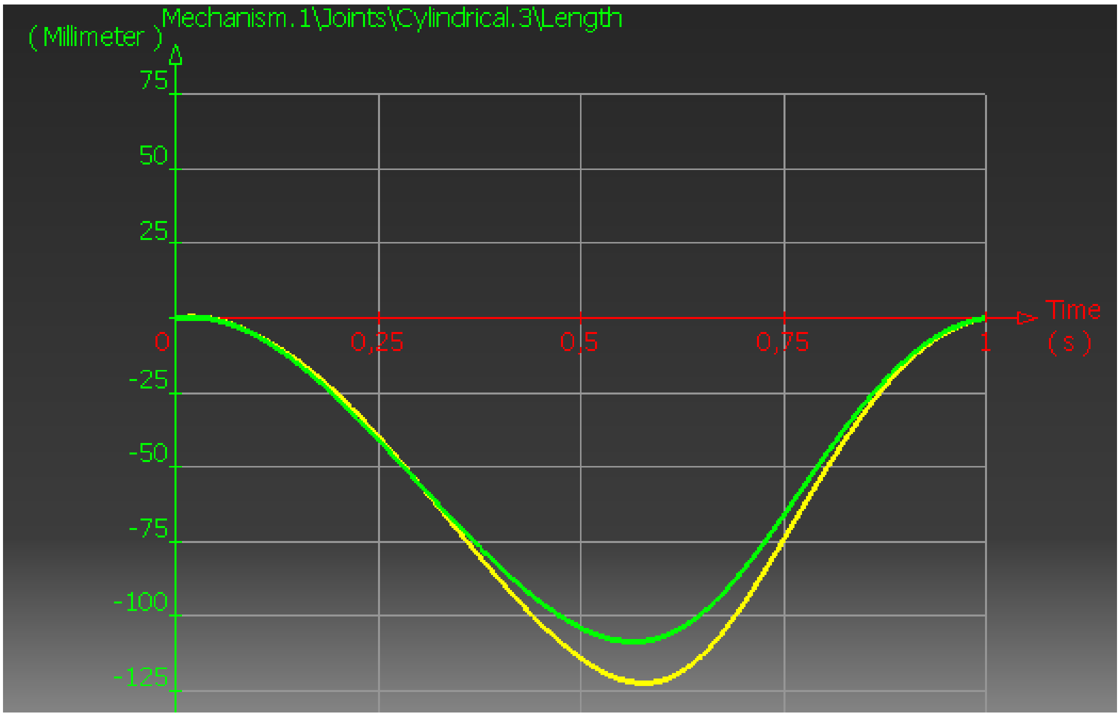

Fig. 9. Variation of linear displacement for the last element in initial mechanism (green) and for the replacing mechanism (yellow)

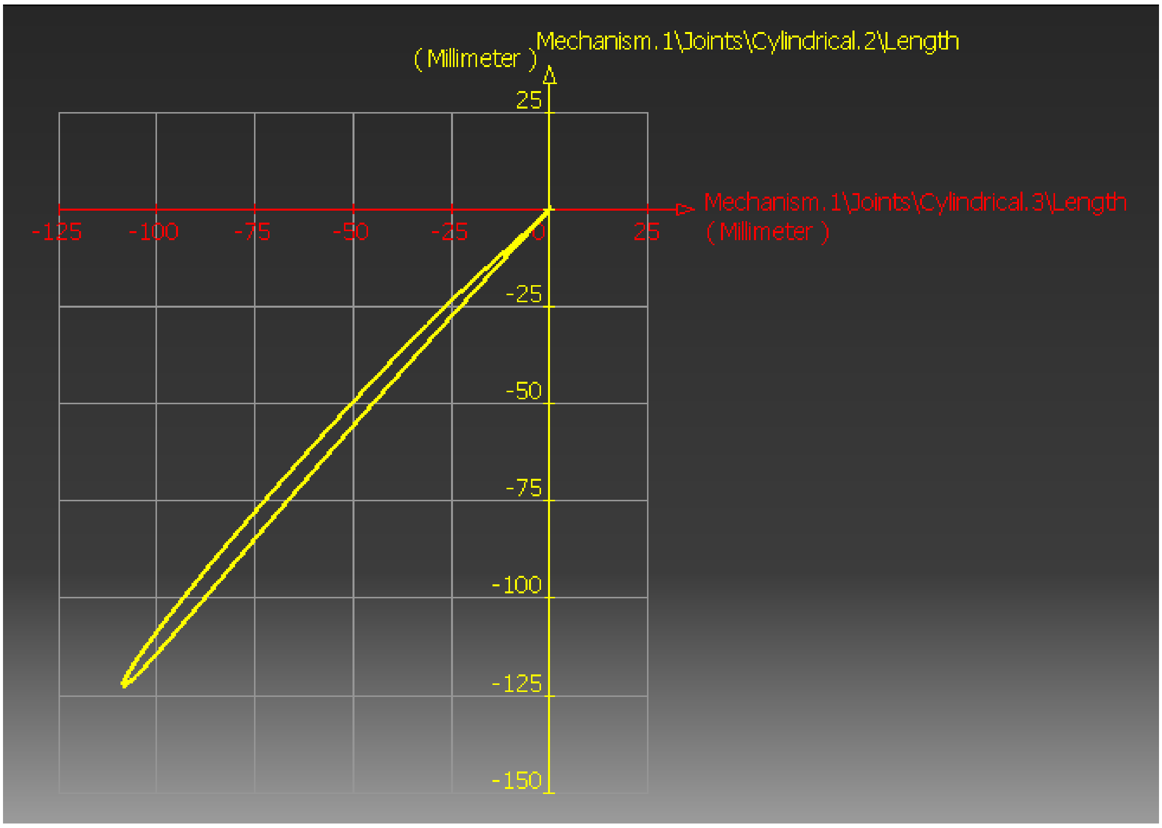

Fig. 10. Variation of linear displacement for the last element for the replacing mechanism versus linear displacement of the initial mechanism

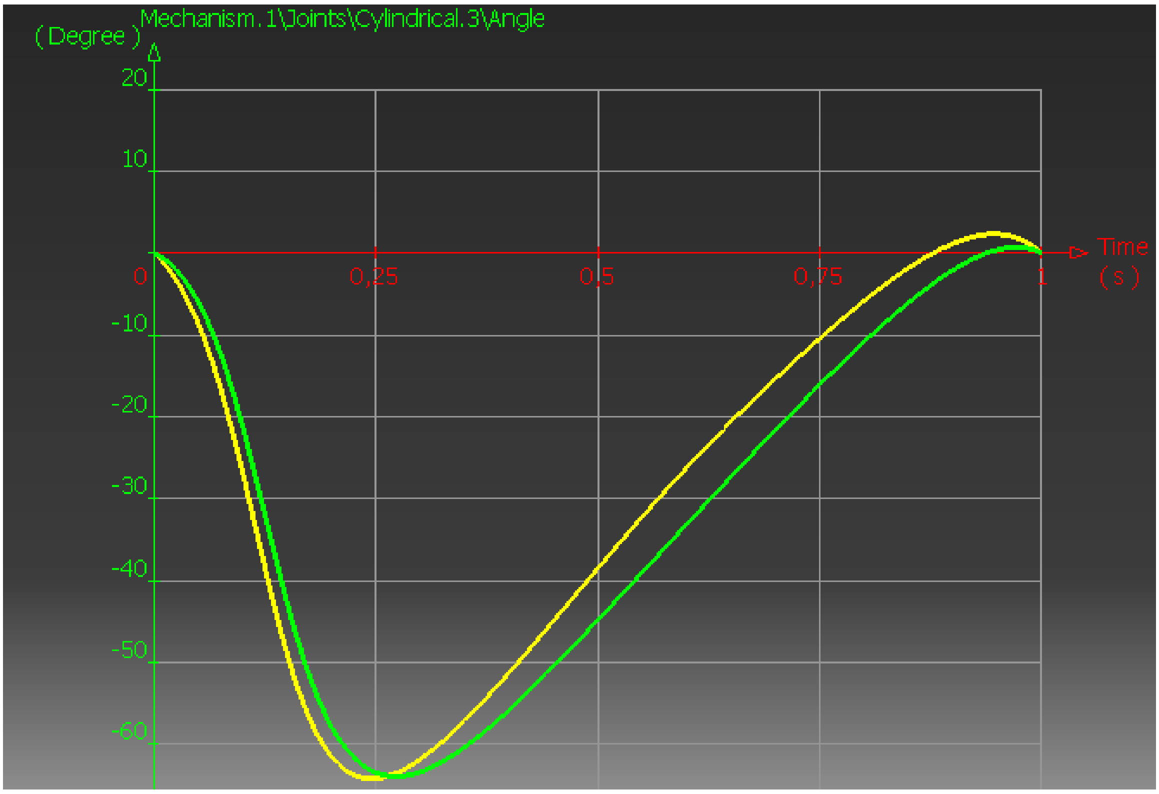

Fig. 11. Variation of revolute displacement for the last element in initial mechanism (green) and for the replacing mechanism (yellow)

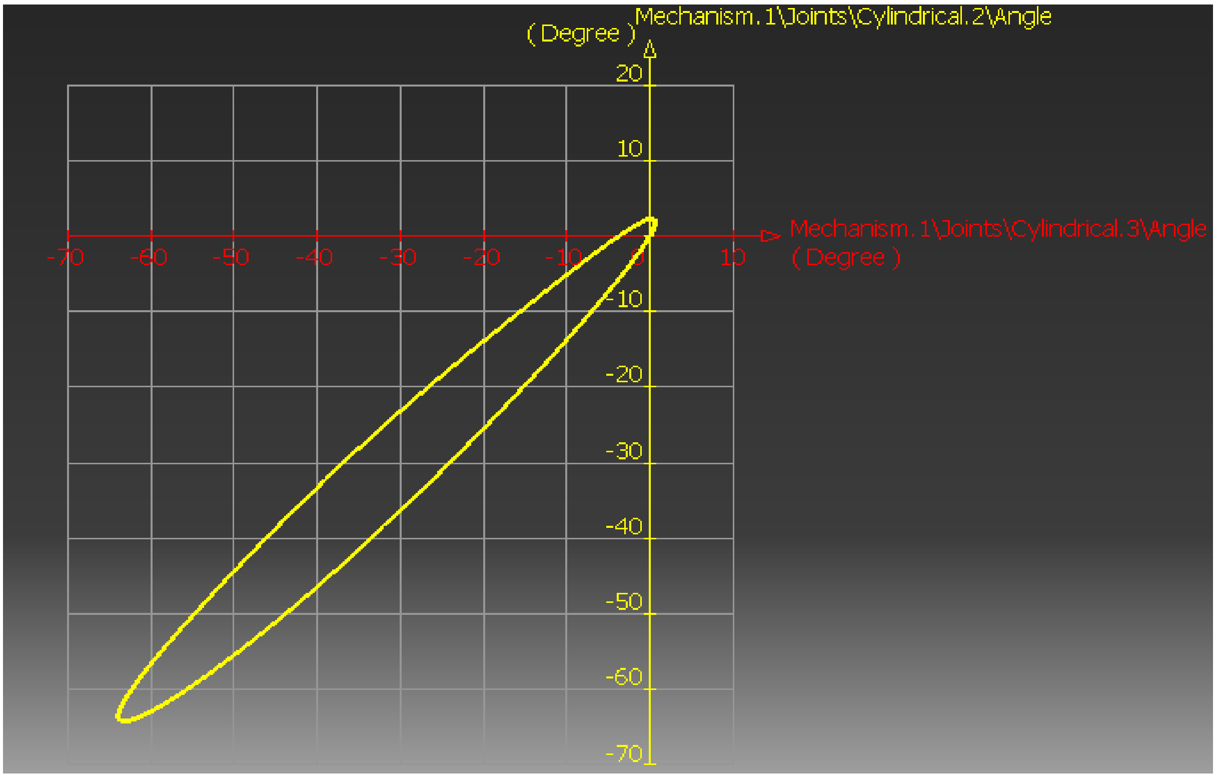

Fig. 12. Variation of revolute displacement for the last element for the replacing mechanism versus revolute displacement of the initial mechanism

5. CONCLUSIONS

The RSRC mechanism is one of the basic spatial mono-contour mechanisms. For this mechanism the authors found the analytical solutions for relative displacements from its joints. In order to improve the mechanism's performances, namely to enlarge the angular amplitude in the spherical joint, the authors offered a method of optimization of outer ring assembly.

The amplitude of spherical motion is though bounded by the geometrical parameters of spherical bearing. With the intention of eliminating this inconvenience, in the present paper, the authors proposed the replacement of the spherical joint with a set of three successive revolute pairs, with orthogonal but not intersecting axes.

Analytical positional analysis for the replacing mechanism is complicated and a numerical method must be applied. To this end, the DMUKinematics module from CATIA environment was involved.

Using this Module, three mechanisms were analyzed, the initial one and two equivalent mechanisms, simultaneously acted by the same element.

From this analysis, it could be observed that only the replacing mechanism having the first equivalent axis of revolution normal to the crank axis presents the mechanical interference phenomenon.

Subsequently, the comparison between relative displacements from cylindrical pairs of the two mechanisms was performed. One can tell that the equivalent mechanism approximates the motion of the real mechanism with a very good agreement. The replacement process can be optimized and this is a concern for future researches.

6. REFERENCES

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