

## MATHEMATICAL MODEL FOR BEARING MODELING

**PATER Sorin, BRATU Ion, POLOJINTEF CORBU Nicolae**

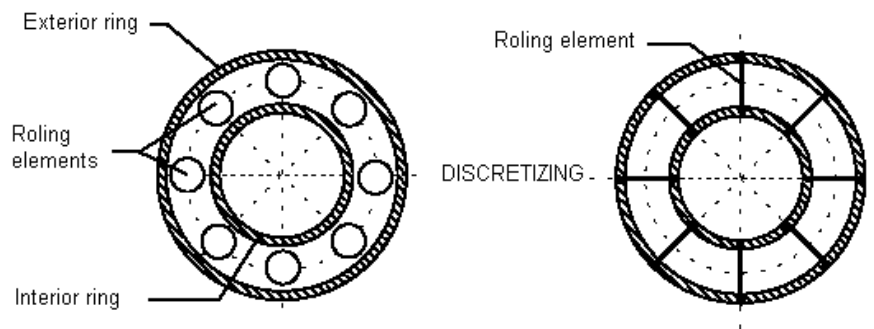
University of Oradea, e-mail: spater@uoradea.ro

**KEY WORDS:** bearings, rings, rolling elements

**ABSTRACT:** The essential idea is to split the bearing into „rolling elements” with balls and to associate to each part a fixed element. The approximation using classic forces method for bearings has been replaced by the displacement method with the aim of using a global method that allows a correlation between bearing calculations and global structure calculations.

### 1. INTRODUCTION

These elements known as „rolling elements” are considered to be a flat board that connects a point from the interior ring with a point in the exterior ring through a rigidity matrix of  $10 \times 10$  in dimension. This matrix provides the connection between the 10 degrees of active freedom ( $2 \times 3$  degrees of translation freedom and  $2 \times 2$  degrees of rotation freedom) and provides the transition of 3 forces and 2 moments. If the degrees of freedom of the bearing rotation are free of all links than no moment is transmitted on that direction.



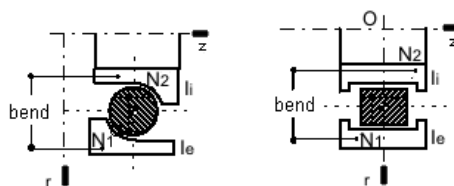
**Figure 1.1. Splitting bearings into “rolling elements”**

The ring split has been verified through the following hypothesis:

- The transverse section of the rings is non-deformable.
- For each rolling element there is a certain point on the interior ring and a point on the exterior ring. These points are situated in a radial plane that contains the center of the rolling body.

### 2. THE PRINCIPLES OF BEARING ELEMENT DETERMINATION

For bearing encampment, where rolls or balls can be assimilated with solids, the principles of bearing element determination trough classic finite element approximation or trough numeric calculation are identical. Element integration consists in the determination, for shifting due to the  $N_1$  and  $N_2$  joints, of the interior and exterior ring (fig.1.2.), elementary shifts of the center  $C_r$  of the rolling element, ensuring its balance in radial plane.



**Figure 1.2. Sections through “the rolling element”**

Indeed, for known shifts of each ring's joints, the rolling element will be subjected to contact forces at the running track level and shoulders. These forces  $Q$  are estimated by Hertz's theory of contact pressure:

$$Q = C_f \delta^n \tag{1.1.}$$

Where,

- $\delta$  is the deformation of elements in contact,  $n$  being an exponent varies based on the nature of the contact
- $C_f$  is the contact rigidity constant defined by Hertz that depends on the bearing's geometry and material characteristics.

For each iteration the forces acting on the rolling elements, the rings and the shoulders are determined. This is done by taking into consideration the bearing's geometric elements. Although balance is insufficient, forces between the rolling elements, rings and shoulders are known, so the tangent rigidity matrix between the rolling elements and the rings is known. An equivalent rigidity matrix can be determined between the joints of the interior and exterior ring of the bearing. This matrix corresponds to the tangent rigidity matrix associated to the rolling element.

The essential difference between the elements taken into consideration is that the radial field oscillations are defined through a tangent rigidity matrix for each rolling element. In each case "rolling element" assembling leads to a classic nonlinear finite element model, where the rigidity matrix depends on the shifts.

The balance position of the system "rings–rolling elements" is calculated on the Newton-Raphson iterative loop.

The Newton-Raphson method consists of iterative determination on the nonlinear phenomenon shift generalization matrix:

$$\{R(\{x^*\})\} = [K(\{x^*\})]\{x^*\} - \{F_{ext}\} = \{0\} \tag{1.2.}$$

Where:

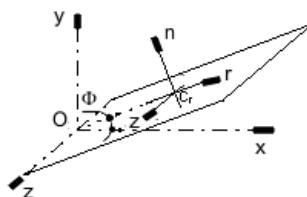
- $\{x^*\}$  is the generalized shift vector;
- $[K(x)]$  is the system rigidity matrix;
- $\{F_{ext}\}$  is the exterior forces applied to the system vector;
- $\{R(x)\}$  is the residual efforts vector.

On each iteration we determine:

- The forces and momentums applied on the rolling elements by the rings and possibly the shoulders;
- The rigidity matrix tangential between the rolling elements and the rings;
- The residual effort vector that acts on the rolling elements.

These calculations are made taking into consideration the lost motion and the geometry of the bearing.

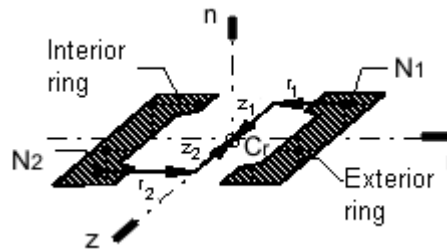
To each bearing we associate a system of axes  $S_1(O,x,y,z)$  in which the  $z$  axis corresponds to the bearing axis. The position of reference of the center  $C_r$  of a rolling element is determined by the  $\Phi$  angle of the axis system  $S_2(C_r,r,n,z)$  tied to the rolling element.



**Figure 1.3. Associated axis systems**

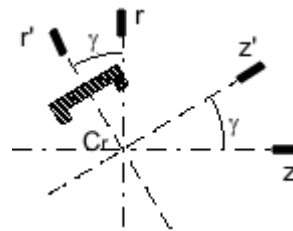
In the radial field  $(C_r, r, z)$  we determine the rolling element's balance. The  $S_2$  system is the reference one in which the rolling element's geometry and  $N_1$  and  $N_2$  joints position in relation to the center  $C_r$  are determined. The shift vectors will be:

$$\left\{ \overrightarrow{N_1 C_r} \right\}_{S_2} = \begin{Bmatrix} r_1 \\ 0 \\ z_1 \end{Bmatrix} \quad \left\{ \overrightarrow{N_2 C_r} \right\}_{S_2} = \begin{Bmatrix} r_2 \\ 0 \\ z_2 \end{Bmatrix} \quad (1.3.)$$



**Figure 1.4. Generalized shift vectors**

For calculating global finite elements, the generalized shift vectors of the  $N_1$  and  $N_2$  joints are calculated each iteration and can be represented in the system  $S_1$ . To determine the balance for the „rolling elements-rings” system we take into account only the relative shift of the rings. We consider the exterior ring as the reference and we will determine the relative shifts of the centers of the rolling elements in a  $S_3(C_r, r', n, z')$  system tied to this ring, but we will take into account the possible oscillations of the exterior ring's section in the reference system  $S_2$ .



**Figure 1.5. Oscillation angle**

The oscillation angle,  $\gamma$ , of the exterior ring in the radial field, is expressed according to the shifts  $\theta_1$  and  $\theta_2$  of the  $N_1$  joint in the reference system  $S_1$  by:

$$\gamma = -\sin \varphi \cdot \theta_x + \cos \varphi \cdot \theta_y \quad (1.4.)$$

In this system the joints  $N_1$  and  $N_2$  position is expressed through the vector:

$$\left\{ \overrightarrow{N_1 C_r} \right\}_{S_3} = \begin{Bmatrix} r_1 \\ 0 \\ z_1 \end{Bmatrix}; \quad \left\{ \overrightarrow{N_2 C_r} \right\}_{S_3} = \begin{bmatrix} \cos \gamma & 0 & \sin \gamma \\ 0 & 1 & 0 \\ -\sin \gamma & 0 & \cos \gamma \end{bmatrix} \cdot \left\{ \overrightarrow{N_2 C_r} \right\}_{S_2} = \begin{Bmatrix} r_2' \\ 0 \\ z_2' \end{Bmatrix} \quad (1.5.)$$

Where:

$$r_2' = (-r_2 \sin \gamma + z_2 \sin \gamma) \quad z_2' = (r_2 \cos \gamma + z_2 \cos \gamma)$$

The geometric matrixes  $[G_1]$  and  $[G_2]$  allow the determination of the plane shifts of  $C_r$  and the interior and exterior rings in the  $S_3$  system, depending on the shifts in 3D of the  $N_1$  and  $N_2$  joints defined in the  $S_1$  system.

$$\left\{ \overrightarrow{dep}(C_r \in I_e) \right\}_{S_3} = \begin{Bmatrix} u_r^1 \\ \theta_n^1 \\ u_z^1 \end{Bmatrix} = [G_1] \left\{ \overrightarrow{dep}(N_1 \in I_e) \right\}_{S_1} \quad (1.6.)$$

$$\left\{ \overrightarrow{dep}(C_r \in I_i) \right\}_{S_3} = \begin{Bmatrix} u_r^2 \\ \theta_n^2 \\ u_z^2 \end{Bmatrix} = [G_2] \left\{ \overrightarrow{dep}(N_2 \in I_i) \right\}_{S_1} \quad (1.7.)$$

These formulas are obtained by calculating the shifts of the  $N_i$  points in the  $S_3$  system, starting from the shifts in the  $S_1$  system.

$$\left\{ \overrightarrow{dep}(N_i) \right\}_{S_3} = \begin{pmatrix} [\mathfrak{R}_{1 \rightarrow 3}] & 0 \\ 0 & [\mathfrak{R}_{1 \rightarrow 3}] \end{pmatrix} \left\{ \overrightarrow{dep}(N_i) \right\}_{S_1} \quad (1.8.)$$

where  $[\mathfrak{R}_{1 \rightarrow 3}]$  is the rotation matrix that enables the switch from the  $S_1$  system to  $S_3$ .

$$[\mathfrak{R}_{1 \rightarrow 3}] = \begin{bmatrix} \cos \gamma \cos \varphi & \cos \gamma \sin \varphi & \sin \gamma \\ -\sin \varphi & \cos \varphi & 0 \\ -\sin \gamma \cos \varphi & -\sin \gamma \sin \varphi & \cos \gamma \end{bmatrix} \quad (1.9)$$

We will formulate the translation and rotation shifts of each ring in the  $C_r$  point by assimilating the ring sections with solid bodies. For each ring we will obtain:

$$\begin{aligned} \left\{ \overrightarrow{T}(C_r) \right\}_{S_3} &= \left\{ \overrightarrow{T}(N_i) \right\}_{S_3} + \left\{ \overrightarrow{C_r N_i} \right\}_{S_3} \times \left\{ \overrightarrow{R}(N_i) \right\}_{S_3} \\ \left\{ \overrightarrow{R}(C_r) \right\}_{S_3} &= \left\{ \overrightarrow{R}(N_i) \right\}_{S_3} \end{aligned} \quad (1.10)$$

where  $\{\overrightarrow{T}\}$  and  $\{\overrightarrow{R}\}$  are the translation and rotation torsos of the ring in question. The geometric matrix, for 3 relative shifts in radial field, is expressed by:

$$[G_i] = \begin{bmatrix} \cos \gamma \cos \varphi & \cos \gamma \sin \varphi & \sin \gamma & -z \sin \varphi & z \cos \varphi & 0 \\ -\sin \gamma \cos \varphi & -\sin \gamma \sin \varphi & \cos \gamma & r \sin \varphi & -r \cos \varphi & 0 \\ 0 & 0 & 0 & -\sin \varphi & \cos \varphi & 0 \end{bmatrix} \quad (1.11)$$

where  $r$  and  $z$  are the coordinates of the  $\{\overrightarrow{N_i C_r}\}$  vector in the  $S_3$  system. The last column, equal to zero, indicates the fact that the rotations around the bearing axis are not taken into consideration.

### 3. BIBLIOGRAPHY

- [1] Choi, I.S., Rigal, J.F. & Play, D., Housing effects on bearing load distributions of power transmission gearboxes. In: Proceedings of 3rd world congress on gearing and power transmissions CMET'92, Paris, 12-13 October, 1992.
- [2] Harris, T.A., Rolling bearing analysis. 2nd edition, New York, John Wiley and Sons, 1984, p 565
- [3] Lim, T.C., Singh, R., Vibration transmission through rolling element bearings. Part I, II, III, System studies, Journal of Sound and Vibrations, 1990, Vol. 139, nr.2, p (179-199, 201-225)
- [4] de Mul, J.M., Vree, J.M., Mass, D.A., Equilibrium and associated load in ball and roller bearing in five degrees of freedom while neglecting friction, Part I and II, Journal of Tribology, Trans. ASME, January 1989, Vol. 111, p (142-148, 149-155)
- [5] Munro, R.G., A review of theory and measurement of gear transmission error, Proc. 1-st IMechE Int. Conf. Gearbox Noise and Vib., Cambridge, 1991, p.(3-10).
- [6] Palmgren, A., Les roulements descriptions, théorie, applications, Paris : SKF, 1967, p120