FEA OF STRESS CONCENTRATOR EFFECT FROM A ROTATING DISK WITH A KEYWAY

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Keywords: stress concentration factor, FEM, keyway, rotating disk

Abstract: Rotating disks are found in numerous industrial applications. For an axially symmetrical disk, with or without a central hole, there are analytical relations between stresses, strains, depending on disk geometry and angular velocity. It is known that in static conditions a keyway induces increased stresses, acting as a stress concentrator. An analytical solution for the case of rotating disk with a keyway cannot be found due to complicated boundary conditions. The FE analysis presented in the paper allows appreciating the SCF of a keyway for the dynamic case.

1. INTRODUCTION

The stress concentration factor (SCF) is one of the most important causes producing machine elements failure during running. The stress concentration occurs mainly in the proximity of the regions where discontinuities of geometrical or material properties can be noticed, as example, inclusions, holes, dislocations etc. Finding the SCF defined as the ratio between the stress values for the case with and without discontinuities, is an important problem in mechanical design.

As reference stress it can be used one of the components of the stress tensor for mono-axial loading and an equivalent stress for the case of multi -axial loading.

Historically, the first analytical solving of the concentration factor was made by Kirsch, [1], who showed that the maximum of the hoop stress on the contour of a circular hole, placed into an elastic plate, uniaxial stretched at infinity, is three times the traction stress. For the present problem, the authors used the Finite Element method (FEM) in order to find the SCF. An experimental validation is difficult and we can mention, for instance: for the strain gauge method, the space is limited due to the dimensions of the keyway and if a photoelastic method is considered, [2], "stress frozen method" must be used as the disk is in rotation.

2. PROBLEM FORMULATION

A homogenous rotating steel disk is considered, with known elastic constants E and ν, and the angular velocity ω is constant and also, known. This option was based on the observation that in the reference work of Pilkey [3], the above problem is not considered. The disk presents a hole with a keyway with dimensions set according to [4], as seen in Fig. 1.

The mechanical characteristics of the steel are $E = 2.1 \cdot 10^{11} Pa$ and $\nu = 0.3$, and the angular velocity is constant, $\omega = 10 rad / s$. One of the assumptions is that during loading, the material presents a linear elastic behaviour and therefore the FEA simulation can be made using CATIA environment.

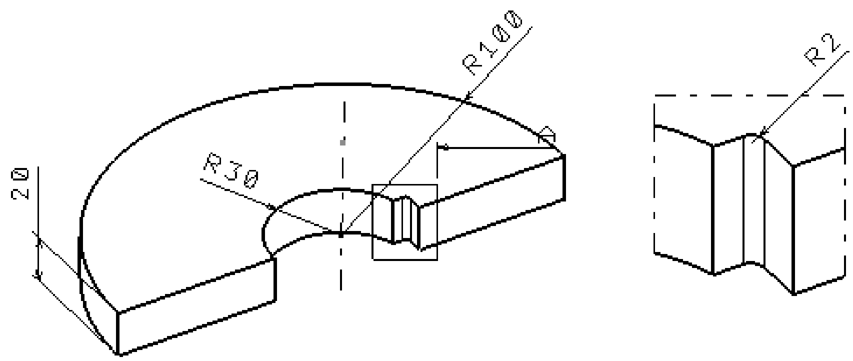


Fig.1 Dimensions of the disk

3. ESTIMATION OF METHOD ERROR

In order to estimate the numerical accuracy, first, the disk was considered without the keyway, as analytical relations are available for stresses calculus. For the hoop stress, the relation is, [5]:

$$\sigma_{hoop}(r) = \frac{3+\nu}{8}\left[(R_1^2 + R_2^2) - \frac{1+3\nu}{3+\nu}r^2 + \frac{R_1^2 \cdot R_2^2}{r^2}\right]\rho\omega^2 \qquad (1)$$

and the radial stress:

$$\sigma_{rad}(r) = \frac{3+\nu}{8}\left[(R_1^2 + R_2^2) - r^2 - \frac{R_1^2 \cdot R_2^2}{r^2}\right]\rho\omega^2 \qquad (2)$$

where $\rho = 7800 kg/m^3$ is the material density, r is the current radius, and R_1, R_2 are the inner and outer disk radii, respectively.

Figures 2 and 3 present on the left side the variation of the above stresses obtained numerically and on the right side, the analytical results for the same stress obtained via relations (1) and (2).

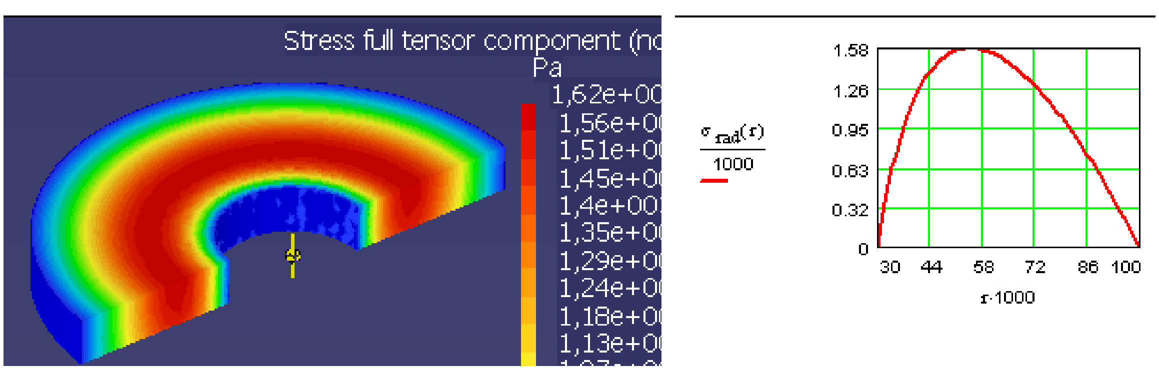


Fig.2. Radial stress variation

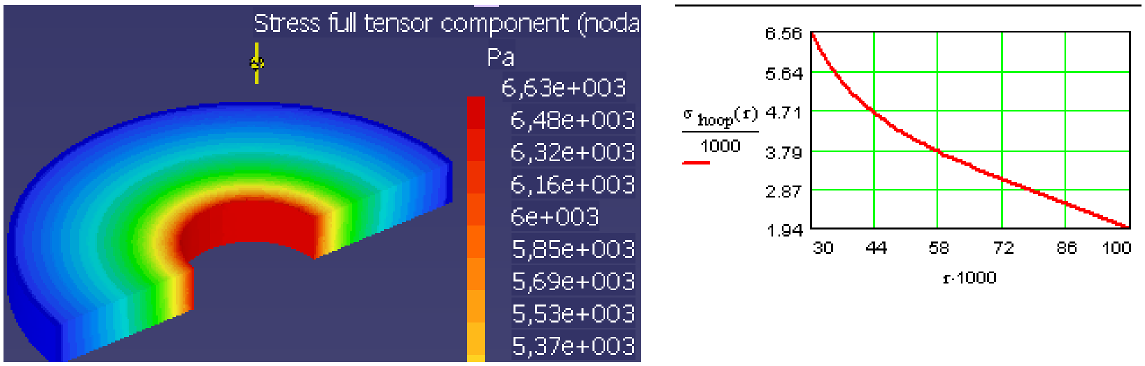


Fig.3. Hoop stress variation

4. FINDING KEYWAY SCF

The chosen stress for SCF calculus was the equivalent von Mises stress. In Fig. 4, the variation of the equivalent stress obtained with FEM is presented.

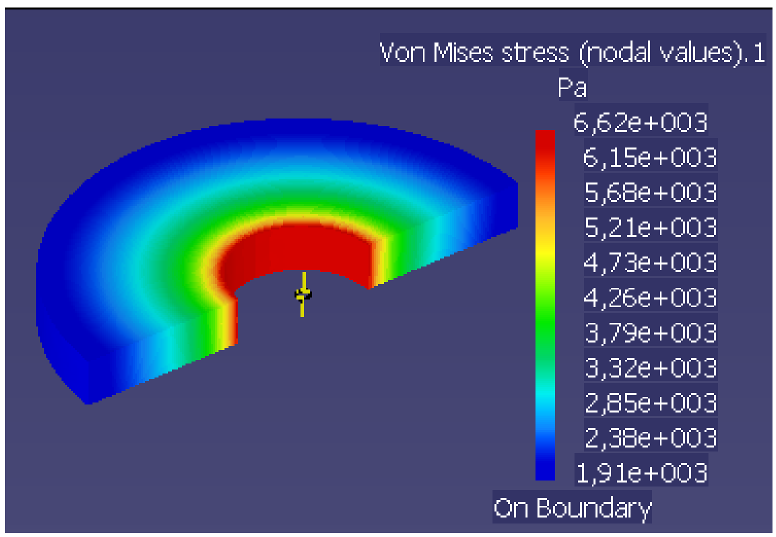


Fig.4. Equivalent von Mises stress

In order to obtain a high accuracy, the meshing parameters were diminished in the vicinity of the keyway, as shown in Fig. 5.

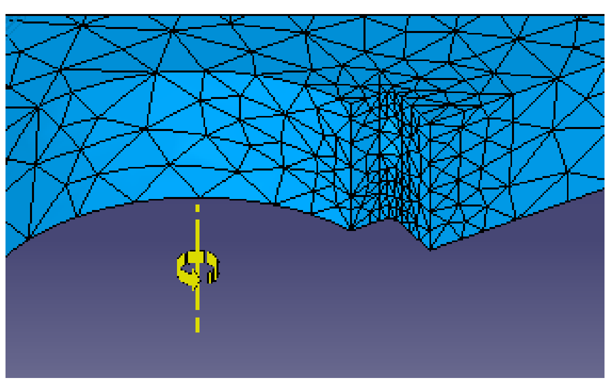
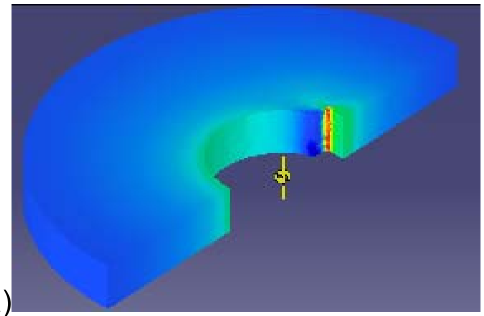


Fig.5. Meshing with variable parameters

The result of the numerical calculus is presented in Fig. 6.a, and the concentrator effect of the keyway and the fact that the maximum stress occurs in fillet region of the keyway is revealed. In fig. 6.b, a detail from the proximity of the keyway and the meshing is presented.

a) 

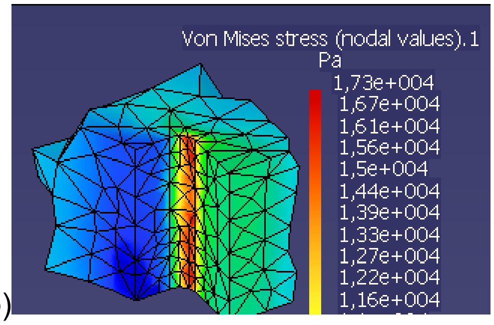
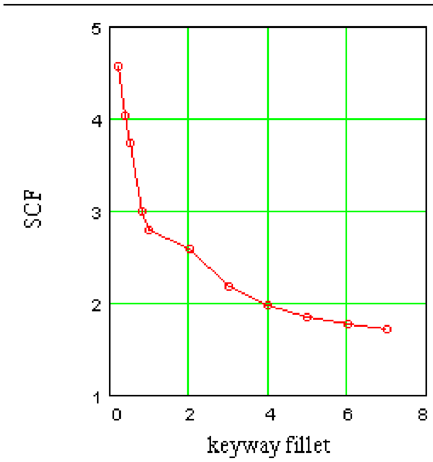
b) 

Fig.6. Stress concentration effect of keyway and maximum equivalent stress

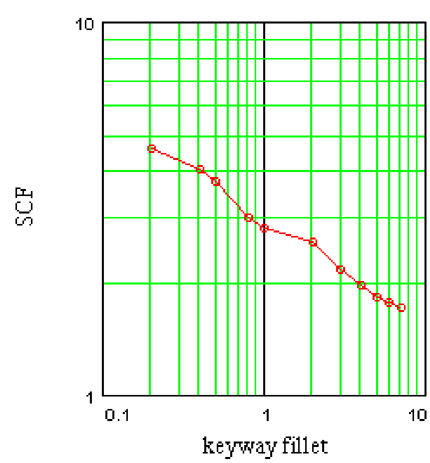
The equivalent von Mises stresses were found for a range of fillet radius values and the results are presented in Table 1. One can observe that for fillet radius values smaller than $0.2mm$, the maximum stress is lower than for other greater radii. This remark shows that the results are not realistic for these small radii, fact also emphasised by the program. The recommendations from [2], explain that for inner disk radius $R_1 = 30mm$, minimum fillet radius is $r_{r\ min} = 0.25mm$ and thus a finer meshing is not justified.

Table 1. Maximum values for von Mises stresses versus fillet radius

	Fillet radius [mm]	*Maximum values for von Mises stress [Pa]*
1.	0,2	3,05 10000
2.	0,4	2,69 10000
3.	0,5	2,49 10000
4.	0,8	2,00 10000
5.	1,0	1,87 10000
6.	2,0	1,73 10000
7.	3,0	1,46 10000
8.	4,0	1,32 10000
9.	5,0	1,23 10000
10.	6,0	1,18 10000
11.	7,0	1,15 10000



a)



b)

Fig.7. SCF versus fillet radius

The results form Table 1 are plotted and presented in Fig. 7. The fig. 7.a is a Cartesian plot and Fig. 7.b is a double logarithmic graph.

The last plot suggests that an interpolation function could take the form $y = Ar^{\alpha}$, to which, in double-logarithmic coordinates, a straight line corresponds. Applying the last square method, the interpolation function was found as it follows:

$$SCF(r) = 1.086 \cdot r^{-0.284} \tag{3}$$

The interpolation function is presented in Fig. 8 for a variation range $r_r = [0.2, 15]$, together with the numerical results from FEM.

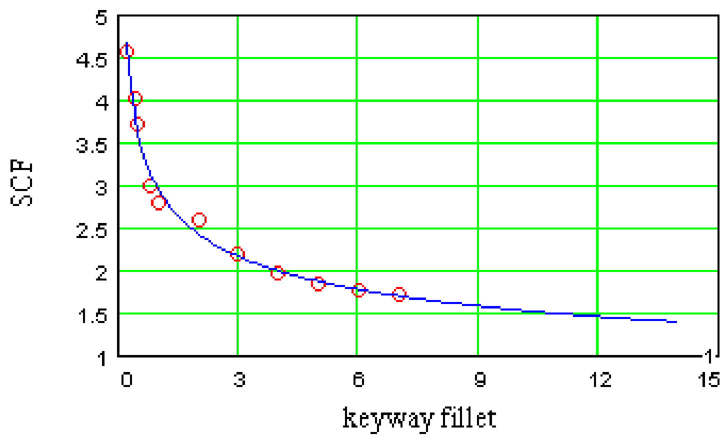


Fig. 8. FEM numerical data and the interpolation function plot

The interpolation function presents two main features:

a) when the fillet radius falls to zero, SCF tends to infinity;
b) for greater fillet radii, SCF tends asymptotically to 1, meaning that the concentrator factor vanishes.

Frocht, [6], presents the results obtained by Nisida concerning the concentrator effect of a keyway from a twisted shaft. Fig. 9 presents the isochromatics pattern from a photoelastic model, obtained using “frozen stress method”.

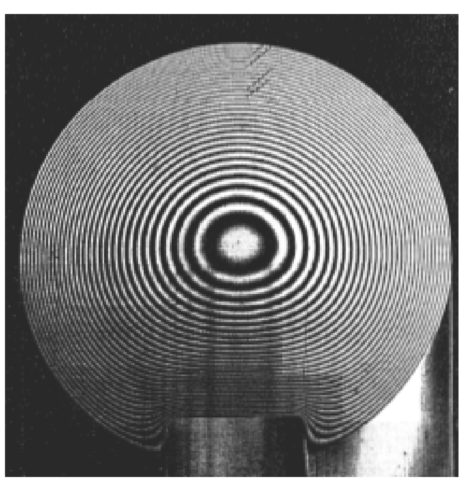


Fig.9. Stress isochromatics pattern from a shaft with a keyway

Nisida also presented the SCF dependence on fillet radius comparatively to other authors, and the result are shown in Fig. 10. It can be seen from the plots that the general shape confirms the observations made for Fig. 8.a and 8.b.

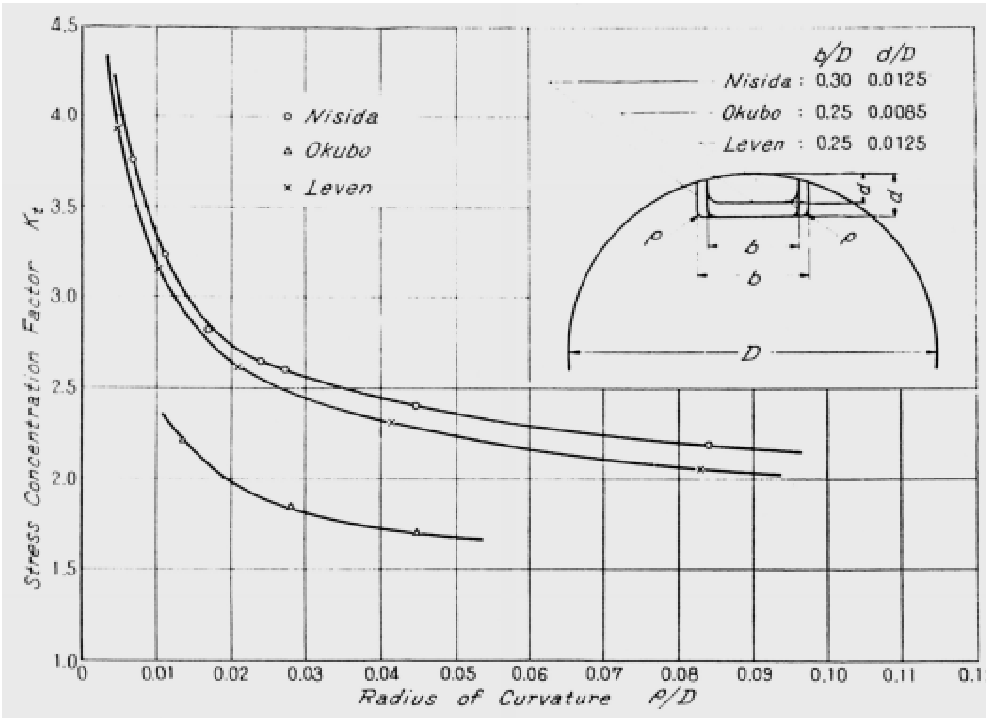


FIG. 15. Variation of stress concentration factor K_t with ρ/D.

Fig.10. Variation of SCF with r / D ***for a circular shaft with a keyway under torsion.***

5. CONCLUSIONS

The paper presents the stress concentrator effect of a keyway from a uniform rotating disk using finite element method.

The analytical relations from literature were used for validation of the method, applied for the case of a disk without keyway. The accuracy of the method was tested and the dimension range of validity.

The SCF was found for different values of fillet radius. For the obtained values, the interpolation function was proposed. The plot obtained using the interpolation function was compared to the plot of experimental SCF for a twisted shaft. One can see that the interpolation function obtained with FEM results presents the same shape as the plots from literature.

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