USE OF THE CLOSED DIFFERENTIAL MECHANISMS TO BUILD AVIATION REDUCERS

TEMPEA Iosif[1], LAZĂR Ionel[2], ADÎR George[1]
[1] University POLITEHNICA of Bucharest, Theory of Mechanisms and Robots Department,
[2] S. C. MNA, Bucharest
iosiftempea@yahoo.com, alexandru.lz@gmail.com

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Abstract: The paper analyses some general problems of using closed differential mechanisms to build aviation reducers.There are introduced structural block-diagrams and some kinematic schemes of closed differential mechanisms. The kinematic analysis of the reducer of the AI-20 turboprop engine is carried out, and there are presented some constructive details of these reducers. Further on, the problems of using closed differential mechanisms in building of the helicopter reducers and the kinematic analysis of some reducers, along with some of their constructive characteristics, are also presented.

1. GENERAL CHARACTERIZATION OF THE CLOSED DIFFERENTIAL REDUCERS

The closed differential mechanisms belong to the composed gear mechanisms group. They are achieved by an elementary *base differential mechanism* (BDM) and a suplementary kinematic chain made by gears, called *closing kinematic chain* (CKC), used to join two elements of the differential mechanism. Thus, the obtained mechanism has the mobility degree $M=1$.

The closed differential mechanisms, usually have big efficiencies (0,93…0,98), explained thru the possibility to separate the transmitted power in two parallel tides, and can achieved big reducing ratios $(i=12...50)$ for reduced overall sizes of the transmission. To provide big efficiencies it has to be avoided the meeting of the power tides inside the mechanism, situation in which it's possible to come out a circulation of power that can produce power looses [14].

In [3] CKC is assimilated to a reverse connection introduced between the two elements of the differential mechanism. The kind of this reverse connection has not a certain significance (CKC can be achieved thru fixed axles gears, planetary mechanisms, thru friction etc), because it is not reflected in the kinematic equations.

By taking into consideration the elements of the differential mechanism tied thru CKC and its role in transmitting the power, we find, just theoretically, 6 possibilities to establish such kind of mechanisms [5].

In Fig. 1 there are introduced the structural schemes of these mechanisms and a kinematic scheme corresponding to each of them, achieved on a differential mechanism with cylindrical gears. In this figure, we have used the following notations:

- BDM – elementary base differential mechanism;
- CKC1 – closing kinematic chain at the central gear 1;
- CKC3 – closing kinematic chain at the central gear 3;
- CKCS – closing kinematic chain at the port-satellite arm *S*.

But, taking into consideration the restriction mentioned above, some authors [1, 12] show only 4 possibilities to achieve closing, respectively, alternatives 1 – 4 from Fig. 1.

The calculus analytical method is based upon establishing of 2 kinematic equations: one expresses the BDM reducing ratio and the second one expresses the CKC reducing ratio. From this system, by knowing the speed of one element, it can be determined the other two ones.

2. REDUCERS ACHIEVED BY USING CLOSED DIFFERENTIAL MECHANISMS FOR TURBOPROPELLER ENGINES

By taking into consideration the advantages mentioned, closed differential mechanisms are used in building of the turboprop engines reducers, to drive the simple or coaxial propellers. As for the reducers used for simple propellers, there aren't distinct problems (it will be analyzed a constructive example below). As for the reducers used for coaxial propellers, appear some specific features. Thus, because the closed differential mechanism has $M=1$, it is kinematically established and therefore the propellers speed will be always equal, independent of their pitch. This feature allows to simplify the command system of the propellers pitch. On the other hand, at the closed differential reducers, the unequal change of the propellers pitch leads to the alteration of the distribution of the torques between propellers and of the loads on CKC. When the torques between the two propellers are distributed as for an open differential reducer, then the power tide is transmitted entirely to the central gears of the differential and the gears of the CKC are not stressed. If it's changed the pitch of a propeller, then thru CKC it takes place a re-distribution of the torques between propellers.

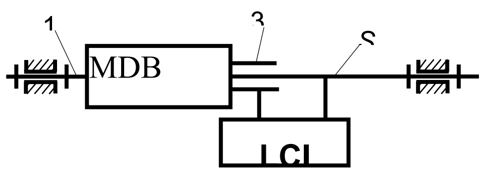
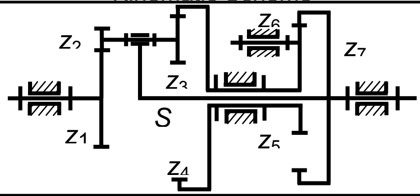
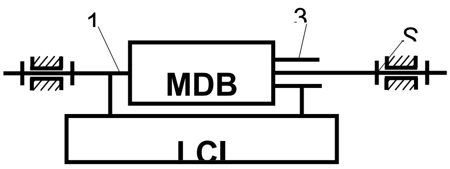
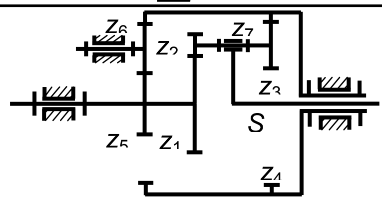
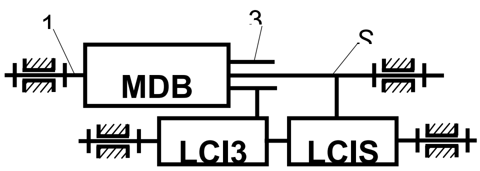
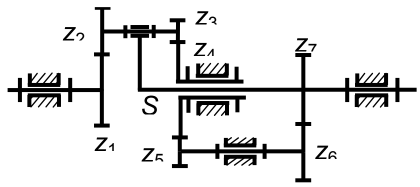
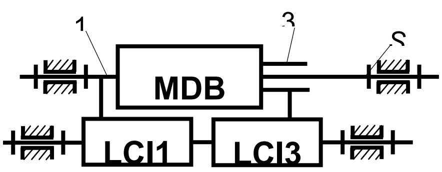
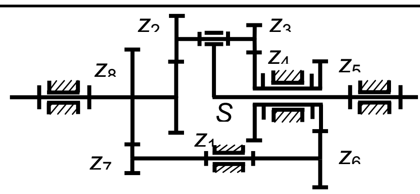
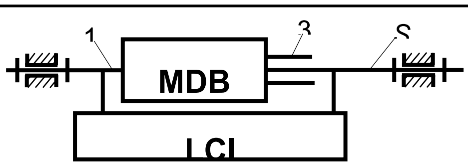
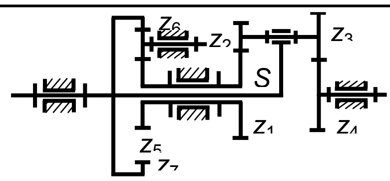
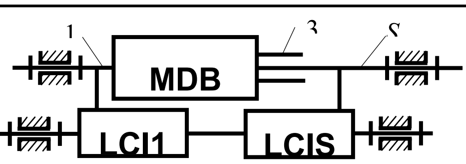
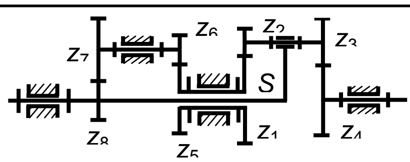
Type	Coupling	Block Scheme	Kinematic Scheme
a	3-S		
b	1-3		
c	3-S		
d	1-3		
e	1-S		
f	1-S		

Fig.1. Structural block schemes and kinematic schemes of the closed differential mechanisms

As regards construction, the alternatives of these reducers are different due to the position of the CKC: 1) CKC installed between the propellers axles (fig. 2*a*); 2) CKC installed between the input axis and a propeller axis (fig. 2*b*). Based on the clasiffication from Fig. 1, the alternative from Fig. 2a is of type *c* (there are connected the driven central gear 3 and the port-satellite arm *S*) and the alternative from Fig. 2*b* is of type *f* (there are connected the leading gear 1 with the port-satellite arm *S*). Based on what we have mentioned above, for the first alternative the reducing ratio of the CKC must be equal to 1 and for the second alternative it must be equal with the reducing ratio of the reducer at equal speeds of the propellers.

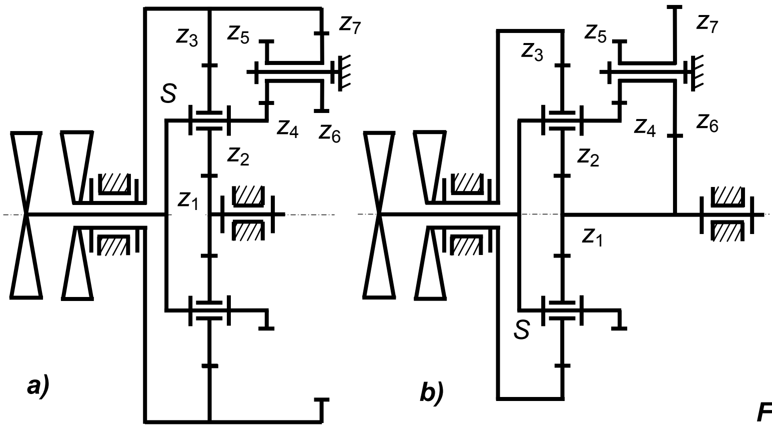


Fig. 2. Closed differ. reducers for coaxial propellers

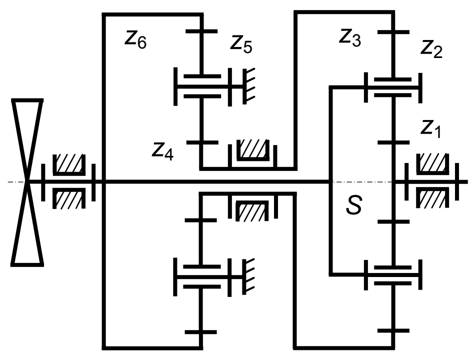


Fig.3. Kinematic scheme of the reducer of the turboprop enginesAI-20 and AI-24

Let's analyze now a constructive solution of a reducer for TPE, achieved by using a closed differential mechanism. Thus, in Fig. 3 is presented the kinematic scheme of a reducer that fits out the turboprop engines AI-20K and AI-24. From viewpoint of the kinematic calculus, the two reducers are distinct thru the teeth numbers of gears and thru the input speeds: 12,300 rot/min, respectively 15,100 rot/min. In these terms, is different also the torque transmitted to propeller: 24,080 N·m, respectively 13,450 N·m. At the reducer of the engine AI-20K there are 4 satellites z_2 and 6 gears z_5. At this engine, the propeller axis speed is $n_{\mathrm{el}} = n_{\mathrm{S}} = 1074\ \mathrm{rot/min}$, therefore the reducer achieved a reducing ratio $i_{\mathrm{red}} = n_{\mathrm{t}} / n_{\mathrm{el}} = 12300/$ $/1074 = 11.4525$.

By applying to the differential mechanism z_1-z_2-z_3-S the Willis' method, we obtain a first kinematic equation:

$$i_{13}^{\mathrm{S}} = (n_1 - n_{\mathrm{S}})/(n_3 - n_{\mathrm{S}}) \tag{1}$$

or

$$n_1 = i_{13}^{\mathrm{S}} \cdot n_3 + n_{\mathrm{S}} \cdot \left(1 - i_{13}^{\mathrm{S}}\right). \tag{2}$$

With the notations $i_{1\mathrm{S}} = n_1/n_{\mathrm{S}}$ and $i_{3\mathrm{S}} = n_3/n_{\mathrm{S}}$, from the above equation results $i_{13}^{\mathrm{S}} = (i_{1\mathrm{S}} - 1)/(i_{3\mathrm{S}} - 1)$, from where

$$i_{1\mathrm{S}} = 1 + i_{13}^{\mathrm{S}} \cdot (i_{3\mathrm{S}} - 1). \tag{3}$$

Based on the kinematic scheme of the differential mechanism we obtain

$$i_{13}^{\mathrm{S}} = (-z_2/z_1) \cdot (z_3/z_2) = -z_3/z_1 . \tag{4}$$

Because the CKC is disposed between the central gear z_3 and the port-satellite arm *S* and it is made by the gears z_4-z_5-z_6, its reducing ratio will be:

$$i_{3S} = i_{46} = (-z_5/z_6)\cdot(z_6/z_5) = -z_6/z_4\,. \tag{5}$$

By introducing the expressions of i_{13}^{S} and i_{3S} in (2), results:

$$i_{1S} = 1 + (z_3/z_1)\cdot(1 + z_6/z_4). \tag{6}$$

For the engines AI-20 and AI-24 the teeth numbers of the gears are accordingly: $z_1 = z_4 = 35/31$; $z_2 = z_5 = 31/29$; $z_3 = z_6 = 97/89$. With these values of the teeth numbers, we obtain at the suitable engines reducers the reducing ratios 11.4522, respectively 12.1134.
The expression (6) can be directly obtained from (2), if we introduce here instead of the ratio n_3/n_S the reducing ratio i_{46} of the CKC.
According to data from [3], at one of the alternatives of the reducer of the engine AI-20, the gears had the following parameters: module $m = 3.8788$ mm; reference profile angle $\alpha_P = 28°$; factor of the reference tooth height $h_P^* = 0.9$; coefficient of the radial clearance $c_0 = 0.28$; the tooth's gap is shaved, with complete racord at the base. In the reducer's modernization process, the reference profile angle has been successively increased from 20° to 27°30′, and then to 28°. In this last case, the teeth are achieved by shaving, the final mechanical operation of shaving being done before the heat treatment. After the final mechanical operation, the working surfaces of teeth are hardened by using shots jet. These measures, together with the increase of the gear module from 3.75 mm to 3.8788 mm, allowed, at an unmodified overall size of the reducer, to increase the transmitted power with almost 1000 HP, at an increase of 3 times of the resource. The external teethed gears are made from low alloy steel 12H2N4A (0.12% C, 2% Cr, 4% Ni, Nitrogen), are case hardened and tempered. The gear rims are made from steel 38HMIuA (38% Cr, Mo, Al, Nitrogen) and they are nitrided.
The power P_{LCI} that solicits the elements of the CKC is given by the expression

$$P_{LCI} = (1/716{,}2)\cdot M_3 n_3\,, \tag{7}$$

where $M_3 = -M_1 i_{13}^{S}$ and it is obtained from the satellite equilibrium condition.
Because from the equilibrium condition of gear with z_3 and z_4 results $M_4 = -M_3$, and the transmitted power is $P_e = (1/716.2)\cdot M_1 n_1$, after their introduction in (7), we obtain $P_{LCI}/P_e = M_1 i_{13}^{S} n_3 / M_1 n_1$. By multiplying and deviding the right member with n_S, it finally results $P_{LCI}/P_e = i_{46} i_{13}^{S} / i_{1S} = z_6 z_3 / z_4 z_1 i_{1S}$.
At the reducer of the turboprop engine AI – 20, the transmitted power thru CKC represents about 70% from the power transmitted to the propeller.

3. CLOSED DIFFERENTIAL REDUCERS FOR HELICOPTERS

Due to the possibility of achieving of a bigger reducing ratio, the closed differential mechanisms are much used in bulding of the main reducers of helicopters. In Fig. 4 is presented the kinematic scheme of the reducer VR-8A, which fits out the helicopter Mi-8. The reducer has 3 steps (the teeth numbers are mentioned below) and achieves a total reducing ratio

$$i_{\text{red}} = n_1/n_{\text{el}} = 12000/192 = 62.5\,,$$

where n_1 is the speed of the free turbine (speed of the input shaft in the reducer), and n_{el} is the speed of the lifting propeller.

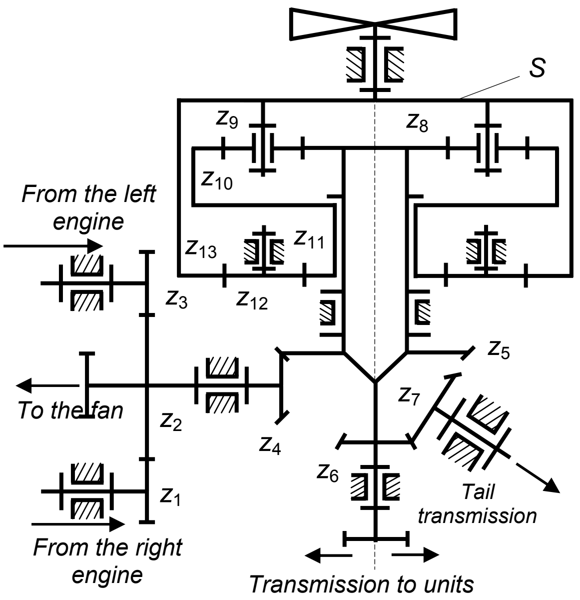


Fig. 4. Kinematic scheme of the reducer VR-8 from the helicopter Mi-8

The gears have the following teeth numbers: $z_1 = z_3 = 33$; $z_2 = 95$; $z_4 = z_7 = z_8 = 31$; $z_5 = 66$; $z_6 = 41$; $z_9 = z_{12} = 29$; $z_{10} = 89$; $z_{11} = 48$; $z_{13} = 106$.

The first step of the reducer is made by a cylindrical gear with the reducing ratio $i_{12} = i_{32} = z_2/z_1 = z_2/z_3 = 95/33 = 2.87(87)$. The second step is made by a conical gear with the reducing ratio $i_{45} = z_5/z_4 = 66/31 = 2.1290$. The third step is made by a closed differential mechanism, in which z_8-z_9-z_{10}-S represent the base differential mechanism and z_{11}-z_{12}-z_{13} is the CKC, that achieves the connection between the central gear z_{10} and the port-satellite arm S (it is about the alternative from Fig. 1c). The kinematic study of this step is similar to that of the mechanism from Fig. 3. By applying the Willis' method to the base differential mechanism, we obtain the expression:

$$i^{\text{S}}_{8,10} = (n_8 - n_{\text{S}})/(n_{10} - n_{\text{S}}) = (-z_9/z_8)\cdot(z_{10}/z_9) = (-z_{10}/z_8)\,. \tag{8}$$

With the notations $i_{8\text{S}} = n_8/n_{\text{S}}$, $i_{10,\text{S}} = n_{10}/n_{\text{S}}$ we re-write the equality from the previous expression under the aspect:

$$i_{8\text{S}} = 1 + i^{\text{S}}_{8,10}\cdot(i_{10,\text{S}} - 1)\,. \tag{9}$$

From the kinematics of the closing chain, we can write the expression:

$$i_{10,\text{S}} = n_{11}/n_{13} = (-z_{12}/z_{11})\cdot(z_{13}/z_{12}) = -z_{13}/z_{11}\,. \tag{10}$$

By introducing the suitable expressions of $i^{\text{S}}_{8,10}$ and $i_{10,\text{S}}$ in (10) result:

$$i_{8\text{S}} = 1 + (z_{10}/z_8)\cdot(1 + z_{13}/z_{11})\,. \tag{11}$$

The values of the teeth numbers being previously mentioned, we obtain: $i_{8\text{S}} = 1 + (89/31)\cdot(1 + 106/48) = 10.2110$. Then, the total reducing ratio of the reducer is:
$i_{1\text{S}} = \; = i_{12}\cdot i_{45}\cdot i_{8\text{S}} = 2.87(87)\cdot 2.1290\cdot 10.2110 = 62.580$, that means the speed of the lifting propeller is $n_{\text{el}} = n_1/i_{1\text{S}} = 12000/62.580 = \; 191.754 \approx 192\ \text{rot/min}$.

The overall size dimensions of the reducer are 1,055x8,80x1,760 mm. The mass of the reducer is 7,50 kg (there is here an interesting aspect, because the mass of the two

engines TV2-117A is 6,60 kg). The reducer transmitts a maximum power of 3,000 HP to the lifting propeller, the working time at this regime being of 5% from the resource.
Based on a similar scheme is achieved also the reducer VR-7 (Fig. 5), that fits out the helicopter Mi-6, the reducing ratio achieved being $i_{\text{red}} = n_1/n_{\text{el}} = \ = 8{,}300/1{,}20 = 69.166$. The teeth numbers of the gears from the transmission are:: $z_1 = 28$; $z_2 = 67$; $z_3 = z_7 = = z_9 = z_{12} = 31$; $z_4 = 139$; $z_5 = 28$; $z_6 = 67$; $z_8 = 65$; $z_{10} = 127$; $z_{11} = 79$; $z_{13} = 141$. The gears z_1, z_2, z_3 are 4 pieces each, the gears z_9 şi z_{12} are 8, respectively 12.
At this reducer the first step is conical and the second is cylindrical, their reducing ratios being $i_{12} = z_2/z_1 = 67/28 = 2.3928$, respectively $i_{34} = z_4/z_3 = 139/31 = 4.4838$.The third step represents a closed differential mechanism in the alternative from Fig. 1*c*.
Therefore, to establish the reducing ratio of this step is still valid the expression (11) (the noting of the gears at the closed differential mechansim is identical to that from Fig. 4), with the suitable values of the teeth numbers previously mentioned obtaining: $i_{8\text{S}} = 1 + (127/65)\cdot(1 + 141/79) = 6.4410$. Then, the total reducing ratio of the reducer is: $i_{1\text{S}} = i_{12} \cdot i_{34} \cdot i_{8\text{S}} = 2.3928 \cdot 4.4838 \cdot 6.4410 = 69.108$, namely the speed of the lifting propeller is $n_{\text{el}} = n_1/i_{1\text{S}} = 8{,}300/69.108 = 120.1018 \approx 120\ \text{rot/min}$.
The reducer has the overall size dimensions 1,852x1,551x2,975 mm. The mass of the reducer is 3,200 kg. The dimensions are not surprising if we take into consideration that Mi-6 is one of the greatest helicopters in the world: its maximum weight at taking off is 42,500 daN, for an useful load of 12,000 daN; the the rotor has 5 proppeler blades and a diameter of 35 m; it is fitted out with 2 turboengines Soloviev D-25M of 5,500 HP each in taking off regime.

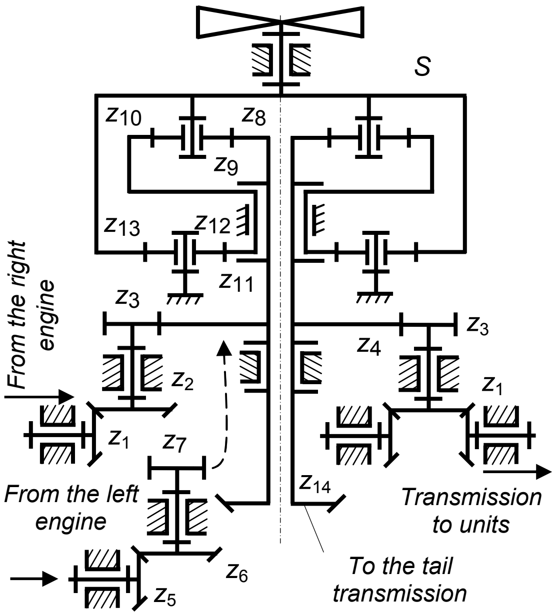


Fig. 5. Kinematic scheme of the reducer VR-7 from the helicopter Mi-6

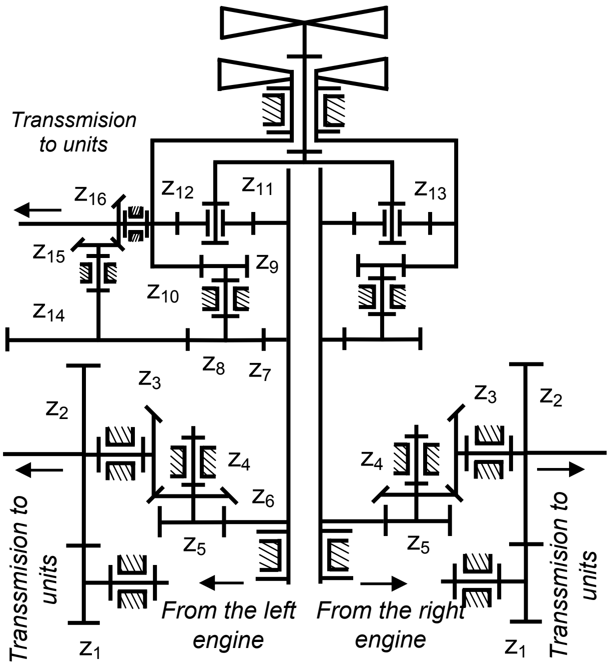


Fig. 6. Kinematic scheme of the reducer RV-3F from the helicopter Kamov Ka-25K

A closed differential mechanism is used, also, in building of the main reducer of the helicopter Kamov Ka-25K. The first 3 steps of the reducer (Fig. 6) are made by elementary cylindrical and conical gears with the folowing reducing ratios: $i_{12} = z_2/z_1 = 89/26 = 3.4230$; $i_{34} = z_4/z_3 = 38/21 = 1.8095$; $i_{56} = z_6/z_5 = 55/24 = 2.2916$. The fourth step is made by a closed differential mechanism in the alternative 4 from Fig. 1, closing being achieved between the central gears z_{11} and z_{13} of the base differential mechanism z_{11}-z_{12}-z_{13}-S thru the kinematic chain z_7-z_8-z_9-z_{10}. By using the Willis' method, for the base differential mechanism we obtain the expression:

$$i_{11,13}^{S} = (n_{11} - n_S)/(n_{13} - n_S) = (-z_{12}/z_{11}) \cdot (z_{13}/z_{12}) = -z_{13}/z_{11} = -88/38 = -2.3157 . \quad (12)$$

From the kinematics of the closing kinematic chain we can write the expression:

$$i_{7,10} = n_7/n_{10} = (-z_8/z_7) \cdot (z_{10}/z_9) = (-54/57) \cdot (107/18) = -5.6315 . \quad (13)$$

From the kinematic scheme is noticed that this is the partial reducing ratio at the fourth step at the inferior propeller. To establish the reducing ratio at the superior propeller we notice that $n_7 = n_1$ and $n_{10} = n_{13}$. From expression (13), it results $i_{7,10} = n_7/n_{10} = n_{11}/n_{13}$, from where $n_{13} = n_{11}/i_{7,10}$. By introducing this expression of n_{13} in the first equality from (3.5), we obtain:

$$i_{11,13}^{S} = [(n_{11} - n_{13}) \cdot i_{7,10}]/(n_{11} - n_S \cdot i_{7,10}) = [(i_{11,S} - 1) \cdot i_{7,10}]/(i_{11,S} - i_{7,10}), \quad (14)$$

where $i_{11,S} = n_{11}/n_S$, this being the partial reducing ratio from the fourth step at the superior propeller.
From (14) results immediately:

$$i_{11,S} = i_{7,10} \cdot (i_{11,13}^{S} - 1)/(i_{11,13}^{S} - i_{7,10}) = [-5.6315 \cdot (-2.3157 - 1)]/[-2.3157 - (-5.6315)] = 5.6313 .$$

By comparison with the value obtained in (13), we notice that the propellers speeds are equal and of opposite sense.
The reducer achieves a total reducing ratio

$$i_{red} = i_{12} \cdot i_{34} \cdot i_{56} \cdot i_{dif} = 3.4230 \cdot 1.8095 \cdot 2.2916 \cdot (\pm 5.6315) = \pm 79.9385 , \quad (15)$$

Where + is for the superior propeller, – for the inferior propeller and i_{dif} represents the value of the reducing ratio of the closed differential mechanism , established with the expression (13), respectively (14).

4. CONCLUSIONS

The closed differential mechanisms are used at the aviation reducers due to some specific advantages: high efficiencies at high reducing ratios, compact construction, good reliability, simple and easy maintenance. If they are used at the reducers of the coaxial propellers of the turboprop engines, they allow to simplify the command system of the propellers pitch, because of some personal functional specific features.

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