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INFLUENCE OF GEOMETRICAL PARAMETERS ON THE MID-ZONE FACTOR Z_{M-B} IN THE ELOID SPIRAL BEVEL GEARS

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Abstract: For stress calculus, the spiral bevel gears are replaced with virtual cylindrical gears corresponding to the median frontal cones. The tension during the contact stress reaches its pitch in a point in the median area of the segment of singular gearing of the virtual cylindrical gear. The correspondence between the contact stress in the driving pole and the one in the point in which this stress is maximal is made through the mid-zone factor Z_{M-B}. The main geometrical parameters which are taken into consideration in the analysis of the variation of the factor Z_{M-B} are: the number of teeth of the pinion z_1; the gear ratio u; the nominal coefficients of the radial profile shifts $x_{c1,2}$; the median division inclination angle of the teething β_m and the median width coefficient of the teething ψ_{Rm}.

1. INTRODUCTION

As compared to the cylindrical gears, in which most often the contact between the teeth is linear, in spiral bevel gears the contact between the teeth takes place on an elliptically shaped pressure surface, due to the bulging out of the teeth, both on their length and on their height [1, 3, 4, 5]. At the eloid spiral bevel gear, the flank line of the plane wheel tooth is an extended epicycloids, and the height of the teeth is constant. The eloid spiral bevel gear is usually processed through the Oerlikon-Spiromatic procedure [5, 10]. For machining, cutter tools head are used, in which the cutters are fixed in groups, on different spirals.

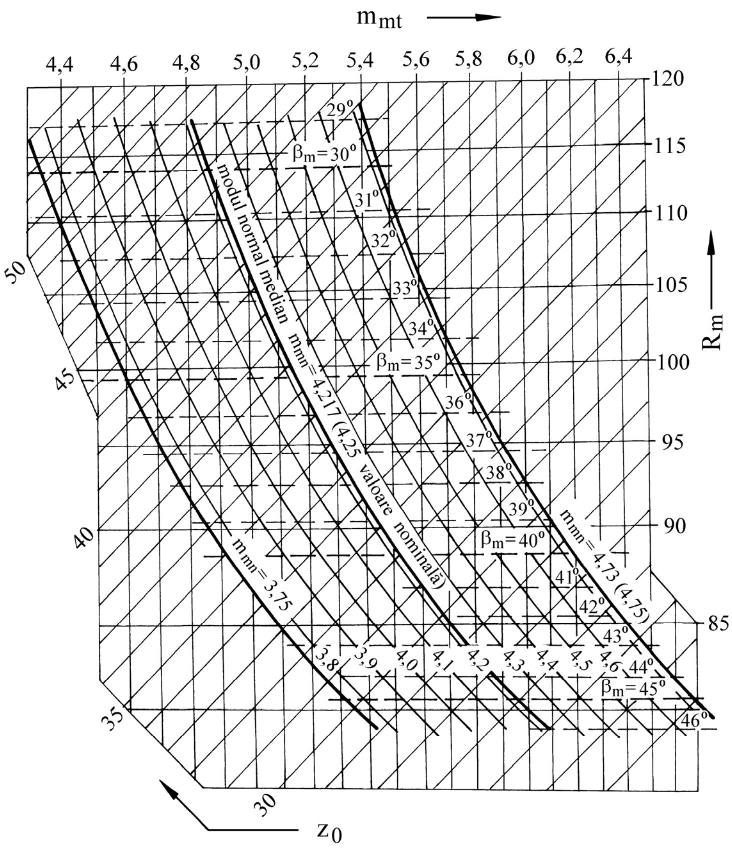

Fig. 1. Diagrama velă

The range of use of every cutter tools head is determined through its *sail diagram* [10] (fig. 1). The dependence among the parameters which characterize the range of use of every cutter tools head is settled through the relation

$$R_m = \frac{m_{mt} z_0}{2} = \frac{m_{mn} z_0}{2\cos\beta}, \qquad (1)$$

in which R_m is the mean cone distance; m_{mt} – mean transverse module; m_{mn} – mean normal module; z_0 –number of teeth of the plane wheel.

Every cutter tools head, in the range of normalized tools, is built for an angle β_m = 35^0, a certain field of

values for the parameters z_0 and R_m and a certain mean normal module, named nominal module of the tool and noted m_c.
For machining an eloid spiral bevel gear, the cutter tools head is chosen such as the coordinates R_m and m_{mt} (or z_0) intersect its sail diagram (see Figure 1) in the central area, against the lateral flanks, which limitate its range of use. This requirement is fulfilled through the modification of the parameters R_m, m_{mt}, z_0 and β_m in relation (1). Following of this requirement leads to the equality $m_{mn}=m_c$.
The contact stress calculation of the eloid spiral bevel gears is carried out in their mean section, the hypotheses and the calculus relations settled for the cylindrical helical gears being applied to the virtual gear, corresponding to the mean frontal cones [2, 5, 6, 8].

2. THEORETICAL CONSIDERATIONS

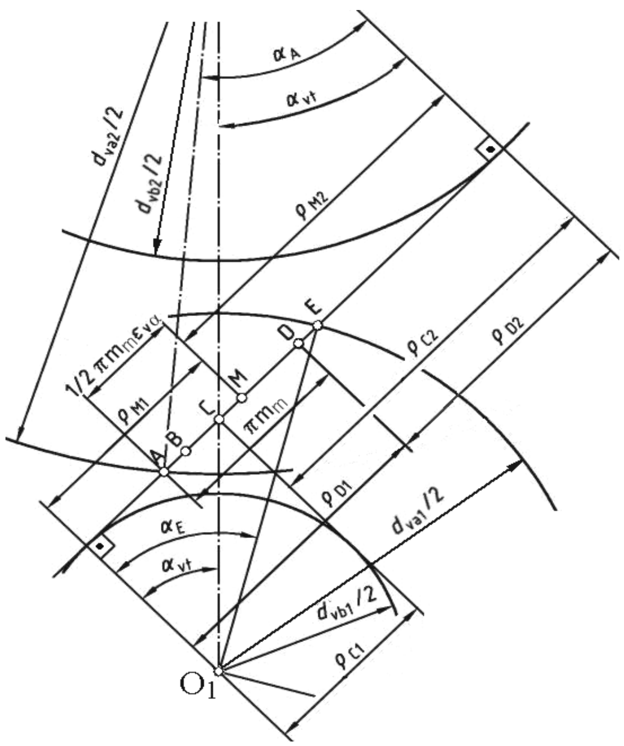

Fig. 2. Radius of curvature of tooth profile for caracteristic points

The design activity in the field of the spiral bevel gears is sustained by the standards elaborated in this respect and by the researches carried out to the purpose of improving the performance of these gears [6, 7, 8, 9]. Considering the contact of the teeth of the virtual cylindrical gear, in the pole of the gearing C (fig. 2), the nominal value of the contact stress for an orthogonal bevel gear (Σ=90º) is determined with relation [9],

$$\sigma_{H0} = \sqrt{\frac{F_{mt}}{d_{m1} l_{bm}} \frac{\sqrt{u^2+1}}{u}} Z_{M-B} Z_H Z_E Z_{LS} Z_\beta Z_K \,, \qquad (2)$$

in which F_{mt} is the nominal tangential force at reference cone at mid-face width; d_{m1} – mean pitch diameter of the bevel pinion; l_{bm} – length of middle line of contact; u – gear ratio of bevel gear; Z_{M-B} – mid-zone factor; Z_H – zone factor; Z_E – elasticity factor; Z_{LS} –load sharing factor; Z_β – helix angle factor for contact stress; Z_K – bevel gear factor (flank). The factor Z_{M-B} is determined [9] with the relation

$$Z_{M-B} = \frac{\tan \alpha_{vt}}{\sqrt{\left[\sqrt{\left(\frac{d_{\upsilon a1}}{d_{\upsilon b1}}\right)^2 - 1} - F_1 \frac{\pi}{z_{v1}}\right]\left[\sqrt{\left(\frac{d_{\upsilon a2}}{d_{\upsilon b2}}\right)^2 - 1} - F_2 \frac{\pi}{z_{v2}}\right]}} \qquad (3)$$

in which $\alpha_{\upsilon t}$ is the transverse pressure angle of virtual cylindrical gear; $d_{\upsilon a1,2}$ – tip diameters of virtual cylindrical gears; $d_{\upsilon b1,2}$ – base diameters of virtual cylindrical gears; $z_{\upsilon 1,2}$ – numbers of teeth of the virtual gears; $F_{1,2}$ – auxiliary factors for mid-zone factor.
For the calculation of the auxiliary factors F_1 and F_2 the following relations [8] are used:

$$F_1 = 2 + (\varepsilon_{v\alpha} - 2)\varepsilon_{v\beta}; \;\; F_2 = 2\varepsilon_{v\alpha} - 2 + (2 - \varepsilon_{\upsilon\alpha})\varepsilon_{\upsilon\beta} \,, \qquad (4)$$

for $0 < \varepsilon_{\upsilon\beta} \le 1$, respectively $F_1 = F_2 = \varepsilon_{\upsilon\alpha}$, for $\varepsilon_{\upsilon\beta}>1$, in which $\varepsilon_{\upsilon\alpha}$ is the transverse contact ratio of virtual cylindrical gear; $\varepsilon_{\upsilon\beta}$ – overlap ratio of virtual cylindrical gear.
Relation (4) was determined through interpolation between the straight bevel gear with $\varepsilon_{\upsilon\beta} = 0$ and the spiral bevel gear with $\varepsilon_{\upsilon\beta}= 1$ [9].
The angle $\alpha_{\upsilon t}$ is determined through relation

$$\alpha_{\upsilon t} = \arctan\left(\tan\alpha_n / \cos\beta_m\right), \tag{5}$$

in which α_n is the normal pressure angle.
In order to determine the diameters $d_{\upsilon a1,2}$ and $d_{\upsilon b1,2}$ the following relations are used:

$$d_{\upsilon a1,2} = d_{\upsilon 1,2} + h_{am1,2}\,;\; d_{\upsilon b1,2} = d_{\upsilon 1,2}\cos\alpha_{\upsilon t}\,, \tag{6}$$

in which $d_{\upsilon 1,2}$ are the reference diameters of virtual cylindrical gear; $h_{am1,2}$ – the mean addendum of the tooth. These elements are determined with relations:

$$d_{\upsilon 1} = \frac{d_{m1}}{\cos\delta_1} = \frac{m_{mt} z_1}{u}\sqrt{u^2+1}\,;\; d_{\upsilon 2} = \frac{d_{m2}}{\cos\delta_2} = m_{mt} z_2\sqrt{u^2+1}\,; \tag{7}$$

$$h_{am1,2} = m_c\left(h_c^* \pm x_{c1}\right), \tag{8}$$

in which $d_{m1,2}$ are the mean pitch diameters of the bevel gears; $\delta_{1,2}$ – the pitch angles of the bevel gears; h_c^* - the nominal coefficient of the reference addendum. For the eloid spiral bevel gear [10], $h_c^* = 1.0$ and $x_{c2} = -x_{c1}$.
The maximal value of the nominal coefficient of the radial profile shift x_{c1} which ensures the avoidance of interface [10] is determined with relation

$$x_{c1} = h_c^* + c_c^* - \left(\frac{d_{i1} y_1 y_2 y_3}{m_c} + \frac{1}{8}\right), \tag{9}$$

in which $c_c^* = 0.3$ is the nominal coefficient of the reference clearance; d_{i1} – the minim reference diameter of the bevel pinion; $y_{1,2,3}$ – correction coefficients.
The diameter d_{i1} is determined [5] with relation

$$d_{i1} = m_{mt} z_1\left(1 - 0.5\psi_{Rm}\right), \tag{10}$$

in which ψ_{Rm}= 0.285…0.38 is the width-face coefficient of the gear and the correction coefficients y_1 and y_2 are obtained from the diagrams presented in fig. 3, respectively fig. 4; the correction coefficient y_3 = 1.0, for the normal division angle α_n = 17º30' and the value y_3 = 11,06 $\sin^2\alpha_n$, for α_n = 20º [10].
Considering the recommendation $m_{mn} \cong m_c$ [5, 10], relation (9) becomes

$$x_{c1} = 1.0 + 0.3 - \left(\frac{z_1\left(1 - 0.5\psi_{Rm}\right) y_1 y_2 y_3}{\cos\beta_m} + \frac{1}{8}\right). \tag{11}$$

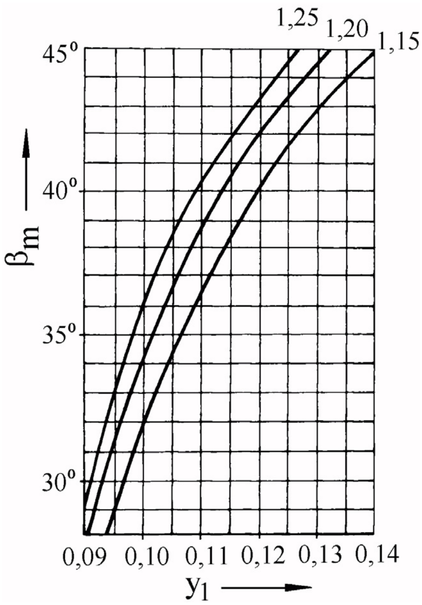

Fig. 3. Variation of y_1

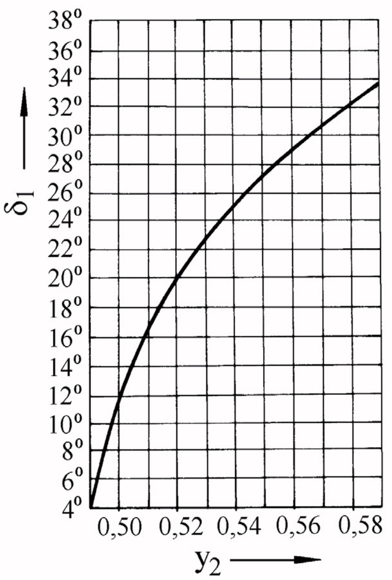

Fig. 4. Variation of y_2

For pinions with a high number of teeth, if the value of x_{c1} is lower than zero, there the profile shift will no longer be applied to ($x_{c1} = x_{c2} = 0$).
Taking into consideration relations (6)…(9), for ratios d_{va1}/d_{vb1} and d_{va2}/d_{vb2} in relation (3), there will be obtained the expressions:

$$\frac{d_{\upsilon a1}}{d_{\upsilon b1}} = \frac{z_1\sqrt{u^2+1} + 2u\left(h_c^* + x_{c1}\right)\cos\beta_m}{z_1\sqrt{u^2+1}\cos\alpha_{\upsilon t}}\,;\;\; \frac{d_{\upsilon a2}}{d_{\upsilon b2}} = \frac{z_1\sqrt{u^2+1} + 2\left(h_c^* - x_{c1}\right)\cos\beta_m}{z_1 u\sqrt{u^2+1}\cos\alpha_{\upsilon t}}\,. \tag{12}$$

In relation (3), the numbers of teeth of the virtual gear wheels are determined with relations:

$$z_{\upsilon 1} = \frac{z_1\sqrt{u^2+1}}{u};\;\; z_{\upsilon 2} = z_1\sqrt{u^2+1} \tag{13}$$

and the transverse contact ratio of virtual cylindrical gear, with relation

$$\varepsilon_{\upsilon\alpha} = \frac{\sqrt{d_{\upsilon a1}^2 - d_{\upsilon b1}^2} + \sqrt{d_{\upsilon a2}^2 - d_{\upsilon b2}^2} - 2a_\upsilon \sin\alpha_{\upsilon t}}{2\pi m_{mn}\cos\alpha_{\upsilon t}}\,, \tag{14}$$

in which

$$a_\upsilon = \frac{d_{\upsilon 1} + d_{\upsilon 2}}{2} = \frac{m_{mt}\sqrt{u^2+1}}{2}\left(\frac{z_1}{u} + z_2\right), \tag{15}$$

is the centre distance of virtual cylindrical gear. Knowing that

$$m_{mt} = \frac{d_{m1}}{z_1} = \frac{2R_m}{z_1\sqrt{u^2+1}} \tag{16}$$

and $m_{mn} = m_{mt} \cos\beta_m$ and $b = \psi_{Rm} R_m$, the calculus relation of the overlap ratio of virtual cylindrical gear becomes

$$\varepsilon_{v\beta} = \frac{\psi_{Rm} z_1 \sqrt{u^2+1}}{2\pi} \tan\beta_m. \tag{17}$$

in which b is the width face.

3. CALCULUS AND CONCLUSIONS

The relations of calculation presented above were implemented in a software program, which allowed obtaining all data necessary for tracing variation diagrams for x_{c1} and the factor Z_{M-B}, according to z_1 and u. There were used the values: z_1=6...16; u=1.5...6; β_m=35º and α_n=20º. These values are characteristic for the eloid spiral bevel gears.

Using relation (13) and the diagrams from Figure 3 and Figure 4, Figure 5 presents the variation of the nominal coefficient of the radial profile shift x_{c1} depending on z_1, for $u=\{1.5; 2; 3; 4; 6\}$ and β_m=35º. From Figure 5, there may be observed that x_{c1} diminishes once with the rise of the number of teeth of the pinion z_1 and with the diminution of the gear ratio u.

Figure 6 presents the variation of the coefficient x_{c1} depending on z_1, for $u=\{2; 5\}$ and limit values of the angle β_m (β_m=30º şi β_m=40º). There may be noted that x_{c1} diminishes once with the rise of the angle β_m and with the reduction of the gearing ratio u.

Figure 7 there presents the variation of the overlap ratio of virtual cylindrical gear $\varepsilon_{\upsilon\beta}$ depending on the number of teeth of the pinion z_1 and on the gear ratio u, for limit values of the median width-face coefficient of the teething ψ_{Rm}. Out of the analysis of the diagram, there may be noted that $\varepsilon_{\upsilon\beta}$ rises once with the augmentation of the number of teeth z_1 and of the gearing ratio u. Likewise, $\varepsilon_{\upsilon\beta}$ rises at high values of the coefficient ψ_{Rm}. For gears with u>4, there may be noted that $\varepsilon_{\upsilon\beta}$ >1, for z_1>6.

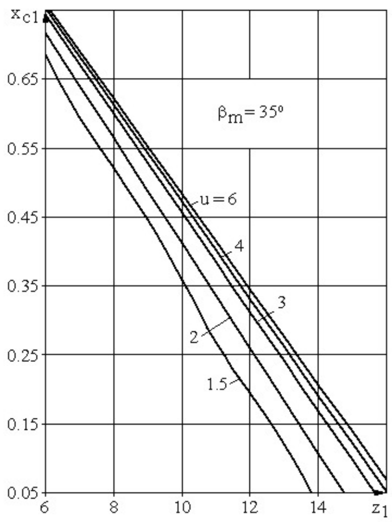

Fig. 5. $x_{c1}=f(u, \beta_m=35º)$

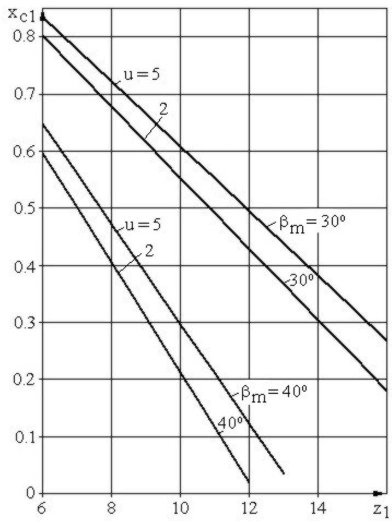

Fig. 6. $x_{c1}=f(\beta_m, u)$

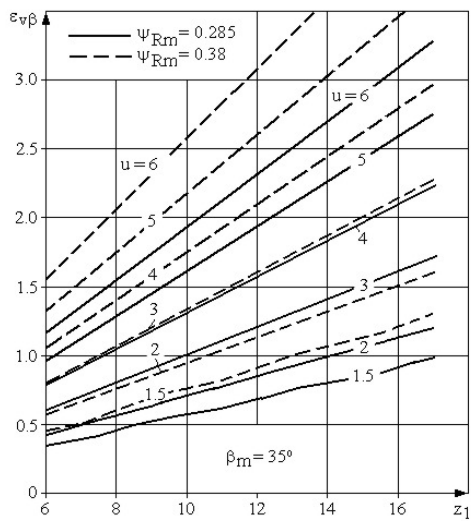

Fig. 7. $\varepsilon_{v\beta}=f(u, \psi_{Rm}, \beta_m=35º)$

The variation of the mid-zone factor Z_{M-B} depending on the gear ratio u and on the number of teeth z_1, for β_m=35º and ψ_{Rm} =0.35 is presented in Figure 8. There may be noted that the factor Z_{M-B} diminishes once with the rise of the value of the gearing ratio u. For u>2.5 and z_1<12, characteristic values for the majority of the eloid spiral bevel gears, there may be noted that Z_{M-B} <1 and it contributes to the reduction of the effective value of the contact stress.

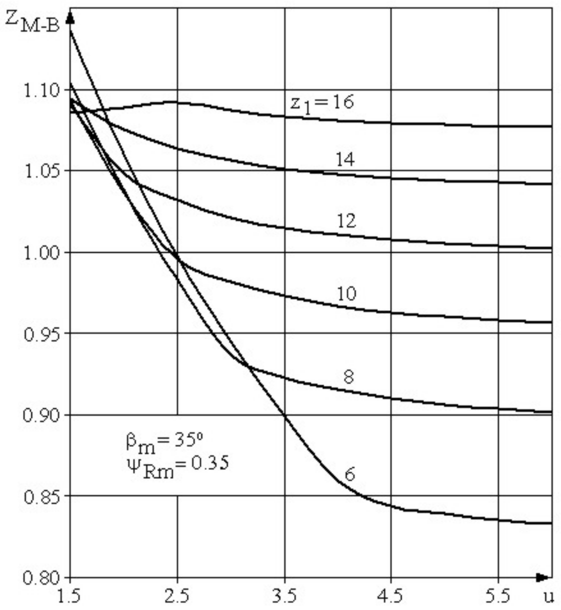

Fig. 8. Z_{M-B}=f(z_1, β_m=35º, ψ_{Rm} =0.35)

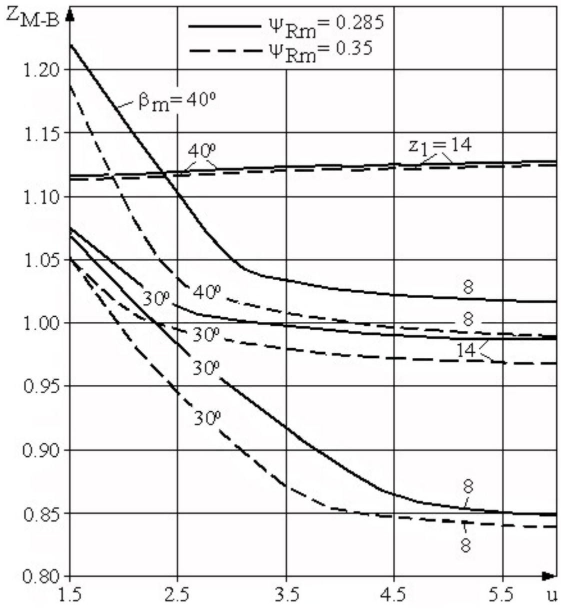

Fig. 9. Z_{M-B}=f(z_1, β_m , ψ_{Rm})

Figure 9, presents the variation of the factor Z_{M-B} depending on the gearing ratio u, for limit values of the division inclination angle of the teething β_m and of the width-face coefficient ψ_{Rm} of the spiral gear.

The results and the conclusions of the paper are useful in the design work, allowing a rapid assessment of the contact stress which may appear in the functioning of the spiral bevel gears.

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