ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume IX (XIX), 2010, NR4

SHAPING ACCURATE GEARING'S CONTROLLED VIBRATIONS

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Keywords: vibrations, gear, damping coefficient, stability conditions
Abstract: Even if the damping coefficient varies from one phase to another contact, but remains constant within these phases, there is only one periodic motion. The stability of this movement is achieved only if conditions (27).

1. CONSTANT DAMPING COEFFICIENT

Gear periodic motion consists of two phases:
- Dual meshing, the term:

$$T_2 = \frac{(\varepsilon - 1)p_b}{r_{b1}p_1} \tag{1}$$

- Engaging single, lengthy:

$$T_1 = \frac{(2 - \varepsilon)p_b}{r_{b1}p_1} \tag{2}$$

In a first approximation be considered a single source of vibration gears, variable stiffness of contact between teeth Gear [2]

a) Double Leverage

Differential equation of motion in this phase is of the form:

$$\ddot{x} + 2\xi\dot{x} + \frac{k_D}{m_{red}}x = F_{(t)} \tag{3}$$

Solution equation (3) is of the form:

$$x = e^{-\xi t}\left(A_1 \cos p_1 t + B_1 \sin p_1 t\right) \tag{4}$$

$$p_1^2 = \frac{k_D}{m_{red}} - \xi^2 \tag{5}$$

If considered initial conditions at t=0, $x=x_0$, $\dot{x}=v_0$, then the constants of integration will be:

$$A_1 = \frac{\xi.x_0 + v_0}{p_1}; \qquad B_1 = x_0 \tag{6}$$

In this case, the solution equation (4) becomes:

$$x = e^{-\xi t}\left(x_0 \cos p_1 t + \frac{v_0 + \xi x_0}{p_1}\right)\sin p_1 t \qquad (7)$$

b) Engaging single

At time t=T_2, the previous pair of teeth out of gear and start engaging single. In this case the differential equation of motion is:

$$\ddot{x} + 2\xi\dot{x} + \frac{k_s}{m_{red}} = \left(1 - \frac{k_S}{k_D}\right)\frac{F_n}{m_{red}} \qquad (8)$$

Having the solution:

$$x = e^{-\xi t}\left(B_2 \cos p_2 t + A_2 \sin p_2 t\right) + F_n\left(\frac{1}{k_s} - \frac{1}{k_D}\right) \qquad (9)$$

$$p_2^2 = \frac{k_s}{m_{red}} - \xi^2 \qquad (10)$$

Integration constants A_2 and B_2 are determined on condition of continuity of motion and speed when moving from double to the single contact. If the equation (9), consider the origin of time at the beginning of contact singular values of x and the relations (7) at t=T_2, are equal to the values of x and $\dot{x}$ the relation (9), at t=0 [2].

$$\begin{cases} A_2 = e^{-\xi T_2}\left(A_1 \cos p_1 T_2 - B_1 \sin p_1 T_2\right) - \dfrac{\xi F_n}{p_2}\left(\dfrac{1}{k_S} - \dfrac{1}{k_D}\right) \\ B_2 = e^{-\xi T_2}\left(A_1 \sin p_1 T_2 + B_1 \cos p_1 T_2\right) - F_n\left(\dfrac{1}{k_S} - \dfrac{1}{k_D}\right) \end{cases} \qquad (11)$$

These relationships, by replacement of A_1 and B_1 in (6) become:

$$\begin{aligned} A_2 &= x_0 a_1 + v_0 a_2 - a_3 \\ B_2 &= x_0 a_4 + v_0 a_5 - a_6 \end{aligned} \qquad (12)$$

$$\begin{cases} a_1 = \dfrac{1}{p_2} e^{\ \xi T_2}\left(\xi \cos p_2 T_2 \quad p_1 \sin p_1 T_2\right); \\ a_2 = \dfrac{1}{p_2} e^{\ \xi T_2} \cos p_1 T_2; \quad a_3 = \dfrac{1}{p_2} e^{\ \xi T_2}\left(\dfrac{1}{k_S} \quad \dfrac{1}{k_D}\right); \\ a_4 = \dfrac{1}{p_1} e^{\ \xi T_2}\left(\xi \sin p_1 T_2 + p_1 \cos p_1 T_2\right); \\ a_5 = \dfrac{1}{p_1} e^{\ \xi T_2} \sin p_1 T_2; \quad a_6 = F_n\left(\dfrac{1}{k_S} \quad \dfrac{1}{k_D}\right) \end{cases} \qquad (13)$$

At the end of single contact, i.e. after the T_Z, where $t=T_1$, $x=x_1$, $\dot{x}=v_1$, we have:

$$x_1 = e^{-\xi T1}(A_2 \sin p_2 T_1 + B_2 \cos p_2 T_1) + F_n(1/K_S - 1/K_D) \quad (14)$$

$$\dot{x}_1 = e^{-\xi T1}[(A_2 p_2 - B_2 \xi) \cos p_2 T_1 - (A_2 \xi + B_2 p_2) \sin p_2 T_1] \quad (15)$$

Substituting the values of A_2 and B_2 in (12) is obtained:

$$x_1 = x_0 b_1 + v_0 b_2 + b_3 = f_1(x_0, v_0) \quad (16)$$

$$\dot{x}_1 = x_0 b_4 + v_0 b_5 + b_6 = f_2(x_0, v_0) \quad (17)$$

$$\begin{aligned} b_1 &= e^{-\xi T1}(a_4 \cos p_2 T_1 + a_1 \sin p_2 T_1) \\ b_2 &= e^{-\xi T1}(a_5 \cos p_2 T_1 + a_2 \sin p_2 T_1); \\ b_3 &= a_6 - e^{-\xi T1}(a_6 \cos p_2 T_1 + a_3 \sin p_2 T_1) \end{aligned} \quad (18)$$

$$\begin{aligned} b_4 &= e^{-\xi T1}[(a_1 p_2 - a_4 \xi) \cos p_2 T_1 - (a_1 \xi + a_4 \omega^*_d) \sin p_2 T_1] \\ b_5 &= e^{-\xi T1}[(a_2 p_2 - a_5 \xi) \cos p_2 T_1 - (a_2 \xi + a_5 p_2) \sin p_2 T_1] \\ b_6 &= e^{-\xi T1}[(a_6 p_2 + a_3 \xi) \cos \omega^*_d T_1 + (a_6 \xi - a_3 p_2) \cos p_2 T_1] \end{aligned} \quad (19)$$

Motion will be periodic, with period T_z, only if the initial conditions satisfy the equality:

$$x_0 = x_1 \text{ and } \dot{x}_0 = \dot{x}_1 \quad (20)$$

Substituting the values of x_1 and $\dot{x}_1$ (16) and (17) in (20) is obtained:

$$x_0 = (b_3 - b_3 b_5 + b_2 b_6)/(1 + b_1 b_5 - b_1 - b_5 - b_2 b_4) \quad (21)$$

$$\dot{x}_0 = (b_6 - b_1 b_6 + b_3 b_4)/(1 + b_1 b_5 - b_1 - b_5 - b_2 b_4) \quad (22)$$

Therefore in this case there is only one periodic motion. For stability this movement is necessary that the matrix [1]:

$$\Delta = \begin{vmatrix} b_{11} - s & b_{12} \\ b_{21} & b_{22} - s \end{vmatrix} \quad (23)$$

$$\begin{cases} b_{11} = \dfrac{\partial f_1}{\partial x_0} = b_1; \quad b_{12} = \dfrac{\partial f_2}{x_0} = b_4; \\ b_{21} = \dfrac{\partial f_1}{\dot{x}_0} = b_2; \quad b_{22} = \dfrac{\partial f_2}{\dot{x}_0} = b_5 \end{cases} \quad (24)$$

To obtain a second degree equation in s:

$$s^2 - (b_1 + b_5)s + (b_1 b_5 - b_2 b_4) = 0 \quad (25)$$

Periodic movements of the system are stable if all roots of characteristic equation are subunit module:

$$|s_i| < 1 \qquad \text{with } i=1,2 \tag{26}$$

These conditions are, in a perfect gear and having c = ct.:

$$\begin{cases} |b_1 b_5 - b_2 b_4| . \langle .1.; \\ |b_1 + b_5| . \langle .1 + (b_1 b_5 - b_2 b_4) \end{cases} \tag{27}$$

Replacing the values of b_1, b_2, b_3, b_4 and b_5 of (18) and (19), we obtain finally:

$$\frac{1}{2}\left\| \begin{bmatrix} \left(2 + \frac{p_1}{p_2} + \frac{p_2}{p_1}\right)\cos(\alpha_1 + \alpha_2) + \\ -\left(\frac{p_1}{p_2} + \frac{p_2}{p_1} - 2\right)\cos(\alpha_1 - \alpha_2) \end{bmatrix} \right\| . \langle . e^{\xi T_z} + e^{-\xi T_z} \tag{28}$$

Which are provided stability?

2. DAMPING COEFFICIENT DIFFERENT FOR THE TWO PHASES OF GEARING

Introducing rigidities of the double and single contact [1] differential equation of vibration Gear:

$$\ddot{x} + 2\xi_{(t)}\dot{x} + \frac{k_{z(t)}}{m_{red}} x = F_{(t)} \tag{29}$$

$$\begin{cases} k_z(t) = k_D .; \\ \xi(t) = \xi_D = D_z \sqrt{\frac{k_D}{m_{red}}} \quad \text{when } 0<t<T_2 \\ F(t) = 0 \end{cases} \tag{30}$$

$$\begin{cases} k_z(t) = k_S .; \\ \xi(t) = \xi_S = D_z \sqrt{\frac{k_S}{m_{red}}} \quad \text{when } T_2<t<T_z \\ F(t) = \frac{F_n}{m_{red}}\left(1 - \frac{k_S}{k_D}\right) \end{cases} \tag{31}$$

Equation (29) can be written for double and single event contact:

$$\ddot{x} + 2D_z p_1 \dot{x} + p_1^2 x = 0 \tag{32}$$

$$\ddot{x} + 2D_z p_2 \dot{x} + p_2^2 x = \frac{F_n}{m_{red}}\left(1 - \frac{k_S}{k_D}\right) \tag{33}$$

Solution equation (32) is of the form:

$$x(t) = A_0 x_1(t) + B_0 x_2(t) \tag{34}$$

Where A_0, B_0 are constants of integration, which is determined by initial conditions, and $x_1(t)$, $x_2(t)$ are fundamental solutions, linearly independent. It is recognized that the two solutions $x_1(t)$ and $x_2(t)$ form a normal key, i.e. at t=0.

$$x_1(0) = 1 \quad \dot{x}_1(0) = 0 \tag{35}$$

$$x_2(0) = 0 \quad \dot{x}_2(0) = 1 \tag{36}$$

If noted, and, where constants of integration will be:

$$A_0 = x_0 \; ; \qquad B_0 = \dot{x}_0 ; \tag{37}$$

Fundamental solutions of the form:

$$\begin{cases} x_1(t) = e^{-\xi_D t}\left(A_1 \cos\alpha_1 t + B_1 \sin\alpha_1 t\right); \\ x_2(t) = e^{-\xi_S t}\left(A_2 \cos\alpha_1 t + B_2 \sin\alpha_1 t\right) \end{cases} \tag{38}$$

$$\alpha_1 = \rho_1 D_1 \qquad D_1 = \sqrt{1 - D_z^2} \tag{39}$$

Integration constants $A_{1,2}$, $B_{1,2}$ are determined in the initial conditions:

$$A_1 = 1; \qquad A_2 = 0; \qquad B_1 = \xi_D / \alpha_1; \qquad B_2 = 1/\alpha_1 \tag{40}$$

If double contact solution becomes:

$$x(t) = e^{-\xi_D t}\left(x_0 \cos\alpha_1 t + \frac{x_0 \rho_1 D_z + \dot{x}_0}{\alpha_1} \sin\alpha_1 t\right) \tag{41}$$

The derivation of this expression is obtained:

$$\dot{x}(t) = e^{-\xi_D t}\left(\dot{x}_0 \cos\alpha_1 t - \frac{x_0 \rho_1 \left(D_z^2 + D_1^2\right) + D_z \dot{x}_0}{D_1} \sin\alpha_1 t\right) \tag{42}$$

General solution contact the singular case is of the form:

$$x(t) = e^{-\xi_S t}\left(A_2 \sin\alpha_2 t + B_2 \cos\alpha_2 t\right) + F_n\left(\frac{1}{k_S} - \frac{1}{k_D}\right) \tag{43}$$

Where: $\alpha_2 = p_2 D_1$

Integration constants A_2 and B_2 are determined from the condition of continuity of motion and speed when moving the double contact to contact and they are singular form:

$$\begin{cases} A_2 = x_0 a_1 + \dot{x}_0 a_2 - a_3.; \\ B_2 = \dot{x}_0 a_4 + \dot{x}_0 a_5 - a_6 \end{cases} \tag{44}$$

$$\begin{cases} a_1 = \dfrac{1}{D_1} e^{\ \xi_D T_2} \left\{ D_z \cos\alpha_1 T_2 \quad \left[\dfrac{p_1}{p_2}\left(1+\dfrac{D_z^2}{D_1^2}\right) \quad \dfrac{D_z^2}{D_1^2} \right] \sin\alpha_1 T_2 \right\}; \\ a_2 = \dfrac{1}{D_1} e^{\ \xi_D T_2} \left[\dfrac{1}{p_2} \cos\alpha_1 T_2 + \dfrac{D_z}{D_1}\left(1 \quad \dfrac{p_1}{p_2}\right) \sin\alpha_1 T_2 \right]; \\ a_3 = \dfrac{D_z}{D_1} F_n \left(\dfrac{1}{k_s} \quad \dfrac{1}{k_D} \right); \\ a_4 = \dfrac{1}{D_1} e^{\ \xi_D T_2} \left(D_1 \cos\alpha_1 T_2 + D_z \sin\alpha_1 T_2 \right); \\ a_5 = \dfrac{1}{D_1} e^{\ \xi_D T_2} \sin\alpha_1 T_2; \quad a_6 = F_n \left(\dfrac{1}{k_S} \quad \dfrac{1}{k_D} \right) \end{cases} \tag{45}$$

If noted, at t = t_z, x(t) =x_1, and $\dot{x}(t)=\dot{x}_1$, will obtain solutions of the form (16) and (17) the coefficients b_j are:

$$\begin{cases} b_1 = e^{-\xi_S T_1}\left(a_1 \sin\alpha_2 T_1 + a_4 \cos\alpha_2 T_1\right).; \\ b_2 = e^{-\xi_S T_1}\left(a_2 \sin\alpha_2 T_1 + a_5 \cos\alpha_2 T_1\right).; \\ b_3 = a_6 - e^{-\xi_S T_1}\left(a_3 \sin\alpha_2 T_1 + a_6 \cos\alpha_2 T_1\right).; \\ b_4 = e^{-\xi_S T_1}\left[\left(a_1\alpha_2 - a_4\xi_S\right)\cos\alpha_2 T_1 - \left(a_1\xi_S + a_4\alpha_2\right)\sin\alpha_2 T_1\right]; \\ b_5 = e^{-\xi_S T_1}\left[\left(a_2\alpha_2 - a_5\xi_S\right)\cos\alpha_2 T_1 - \left(a_2\xi_S + a_5\alpha_2\right)\sin\alpha_2 T_1\right]; \\ b_6 = e^{-\xi_S T_1}\left[\left(a_3\xi_S + a_6\alpha_2\right)\sin\alpha_2 T_1 - \left(a_3\alpha_2 + a_6\xi_S\right)\cos\alpha_2 T_1\right] \end{cases} \tag{46}$$

ξ, damping coefficient, vary as the stiffness k (z), as Figure 1.

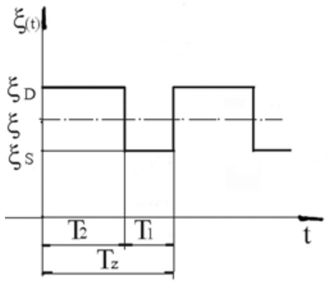

Fig. 1 Variation of damping coefficient [1]

So, even if the damping coefficient varies from one phase to another contact, but remains constant within these phases, there is only one periodic motion. The stability of this movement is achieved only if conditions (27). In this case the stability conditions will be:

$$/b_1 b_5 - b_2 b_4 / = e^{-2\left(\xi_D T_2 + \xi_S T_1\right)} < 1. \tag{47}$$

Since $\xi T_z = \xi_D T_2 + \xi_S T_1$, finally provided stability is:

$$\left\{\begin{array}{l}\left[\frac{\alpha+1}{2\sqrt{\alpha}}\left(1+\frac{D_z^2}{D_i^2}\right)+1\right]\cos\beta_1 - \\ \left[\frac{\alpha+1}{2\sqrt{\alpha}}\left(1+\frac{D_z^2}{D_i^2}-1\right)\cos\beta_2\right]\end{array}\right\}\langle e^{\frac{D_z}{D_i}\beta_1} + e^{-\frac{D_z}{D_i}\beta_2} \tag{48}$$

Where:

$$\begin{cases}\alpha = \dfrac{k_D}{k_S}.; \\ \beta_1 = p_2 D_1 T_z\left[(\varepsilon-1)\sqrt{\alpha}+(2-\varepsilon)\right];. \\ \beta_2 = p_2 D_1 T_z\left[(\varepsilon-1)\sqrt{\alpha}-(2-\varepsilon)\right]\end{cases} \tag{49}$$

If damping is neglected, then D_z=0 and D_I=1.

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