ECONOMETRIC ANALYSIS OF RON/EUR EXCHANGE RATE EVOLUTION IN THE PERIOD 2005-2011

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Abstract: The paper analyzes the evolution of RON/EUR exchange rate from March 2005 to February 2011, a period marked by increases and decreases of the Romanian economy and European economies alike. In the first part of the paper are summarized the main features of the series of linear discrete time of random processes, characteristics taken into consideration throughout the paper. Also, are presented the models of the linear stationary random processes (AR, MA, ARMA) and linear no stationary (ARIMA).

In the second part of the paper is analyzed the evolution of RON/EUR exchange rate between March 2005 and February 2011. Seasonality and trend of the series are tested. Because the data series analyzed is non-stationary, is applied time finite difference operator \(\Delta^{m+1}\), resulting a stationary process. Series of stationary random process is analyzed as mixed linear stationary ARMA(p,q).

Finally is determined a model, that approximates the dynamics of RON/EUR exchange rate during the analyzed period.

1. BRIEF METHODOLOGICAL CONSIDERATIONS

Characterization of the time evolution of business processes can be done by developing dynamic models. Of these many types, for the analysis presented in this paper, we stopped, at first, on autoregressive models with the general form:

\[ y = f(x_t, y_{t-k}) + \varepsilon \]

(1)

Autoregressive models are models that allow description of the evolution of a stationary random process function according to its previous values.

A stationary series \(\{y_t\}_{t \in Z}\) follows a \(\mathbb{A}\mathbb{R}(p)\) process where is met the condition:

\[ y_t - \sum_{k=1}^{p} \phi_k y_{t-k} = \varepsilon_t, \quad \forall t \in Z \]

(2)

where \(\varepsilon_t \sim N(0, \sigma^2)\) is a stationary series, with:

\[ M(\varepsilon_t) = 0, \quad M(\varepsilon_t^2) = \sigma^2, \quad \text{si} \quad \text{cov}(\varepsilon_t, \varepsilon_i) = 0 \quad \forall t \neq i. \]

(3)

In (2), \(p\) is the number of previous values of \(y\) which are taken into account to predict its current value, and the term \(\varepsilon_t\) represents that part of the variable \(y_t\) which is not predictable.

Using the delay operator \(L\) and noting with

\[ \Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p \]

(3)

relationship (2) may be made in the form:

\[ \Phi(L)y_t = \varepsilon_t \]

(4)
Characteristic polynomial attached AR(p) process is:

\[ P(\lambda) = \lambda^p - \phi_1 \lambda^{p-1} - \phi_2 \lambda^{p-2} - \ldots - \phi_p \]  

(5)

The process is stationary if the absolute values of its characteristic polynomial roots are strictly less than 1.

In addition to the series that can be modeled as autoregressive processes, there are time series following other types of processes. Some of these are processes of moving averages (MA). A set of data \( \{y_t\} \) follows a moving average process of order q, if defined by the equality:

\[ y_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q} \]  

(6)

where \( \varepsilon_t \sim N(0, \sigma^2) \) is a stationary series.

A significant improvement of methods of analysis can be obtained by combining AR and MA processes.

Thus, we obtain a generalized model, the autoregressive moving average ARMA. The equation of such model with p autoregressive terms and q moving average terms, denoted ARMA (p, q) is:

\[ y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \]  

(7)

If a random process \( \{x_t\}_{t \in \mathbb{Z}} \) has a uniform comportment highlighting, for example, a polynomial trend, we are dealing with a stochastic process whose average depends on the time. In these circumstances, removing trend and process stationary may be obtained, for example, through differentiation.

No stationary process \( \{x_t\}_{t \in \mathbb{Z}} \) for which \( d \in \mathbb{N} \) exist, so that the transformed process \( y_t = \Delta^d x_t \) is a stationary ARMA (p, q), process is called ARIMA(p, d, q).

Reconstitution of no stationary process \( x_t \) is obtained from the corresponding \( y_t \) stationary, by operation of integration:

\[ x_t = S^d y_d \]  

(8)

where \( S = \Delta^{-1} \) is the operator of integration.

This approach is particularly useful because it allows the study of a no stationary process \( x_t \) using models ARMA(p, q) applied to the stationary transform \( y_t \), which by integrating (8) is \( x_t \).


Analysis of exchange rate RON/EUR was performed on a series containing 1586 records. Graphic exchange rate is shown in Figure 1. As can be seen, in the graphic representation of the series, a significant change in the exchange rate, in terms of its increasing, was in mid-2007. After a slight recovery, recorded in autumn 2008, the exchange rate continued to increase, exceeding in 2009, and threshold of 4 RON.
Series has a polynomial trend which leads to the conclusion that this is a stochastic process whose average depends on the time.

Given the value of the parameter $R^2 = 0.8856$, the polynomial function:

$$EUR = 4 \cdot 10^{-15} \cdot t^5 - 2 \cdot 10^{-11} \cdot t^4 + 3 \cdot 10^{-8} \cdot t^3 - 2 \cdot 10^{-5} \cdot t^2 + 0.0026 \cdot t + 3.5573$$

is a good approximation of the trend series.
3. THE ANALYZING OF RON/EUR RATE AS ARMA STATIONARY PROCESS

Before attempting to identify an as best ARMA model, we checked whether the series is stationary applying Dickey-Fuller Unit Root Test (Figure 3).

![Figure 3 Dickey-Fuller test applied to the EUR series](image)

It is noted that the calculated value of $t_{-Stat} = -0.544682$ test in relation to any relevant critical values of significance thresholds of 1%, 5% and 10% (-3.43, -2.86, -2.57), leading to acceptance of the null hypothesis, which means that it is a no stationary process.

![Figure 4 The correlogram of EUR series](image)
The same results analyzing the 0.87 probability value corresponding to $t_{\text{Statistic}} = -0.544682$ and from correlogram (figure 4).

Since the original series (EUR) is not stationary, was generated first-order difference of this series (denoted D_EUR). D_EUR series was subjected to the same checks as the original series. Test result is presented in Figures 5.

Figure 5 The graph of D_EUR series

Figure 6 Dickey-Fuller test for D_EUR
Also, we checked whether the series is stationary applying Dickey-Fuller Unit Root Test (Figure 6). For D_EUR series, the calculated value of $t_{Statistic} = -26.17067$, leading to reject the null hypothesis and accept the hypothesis: D_EUR series is a stationary process.

The same conclusion of no stationary series, results analyzing from correlogram (figure 7). We can used the correlogram of D_EUR series for obtain a initial ARMA model.

### Correlogram of D_EUR

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.163</td>
<td>74.297</td>
<td>0.000</td>
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</tr>
<tr>
<td>2</td>
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<td>0.345</td>
<td>94.902</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
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<td>-0.104</td>
<td>133.07</td>
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<td>134.79</td>
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<tr>
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<td>-0.013</td>
<td>135.21</td>
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<td>135.29</td>
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<td>-0.002</td>
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<td>137.48</td>
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<td>0.004</td>
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<td>0.000</td>
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<tr>
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<td>-0.009</td>
<td>138.55</td>
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<tr>
<td>11</td>
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<tr>
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<tr>
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<td>0.000</td>
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</tr>
<tr>
<td>14</td>
<td>-0.016</td>
<td>0.021</td>
<td>149.96</td>
<td>0.000</td>
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</tr>
<tr>
<td>15</td>
<td>-0.015</td>
<td>-0.038</td>
<td>150.32</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 7 The correlogram of D_EUR series*

Since D_EUR series is stationary, we test various ARMA models to identify an acceptable model to describe exchange rate RON/EUR as a stochastic process. After testing several models, and taking into account the values of Ficher and Student tests, was chosen two models, ARMA(4,5) and AR(3), presented in Table 1.

### Table 1 Comparative presentation of ARMA(4,5) and AR(3) models

<table>
<thead>
<tr>
<th>Dependent Variable D_EUR</th>
<th>ARMA(4,5)</th>
<th>AR(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000325</td>
<td>0.4744</td>
</tr>
<tr>
<td>AR (1)</td>
<td>0.235219</td>
<td>0.4744</td>
</tr>
<tr>
<td>AR (2)</td>
<td>0.739793</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR (3)</td>
<td>-0.317843</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR (4)</td>
<td>0.130924</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA (2)</td>
<td>-0.887529</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA (5)</td>
<td>0.106616</td>
<td>0.0000</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>4167.447</td>
<td>4163.144</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.992300</td>
<td>1.995542</td>
</tr>
<tr>
<td>Schwarz criterion</td>
<td>-5.239300</td>
<td>-5.244515</td>
</tr>
<tr>
<td>F-statistic</td>
<td>26.08294</td>
<td>47.09827</td>
</tr>
<tr>
<td>Prob(F-statistic)</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>
Considering the data presented in Table 1, we can conclude that both models are valid. Thus, the $F_{\text{statistic}} = 26.08294$ for ARMA (4,5) model, and $F_{\text{statistic}} = 47.09827$ for AR (3) as well as $\text{Prob}(F_{\text{statistic}}) = 0.00$, for both models, we reject the null hypothesis ($H_0$) and we accept the alternative hypothesis ($H_1$): both the models are statistically valid.

Also all the parameters of the models are statistically significant because both the values of corresponding probabilities lead to the rejection of the null hypothesis ($H_0$: parameter values are not significantly different from zero) and accepting the alternative hypothesis ($H_1$: parameter values are significantly different from zero, so they are statistically significant). The estimation equation with substituted coefficients for AR(3) is:

$$D_{\text{EUR}} = 0.00023 + \{\text{AR(1)}=0.23498, \text{AR(2)}=-0.13629, \text{AR(3)}=-0.10411\} \quad (10).$$

Among these models we chose, given the simplicity, the AR(3) model (equation 10).

4. FORECASTING RON/EUR RATE FOR THE PERIOD JANUARY 2011 - MAY 2011

To use the model to forecast exchange rate RON/EUR for the coming period is necessary to test the linear regression model assumptions. To test the errors autocorrelation assumption was used Serial Correlation Breusch-Godfrey LM Test. The results of the test are illustrated in Figure 8.

![Figure 8 The Breusch-Godfrey LM Test for AR(3) model of D_EUR series](image)

The statistic labeled "Obs*R-squared" is the LM test statistic for the null hypothesis ($H_o$) of no serial correlation. The value 0.768586 of probability is much higher than 0.05, leading to the conclusion that the null hypothesis is accepted. Errors are not in correlation.

The assumption of normality distribution errors, as shown in the figure 9, is also satisfied. Distribution of errors is relatively symmetric ($Skewness = 0.049688$), with the average (effectively) equal to 0 ($Mean = -0.87 \cdot 10^{-18}$) and $\text{Std.Dev.} = 0.017419$. The value of $Kurtosis = 13.35659$ indicates a leptokurtic distribution pattern.

To test the presence of heteroskedasticity in the residuals was used ARCH Test (Figure 10).
As can be seen, in this case, Obs*R-squared=50.25253 and Probability=0.000000, values strongly indicates the presence of autoregressive conditional heteroskedasticity (ARCH) in the residuals.

To remove heteroskedasticity we reformulate the model by integration. The new estimation equation of model is:

\[ D_{EUR} = -0.09326 - 0.00268 \times (\text{LOG}(D_{EUR}(-1))) - 0.00821 \times (\text{LOG}(D_{EUR}(-2))) - 0.00582 \times (\text{LOG}(D_{EUR}(-3))) + [\text{AR}(1) = 0.65478] \]

In this case, the null hypothesis \((H_0)\): the presence of heteroskedasticity in the residuals is rejected and the alternative hypothesis \((H_1)\) is accepted (Figure 11).
Now, the model can be used to forecast exchange rate RON/EUR for the period January 2011 - May 2011. The results are shown in Figure 12.

According to the model used, in the period March-May 2011, the exchange rate will have a tendency to decrease to the 4.13, 4.14 RON / EUR.

5. CONCLUSIONS

In the period January 2005 - March 2011 exchange rate RON/EURO has fluctuated between a minimum of 3.1112 recorded July 2, 2007 and a maximum of 4.3688 recorded on June 30, 2010. If in the period January 2005 - June 2007 the fluctuations had a general downward trend in the period June 2007 - January 2009 exchange rate had an almost explosive tendency, exceeding the threshold 4 RON/EUR. During the years 2009 and 2010 the exchange rate fluctuated around 4.2 RON/EURO, with a minimum of 4.0296 on January 5, 2009 and a maximum of 4.3688 on June 30, 2010.

After analyzing the data series, we obtained two models, one type ARIMA(4,5) and the second AR(3) type. For the forecast exchange rate RON/EURO in the period January to March 2011 was chosen AR(3) model, which, after the necessary transformations to
meet assumptions of linear regression model is a good approximation of the exchange rate throughout the period analyzed.

With this model, for the period April-May 2011, the exchange rate forecasted will oscillate around 4.14 RON/EUR. Of course, this forecast is valid as long as the foreign exchange market will not be influenced by particular events.

References