ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume X (XX), 2011, NR1

DYNAMIC ASPECTS OF DAMPED ELASTIC IMPACT PART 1. INFLUENCE OF COEFFICINT OF RESTITUTION

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Keywords: impact mechanics, coefficient of restitution, hysteretic loop, analytical and numerical integration

Abstract: The paper presents kinematical and dynamical aspects concerning damped elastic collision between two spheres. The characteristic parameters of system dynamic behaviour are the coefficient of restitution and the initial indentation velocity. The presented results are obtained by integrating a non-linear ordinary differential equation, made in two stages: the first step was an analytical integration followed by a numerical one. The work is parted in two: the first part presents the influence of the coefficient of restitution and the second part is dedicated to the influence of initial indentation velocity upon kinematical and dynamical system parameters.

1. INTRODUCTION

There are two different modalities to approach the study of collision in the case of mechanical systems considered as multibody systems:

a) considering that the phenomenon takes places instantaneously and by maintaining the rigid body assumption, by introducing the restitution coefficient e, as the characteristic impact parameter:
b) considering that the impact phenomenon displays a continuous evolution in time, of finite duration an the bodies are assumed deformable.

The second hypothesis achieved numerous sustainers. The modern study of impact mechanics is based on Goldsmith's monographic work, [1]. The researchers who continued this approach were, at early stages, Dubowsky and Freudenstein [2], and then carrying on by Hunt and Crossley, [3] and more recently, the works of Lankarani and Nikravesh, [4] and Flores et al., [5]. We must also remind the quite recent monograph written by Stronge, [6].

From the beginning it must be outlined that within the frame of continuous impact phenomenon approach, a few distinct directions are perceived:

- considering only the elastic deformation domain for the entire phenomenon;
- considering both the elastic and the plastic deformations;
- considering the dissipated energy by waves.

In order to emphasise the complexity of the phenomenon one can remind that one of the most advanced software for dynamic analysis, namely ADAMS, performs simulations working only in the first of the above hypothesis. The present work aims to outline the influence of coefficient of restitution (Part I) and the impact velocity (Part II) upon different kinematical and dynamical parameters occurring during centric elastic collision of two identical spheres.

2. THEORETICAL CONSIDERATIONS

The simplest elastic impact model is the one that disregard the damping. This model is based on the theory of Hertzian elastic contact, as shown by Johnson, [7]. According to this model, the dependence between normal load and normal approach is given by a relation having the form:

$$F = K\delta^n \tag{1}$$

where for the point contact $n = 3/2$ and K is a constant considering both the elastic characteristics of the bodies materials -namely the Young moduli, $E_{1,2}$ and the Poisson coefficients, $\nu_{1,2}$, and the local geometry from the vicinity of the contact point:

$$K = \frac{4}{3(\eta_1 + \eta_2)}\left[\frac{R_1 R_2}{R_1 + R_2}\right]^{1/2}$$

The stiffness of the spheres is expressed as:

$$\eta_{1,2} = \frac{1 - \nu_{1,2}^2}{E_{1,2}}$$

The relation (1) is a nonlinear equation describing the behaviour of the system obtained from the two colliding spheres.

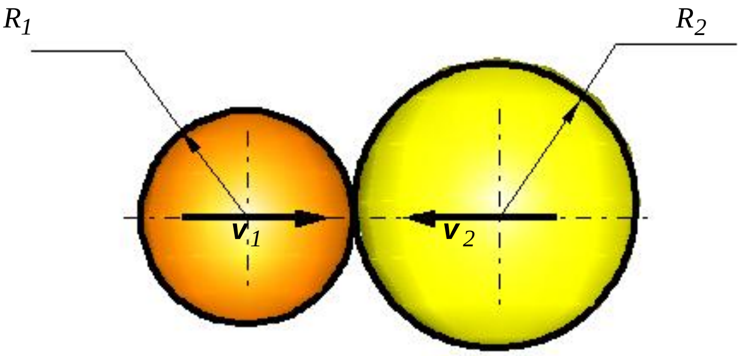

Figure 1. Centric collision of two elastic spheres

The two balls are launched one towards the other one and therefore at the moment of contact the relative velocity is v_0. Two phases can be distinguished: one of approaching and the other one of detaching. The moment separates the two phases is characterised by reaching the minimum of the distance between the centres. The normal force increases and, according to relation (1), attains its maximum when the deformation reaches a maximum too. Based on the non-damping hypothesis, the compression and the relaxation durations, t_c and t_d respectively, should be the same. Following Timoshenko's relations, [8], Goldsmith found the expressions for the above parameters, [1]. Consequently, the maximum normal approach, δ_m , and the contact time, $t = t_c + t_d$, are worked out using the following relations:

$$\delta_m = \left(\frac{5}{4}\frac{v_0^2}{\alpha}\right)^{2/5} = \left(\frac{5}{4}\frac{v_0^2}{K}\frac{m_1 m_2}{m_1 + m_2}\right)^{2/5} \tag{2}$$

$$t = 2\frac{\delta_m}{v_0}\int_0^1 \frac{dx}{\sqrt{1 - x^{5/2}}} = 2\frac{\delta_m}{v_0}\frac{2}{5}\sqrt{\pi}\frac{\Gamma(2/5)}{\Gamma(9/10)} \cong 2.4932\frac{\delta_m}{v_0} \tag{3}$$

where Γ is the Euler's function of second kind.

The maximum impact force is found from relation (1) by replacing the normal approach δ with the maximum approach, δ_m.

The damping phenomenon was first considered in impact studies by Hunt and Crossley, [3]. They considered the system made from the colliding bodies as a Kelvin-Voigt model, consisting of a purely elastic spring having k the elastic constant, connected in parallel with a purely viscous damper, having μ the damping factor. The equation describing the model is a linear second order differential equation:

$$m\ddot{x} + \mu\dot{x} + kx = 0 \qquad (4)$$

where by x was denoted the distance between the centres of the spheres.

In Figure 2 there are represented the elastic force, F_e (blue), the damping force F_a (black), and the collision force, F (red), for two sets of values for the parameters m, c and k.

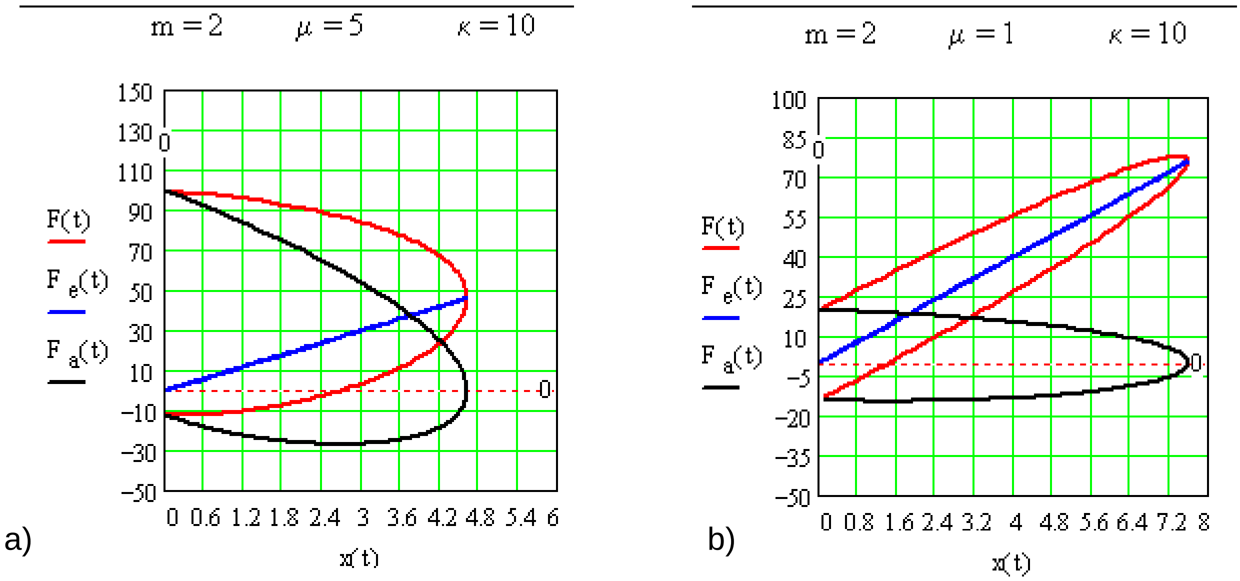

Figure 2. Forces from the Kelvin Voigt model: inertial force, F (red), elastic force F_e (blue), and damping force F_a (black)

The model has the advantage of presenting the hysteresis loop, but the inconvenience consists in the fact that the loop is open in the origin. According to the model, before detachment, the force occurring between the two bodies is an attraction force and not a rejection force. In order to eliminate this difficulty, the two mentioned authors proposed a different form for the model, namely, the coefficients μ and k are not constants but functions depending on the approach x. The behaviour of the proposed model is described by the differential equation having the form:

$$m\ddot{x} + \mu x^n \dot{x} + k x^n = 0 \qquad (5)$$

After some computations, not including details, the final form of the equation is:

$$m\ddot{x}+k\,x^{n}\left[\left(\frac{3}{8}\alpha\sqrt{2\frac{m}{k}(n+1)}\right)\dot{x}^{2}+1\right]=0 \tag{6}$$

where the α parameter, is a factor of proportionality between the hysteresis loop are and the work of damping force.

The Lankarani-Nikravesh model, [4], considers the same two spheres subjected to a centric elastic collision, as represented in Figure 3. At the initial contact moment, the velocities are $V_i^{(-)}$ and $V_j^{(-)}$.

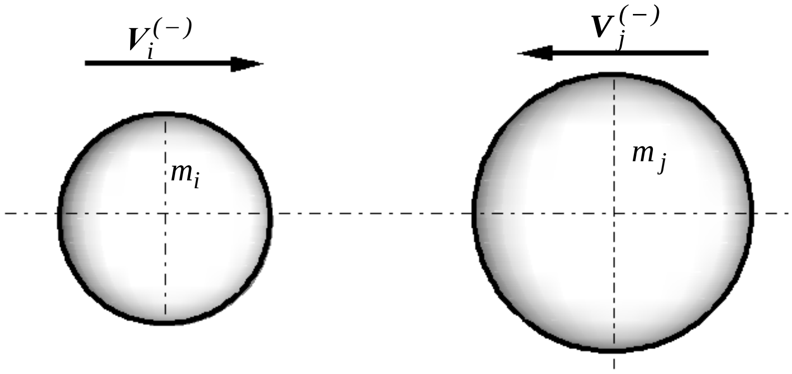

Figure 3. The Lankarani-Nikravesh model of centric collision of two spheres, [4]

At the end of impact, the velocities of the bodies are $V_i^{(+)}$ and $V_j^{(+)}$. The coefficient of restitution, e is defined as:

$$e=-\frac{V_i^{(+)}-V_j^{(+)}}{V_i^{(-)}-V_j^{(-)}} \tag{7}$$

The unknowns of the problem are the velocities after impact, $V_i^{(+)}$ şi $V_j^{(+)}$, and the kinetic energy variation of the system, ΔT. Three equations are required to find these values. The first two equations are given by equation (7) and the impulse-momentum theorem. For the third equation, the authors assume that the energy lost by dissipation is minor compared to the energy accumulated by elastic deformation. Based on this hypothesis, the equation describing the collision process is obtained, namely:

$$F=K\delta^{n}\left[1+\frac{3(1-e^{2})}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{8}$$

where K has the same significance as in equation (1) and $\dot{\delta}^{(-)}$ represents the initial relative velocity.

$$\dot{\delta}^{(-)}=V_i^{(-)}-V_j^{(-)} \tag{9}$$

The equation (8) is a nonlinear second order differential equation and the authors mention that it was numerically integrated, without providing details upon integration method.

3. INFLUENCE OF COEFFICIENT OF RESTITUTION ON IMPACT PARAMETERS

The equation (8) reveals that for a system consisting of two spheres implicated in centric elastic collision, the parameters influencing the process are:

- the coefficient of restitution, (COR), e;
- the initial impact velocity.

So as to study the way these two parameters influence other dynamic or kinematic parameters, the integration of equation (8) is essential. The integration was made in two steps. By writing the equation in a convenient manner, it could be integrated analytically. The following boundary conditions were considered:

$$\begin{aligned} t = 0, \quad \dot{\delta} = \dot{\delta}^{(-)} \\ t = t_m, \quad \dot{\delta} = 0 \end{aligned} \tag{10}$$

The above conditions express the fact that at initial moment the impact velocity is $\dot{\delta}^{(-)}$ and that at the time of maximum normal approach, $t = t_m$, the distance between the bodies reaches a minimum, $\dot{\delta} = 0$.

The rightness of integration of equation (8) is proved by the remark made by Lankarani and Nikravesh, [4], concluding that the equation (8) gives different values for the coefficient of restitution. The mentioned authors present a plot of dependence for the obtained coefficient of restitution e_{out} as a function of the initial coefficient of restitution, e_{in}. In figure 4 there are presented these variations obtained both by the authors of the present paper and in [4].

For the study of the influence of coefficient of restitution, two identical steel balls were considered, having the mass $m_1 = m_2 = 1kg$, the Young modulus $E_1 = E_2 = 2.1 \cdot 10^{11} Pa$ and the Poisson coefficient $\nu_1 = \nu_2 = 0.3$. The initial velocity has the value $v_0 = 5m/s$. For the coefficient of restitution, e, the following values were considered: $\{0.0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 0.95; 0.99; 0.999\}$

The authors considered significant the comparison between the obtained values and the values characteristic to the non-damped contact. A correct integration of equation (8) should give, for the case $e \to 1$, the values obtained for the non-damped contact.

For the case $e = 1$, the collision is perfectly elastic. In this situation, the maximum approach time, in the absence of damping force, is found with relation (3) and denoted by t_H. This parameter is found $t_H = 6.80233 \cdot 10^{-5}\ sec$ and corresponds to the maximum approach, denoted x_H and found to have the value $x_H = 2.31124 \cdot 10^{-4}\ m$.

For these values of coefficient of restitution, there were traced the following:

- the hysteresis curves, Fig. 5;
- the curves of contact forces variation versus time, Fig. 6;
- the curves of normal approach variation in time, Fig. 7;
- the curves of relative velocity variation with respect to time, Fig. 8;
- the curves of acceleration variation. Fig. 9
- the Poincaré's map, Fig. 10.

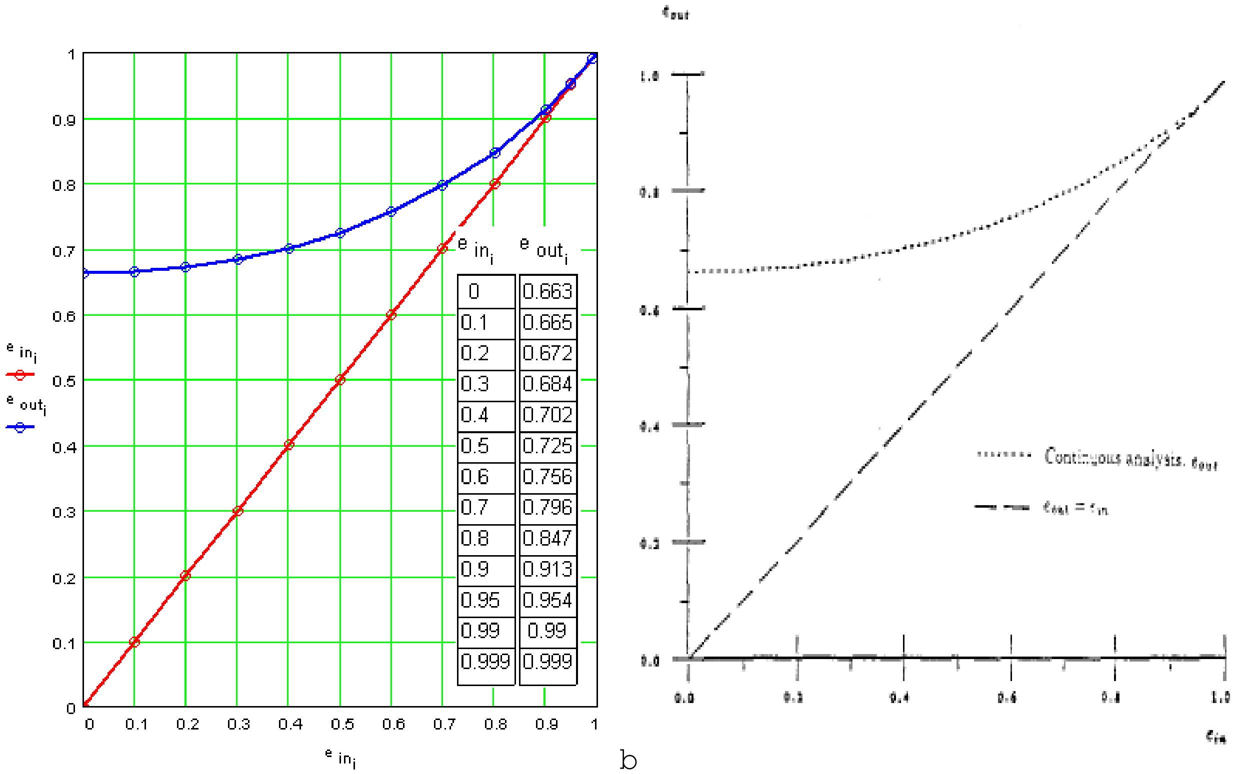

Figure 4. Variations of coefficient of restitution $e_{out} = e_{out}(e_{in})$***, left- present work ; right- [4]***

There were also represented plots for variation of the maximum approach time and detaching time, Fig. 11 and the curve of variation of the ratio between the detaching time and approaching time, Fig. 12. Figure 13 presents the graph of variation, as a function of the coefficient of restitution, of the ratio between the maximum force from the damped model, reached during the compression phase, and the maximum Hertzian force, having the value $F_H = 6.7604 \cdot 10\,N$. Figure 14 presents the plots of the variation of work of dissipative forces, completed during the two phases.

From the above diagrams a general conclusion can be drawn, that is: the *Lankarani-Nikravesh model can be applied for values of the coefficient of restitution ranging in the domain* $0.80 \le e \le 1.00$. Though the constraint appears to be quite strict, there are many situations in engineering applications when the model can be used. For example, the case of collision of bodies made of hardened steel or other metallic bodies colliding at low speeds. The model allows that, given two impacting bodies of known local geometry in the vicinity of contact point, the elastic parameters and the coefficient of restitution, the maximum impact velocity, for a feasible model, can be determined.

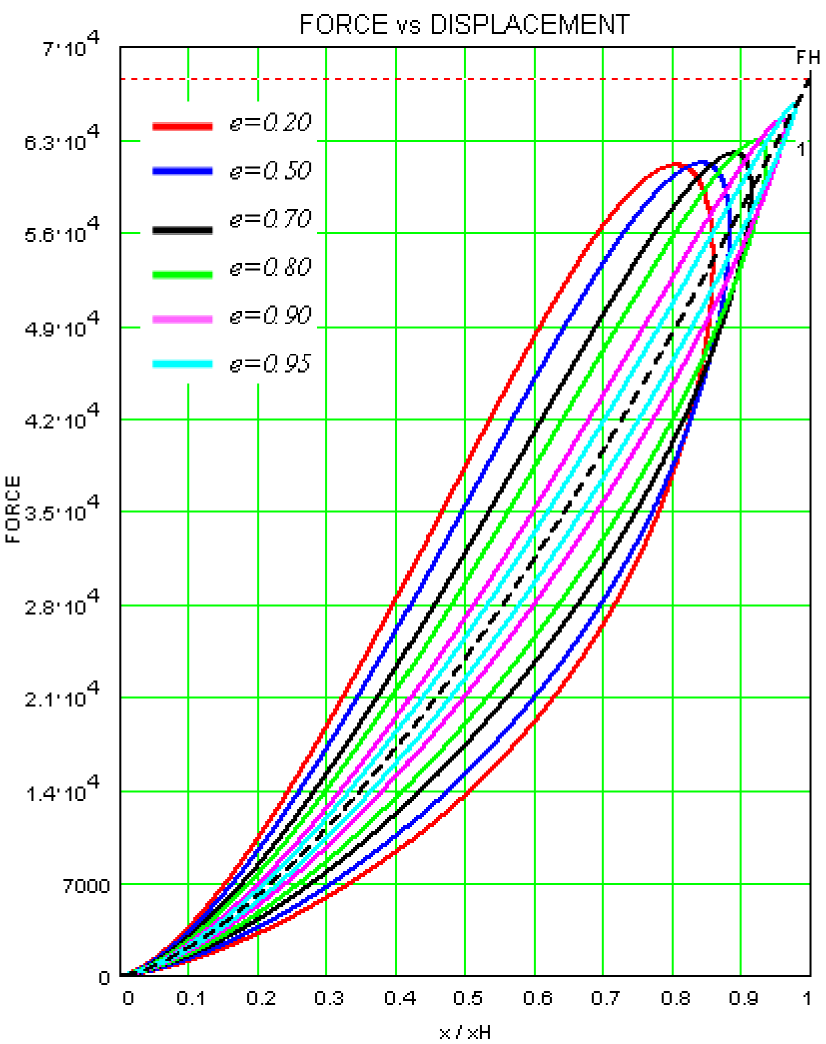

Figure 5. Hysteresis curves

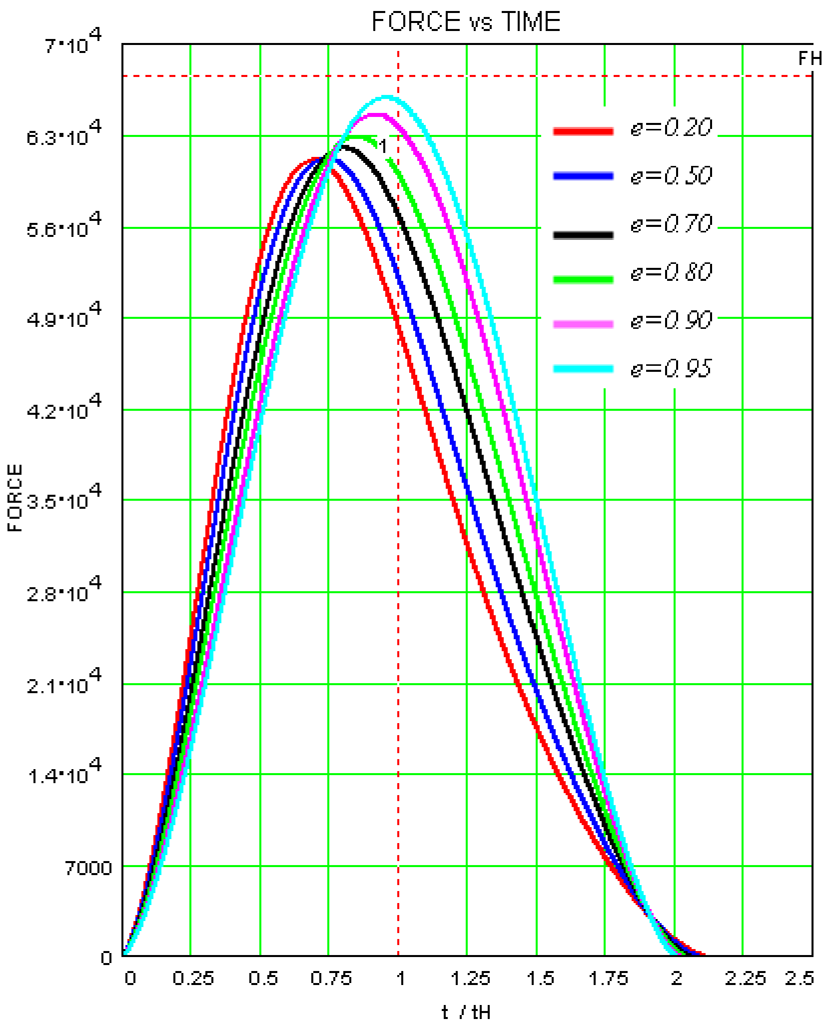

Figure 6. Contact force variation versus time

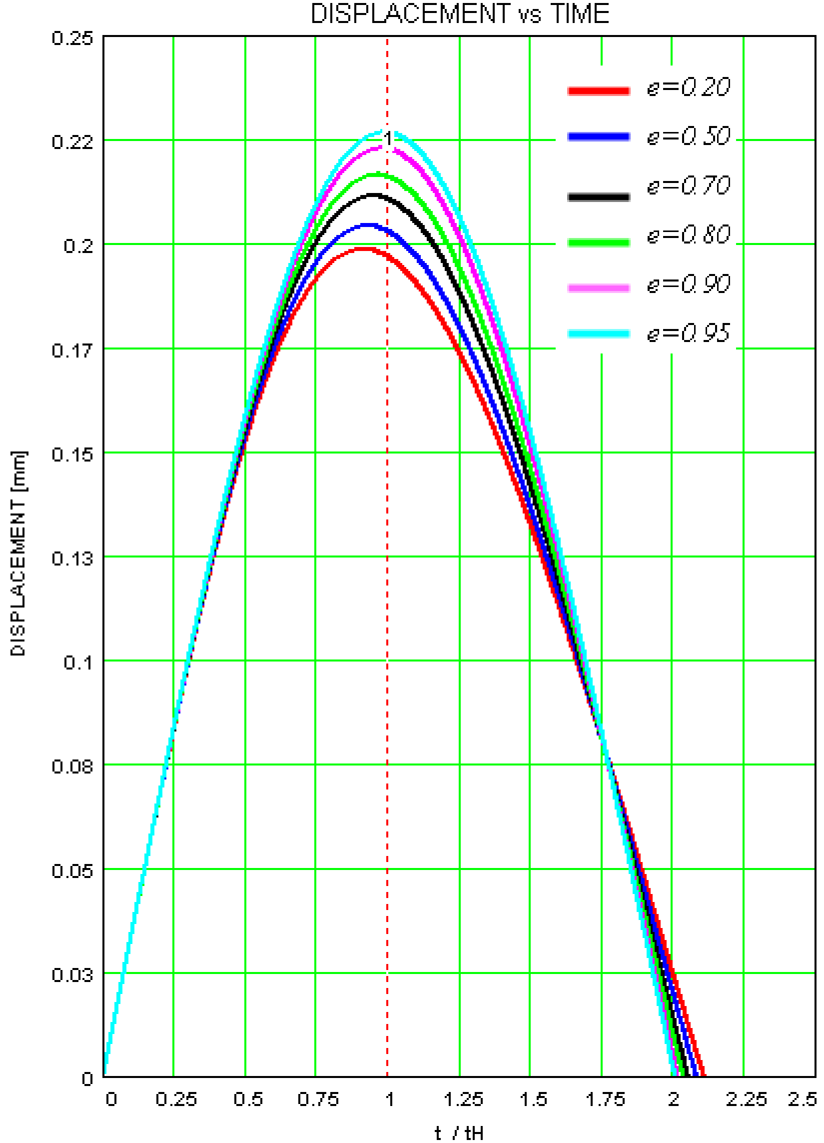

Figure 7. Normal approach variation versus time

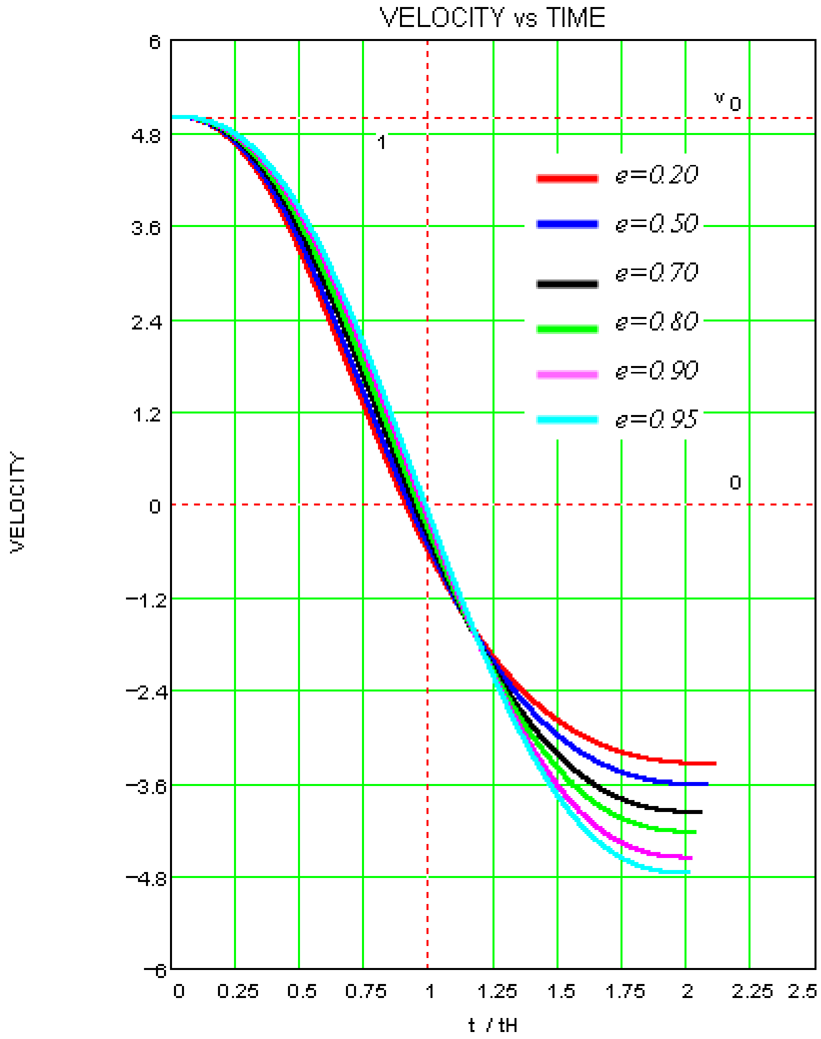

Figure 8. Relative velocity variation versus time

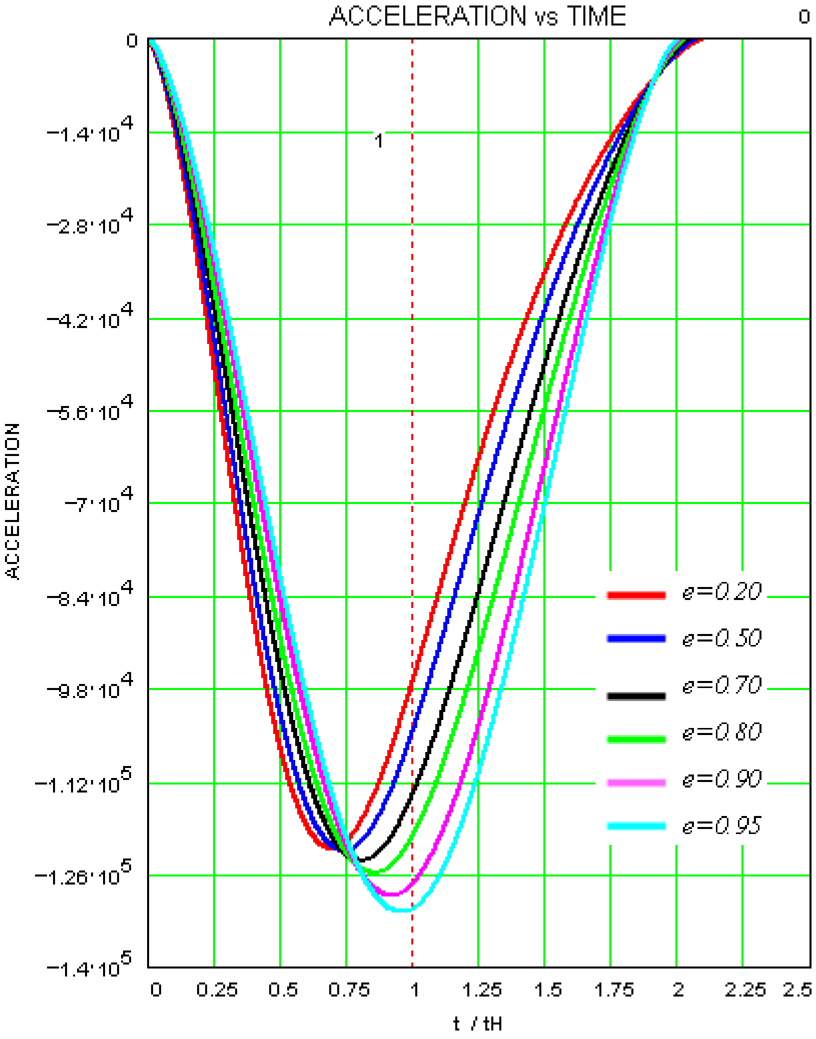

Figure 9. Acceleration variation with time

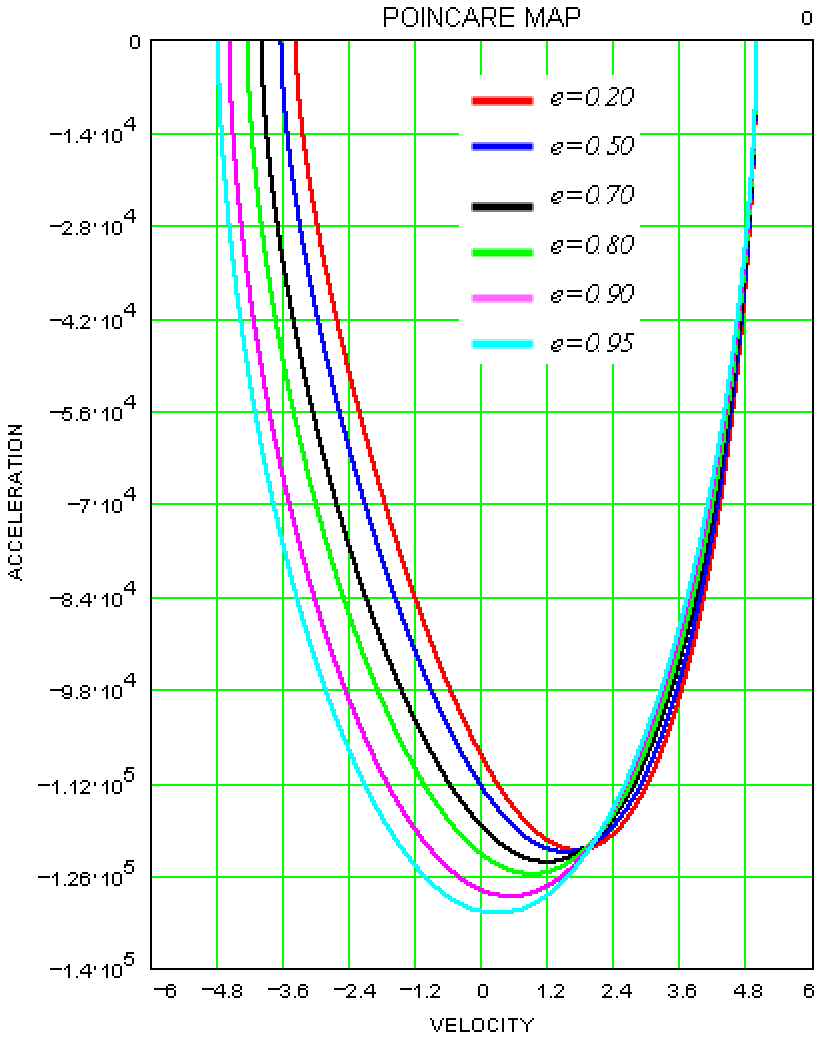

Figure 10. Poincaré's map

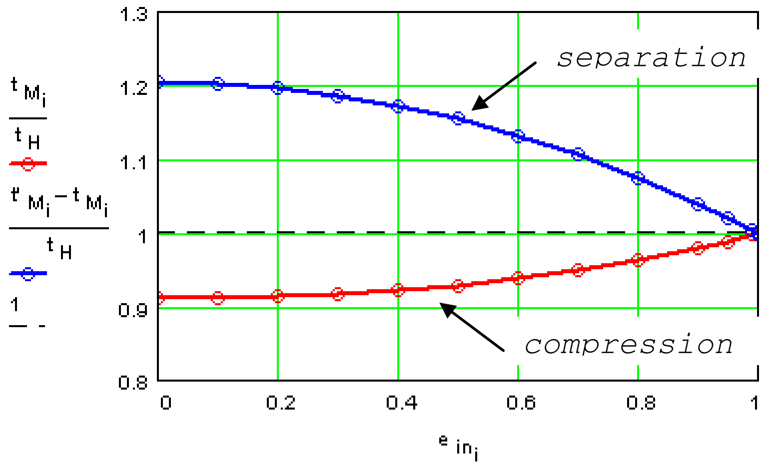

Figure 11. Approaching time and detaching time versus initial coefficient of restitution

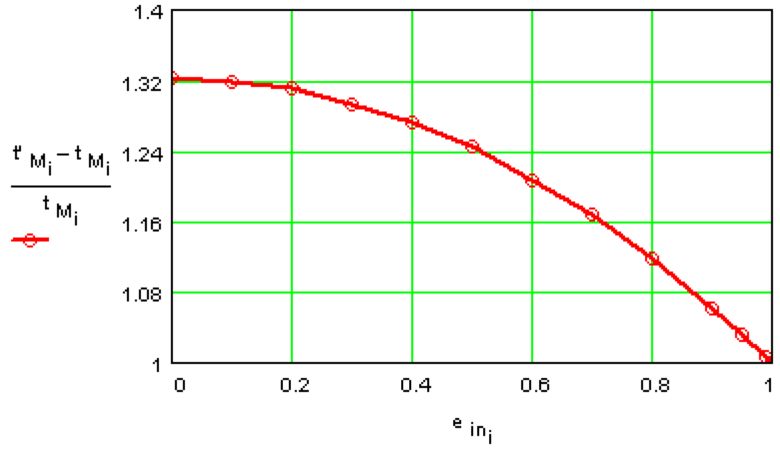

Figure 12. Dependence of the ratio between detaching time and approaching time versus the coefficient of restitution

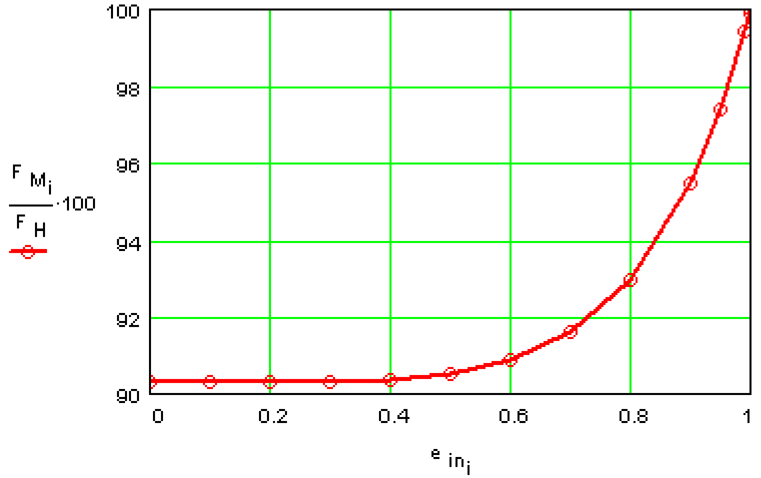

Figure 13. Variation of the ration between maximum force and maximum Hertzian force

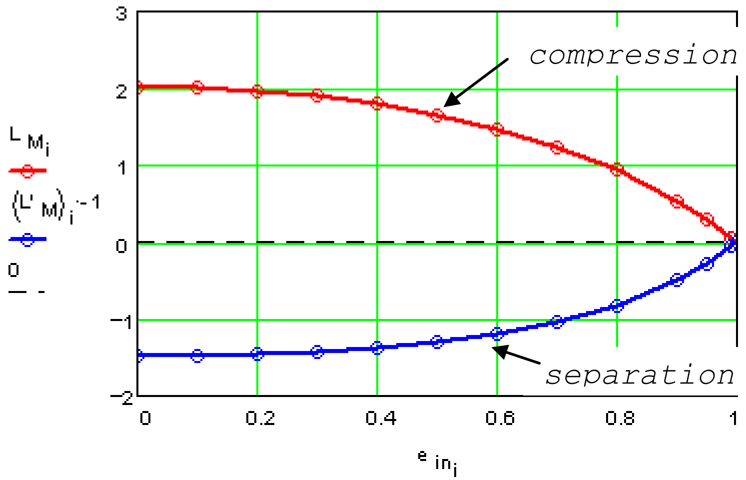

Figure 14. Variation of dissipative forces work for approaching (red) and separation (blue) phases

4. CONCLUSIONS

From the presented results, it can be observed that the area of hysteresis loop increases with increasing coefficient of restitution (COR). At the same time, with increasing COR, the maximum impact force decreases.

For un-damped collision, the approaching time and detaching time are equal, but for damped impact, the augmentation of COR leads to decreasing approach time and increasing separation time. The total impact time increases together to increasing COR.

In addition the raise of COR leads to smaller maximum approach and decreased final separation velocity.

For elastic un-damped collision, $e = 1$, the approaching and separation times have the same value, t_H, and can be obtained using the simplified Hertz collision model. For $e=0$, the compression time is $\cong 92\%$ from t_H while the separation time is with 20% greater than t_H. For the case $e=0$, the ratio between the separation time and the compression time is 1.32.

The increased COR leads to decreasing maximum contact force due to the fact that the initial kinetic energy of the system is not entirely changed into elastic energy. As a result, when COR rises, the ratio between the maximum force from the damped model and the Hertzian force decreases to a value around 0.9. This effect is more pregnant in the vicinity of $e=1$ and is almost constant for $e<0.5$.

With reference to damping lost energy, it increases with decrease of COR and the lost work is greater for approaching period.

The above conclusions should be considered together to the remark that the Lanakarni-Nikravesh model may be applied for values of COR greater than 0.8.

AKNOWLEDGEMENT: This paper was supported by the project " Progress and development through post-doctoral research and innovation in engineering and applied sciences– PRiDE - Contract no. POSDRU/89/1.5/S/57083", project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

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