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BEVEL GEARS GEOMETRY INFLUENCE ON THE LOAD SHARING FACTOR Z_{LS}

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Abstract: The bevel gear contact stress calculus main objective is to avoid the pitting appearance on tooth flank. According to ISO 10300, which refers to bevel gear calculations, the contact strength depends by the tooth geometry, gear manufacturing accuracy, tooth stiffness, deflection of gear-supporting housings, shafts, bearings and load. The load sharing factor Z_{LS} corrects the contact calculus hypothesis which refers to one pair of mesh gear. In this paper the influence of the tooth geometry on this factor value is presented. The conclusions of this analysis are useful for design engineers to obtain reduced overall dimensions bevel gears.

1. INTRODUCTION

Bevel gears with straight or spiral teeth are gearing octoidale as such are not strictly involute flanks tool because they are not flat but curved surfaces. For this reason the bevel gears are manufactured with no profile modification (x_{hm1}=0 and x_{hm2}=0) or displaced zero (x_{hm2}= -x_{hm1}). The octoidale bevel gears are spherical, for which the stress calculation use the Tredgold approximation [6, 7], replacing the bevel gear with cilindrical virtual gear, with spur or helical, in which case the slope of cylindrical teeth is equal to the mean spiral angle β_m for spiral bevel teeth.

2. THEORETICAL CONSIDERATIONS

Calculation of contact stress is based on Hertz's relation applied to virtual gear. Since bevel teeth are crowned manufactured, the contact between the teeth of spiral bevel gears is bounded by an ellipse [2, 4, 5], whose major axis is equal to the tooth width b and the minor axis is equal to the real gearing segment AE=$g_{v\alpha}$=$\pi m_{mt}\varepsilon_{\upsilon\alpha}$. In these hypothesis, for the contact stress calculation, is considered that the load sharing occurs on average length of the contact line l_{bm} (Figure 1 [6]).

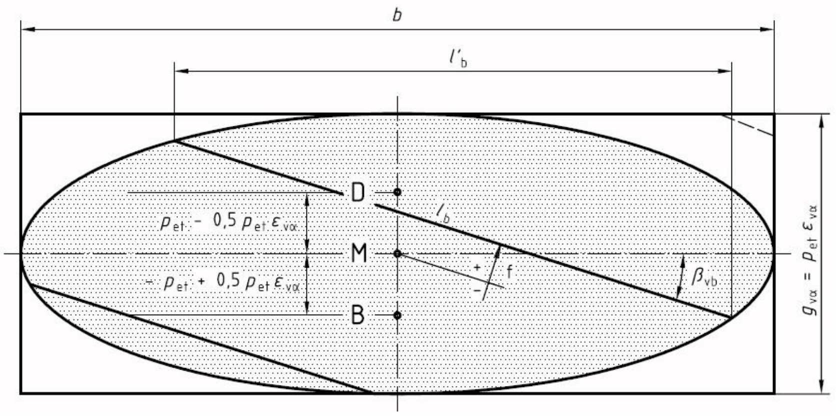

Fig. 1. General definition of lines of contact length [6]

Effective contact stress, depending by the torque on the bevel gear pinion T_1 is determined by the relation [1]

$$\sigma_H = Z_E Z_{LS} Z_K Z_g \sqrt{\frac{T_1 K_A K_\upsilon K_{H\beta} K_{H\alpha}}{2R_m^2}} \le \sigma_{HP}\,, \tag{1}$$

where K_A is the application factor, K_v - the dynamic factor, $K_{H\beta}$ - uneven distribution of the load sharing factor on the contact length between the teeth, $K_{H\alpha}$ – transverse load factor for contact stress; u – gear ratio; Z_E - elasticity factor; Z_{LS} - load sharing factor; Z_K - bevel gear factor (flank); Z_g - global geometric factor for straight bevel gear, for the contact stress calculus, as defined in [1].

The load sharing factor Z_{LS} takes into account the load distribution on two or more pairs of teeth that are simultaneously engaging and it is determined, for bevel gears, depending on the total contact ratio $\varepsilon_{\upsilon\gamma}$ and additional contact ratio $\varepsilon_{v\beta}$. For a straight bevel teeth that $\varepsilon_{\upsilon\gamma}= \varepsilon_{\upsilon\alpha}<2.0$ result $Z_{LS}=1.0$.

For curved toothed bevel gears the load sharing factor is determined by the relations [7]:

$$Z_{LS} = 1.0\,,\ \text{if } \varepsilon_{\upsilon\gamma}\le 2; \tag{2}$$

$$Z_{LS} = 1 \Bigg/ \sqrt{1+2\left[1-\left(\frac{2}{\varepsilon_{\upsilon\gamma}}\right)^{1.5}\right]\sqrt{1-\frac{4}{\varepsilon_{\upsilon\gamma}^2}}}\,,\ \text{for } \varepsilon_{\upsilon\gamma}>2 \text{ and } \varepsilon_{v\beta}>1; \tag{3}$$

For bevel gear with curved teeth, there are cases when total coverage $_{\varepsilon}$ $\varepsilon_{\upsilon\gamma}$ >2.0 and $\varepsilon_{v\beta}<1$. For such bevel gearing, ISO 10300-2 [7] and ANSI/AGMA 2003-B97 [4] propose a new model for calculating the Z_{LS} factor based on the following simplifying assumptions: the distribution of load on the length of the line of contact between teeth is after ellipse, the maximum load on the contact line length distribution, is on the real gear segment length $g_{v\alpha}$, as a parable, with exponent 1.5, these distributions are presented in Fig. 2 [7].

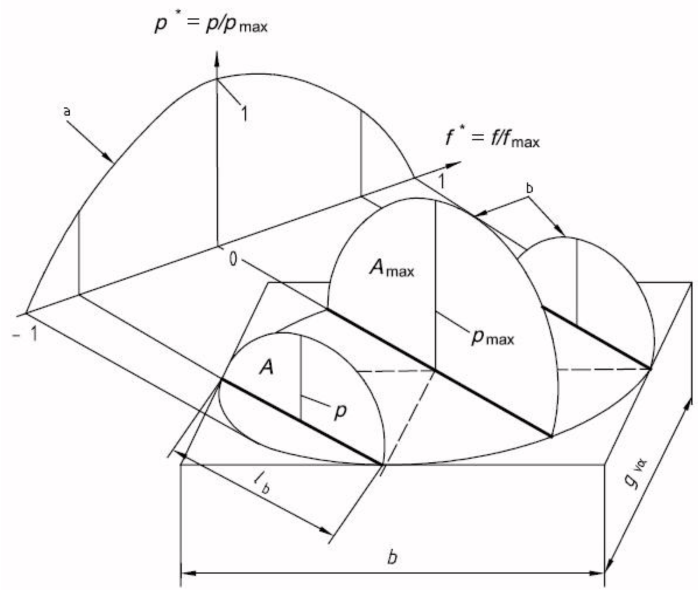

Fig. 2. Load distribution in the contact area [7]

Specific loadf p^* corresponding to a point of contact between the teeth, is determined as the ratio between the load p corresponding to that point and the maximum load p_{max}, resulting

$$p^* = \frac{p}{p_{\max}} = 1 - \left(\frac{|f|}{|f_{\max}|} \right)^{1,5} \geq 0, \tag{4}$$

where f being the distance between the point M (located in the middle of real engagement segment AE) and the contact line between teeth [6].The maximum value of this distance is determined by the relation

$$f_{\max} = \frac{\pi}{2} m_{mt} \varepsilon_{\upsilon\gamma} \cos\alpha_{vt} \cos\beta_{vb}. \tag{5}$$

With these relations the surface area above the contact line can be calculate

$$A^* = \frac{1}{2} \cdot \frac{1}{2} \pi p^* l_b, \tag{6}$$

respectively the load sharing factor on teeth pairs that are simultaneously in engagement, for the situation where $\varepsilon_{\upsilon\gamma}$>2.0,

$$Z_{LS} = \sqrt{\frac{A_m^*}{A_t^* + A_m^* + A_r^*}}, \tag{7}$$

where $A^*_{t,m,r}$ is the area above the tooth contact line to the teeth head (tip t), from the tooth middle (m), respectively, at the tooth foot (r), measured in mm^2 , l_b - length of contact line, determined for tooth head (l_{bt}), the middle tooth (l_{bm}), respectively the tooth foot (l_{br}).

Table 1 presents, for the calculation of the contact bevel gear with curved teeth, relations for determining the parameters f, p *, l_b and A *.

Parameters involved in the paper are determined with the relations below [3, 6].

- Center distance for the virtual gear

$$a_\upsilon = \frac{m_{mt}}{2} \sqrt{u^2 + 1} \left(\frac{z_1}{u} + z_2 \right); \tag{8}$$

- Real gearing segment length

$$g_{\upsilon\alpha} = \frac{1}{2} \left(\sqrt{d_{\upsilon a1}^2 - d_{\upsilon b1}^2} + \sqrt{d_{\upsilon a2}^2 - d_{\upsilon b2}^2} \right) - a_\upsilon \sin\alpha_{\upsilon t}. \tag{9}$$

- Contact ratio in the frontal plane for the virtual gear

$$\varepsilon_{\upsilon\alpha} = \frac{g_{\upsilon\alpha}}{\pi m_{mt} \cos\alpha_{\upsilon t}}, \tag{10}$$

- Pressure angle for the virtual gear

$$\alpha_{\upsilon t} = \arctan \frac{\tan\alpha_n}{\cos\beta_m}; \tag{11}$$

- Supplementar contact ratio for the virtual gear, due to tooth inclination

$$\varepsilon_{\upsilon\beta} = \frac{b \sin\beta_m}{\pi m_{mn}} = \frac{\psi_{Rm} R_m}{\pi m_{mt}} \tan\beta_m = \frac{\psi_{Rm} z_1 \sqrt{u^2 + 1}}{2\pi} \tan\beta_m. \tag{12}$$

- Total gear ratio

$$\varepsilon_{\upsilon\gamma} = \sqrt{\varepsilon_{\upsilon\alpha}^2 + \varepsilon_{\upsilon\beta}^2}. \tag{13}$$

3. CALCULUS AND CONCLUSIONS

Based on the previously calculation relations and those presented in Table 1 has developed a computer program with which were drew diagrams of variation of the factor Z_{LS}. The main parameters taken in the analysis are: number of teeth of pinion z_1, the gearing ratio u, the mean spiral angle of teeth; the radial profile shift coefficient x_{hm1}, with the condition $x_{hm2}=-x_{hm1}$; width coefficient of the wheels, ψ_{Rm}; it was considered that the curved bevel gear teeth pressure angle is $\alpha_n=20^\circ$ and the tooth head height addendum is $h^*_{an}=1$, and the tooth foot height dedendum is $h^*_f=1.25$.

Table 1. The parameters for calculus of Z_{LS} factor [6]

Symbol		$0<\varepsilon_{v\beta}<1$	$\varepsilon_{v\beta}\geq 1$
$f=$	$f_t=$	$\pi m_{mt}\cos\alpha_{vt}\cos\beta_{vb}\left[1-\left(1-0.5\varepsilon_{v\alpha}\right)\left(1-\varepsilon_{v\beta}\right)\right]$	$\pi m_{mt}\cos\alpha_{vt}\cos\beta_{v\beta}$
	$f_m=$	$-\pi m_{mt}\cos\alpha_{vt}\cos\beta_{vb}\left(1-0.5\varepsilon_{v\alpha}\right)\left(1-\varepsilon_{v\beta}\right)$	0
	$f_r=$	$-\pi m_{mt}\cos\alpha_{vt}\cos\beta_{vb}\left[1+\left(1-0.5\varepsilon_{v\alpha}\right)\left(1-\varepsilon_{v\beta}\right)\right]$	$-\pi m_{mt}\cos\alpha_{vt}\cos\beta_{v\beta}$
$p^*=$		$p/p_{\max}=1-\left(\lvert f\rvert/\lvert f_{\max}\rvert\right)^{1,5}$ for	
$p^*_t=p^*_{f=f_t}=$		$1-\left\{2\left[1-\left(1-0.5\varepsilon_{v\alpha}\right)\left(1-\varepsilon_{v\beta}\right)\right]/\varepsilon_{v\gamma}\right\}^{1.5}$	$1-\left(2/\varepsilon_{v\gamma}\right)^{1.5}$
$p^*_m=p^*_{f=f_m}=$		$1-\left[2\left(1-0.5\varepsilon_{v\alpha}\right)\left(1-\varepsilon_{v\beta}\right)/\varepsilon_{v\gamma}\right]^{1.5}$	1
$p^*_r=p^*_{f=f_r}=$		$1-\left\{2\left[1+\left(1-0.5\varepsilon_{v\alpha}\right)\left(1-\varepsilon_{v\beta}\right)\right]/\varepsilon_{v\gamma}\right\}^{1.5}$	$1-\left(2/\varepsilon_{v\gamma}\right)^{1.5}$
$l_b=$	$l_{bt}=$	$b g_{v\alpha}\dfrac{\sqrt{g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}-4f^2_t}}{g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}}$, if $g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}-4f^2_t>0$ 0, if $g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}-4f^2_t\leq 0$	
	$l_{bm}=$	$b g_{v\alpha}\dfrac{\sqrt{g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}-4f^2_m}}{g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}}$, if $g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}-4f^2_m>0$ 0, if $g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}-4f^2_m\leq 0$	
	$l_{br}=$	$b g_{v\alpha}\dfrac{\sqrt{g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}-4f^2_r}}{g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}}$, if $g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}-4f^2_r>0$ 0, if $g^2_{v\alpha}\cos^2\beta_{vb}+b^2\sin^2\beta_{vb}-4f^2_r\leq 0$	
$A^*=$	$A^*_t=$	$\pi p^*_t l_{bt}/4$	
	$A^*_m=$	$\pi p^*_m l_{bm}/4$	
	$A^*_r=$	$\pi p^*_r l_{br}/4$	

The Z_{LS} factor variation depending by the number of teeth of bevel gear pinion z_1 with mean spiral angle of teeth $\beta_m=35^\circ$ is represented in Fig. 3, and depending on the gearing ratio u is represented in Fig. 4. Analysing these diagrams leads to the following conclusions:

- Z_{LS} factor value decreases with increasing number of pinion teeth and increases as the radial profile shift coefficient;

- the values of Z decreases with increasing the gearing ratio and the radial profile shift coefficient. If the gear ratio changes, the values of Z_{LS} factor decreases with increasing the gearing ratio and the radial profile shift coefficient;
- maximum value Z_{LS}=1.0 is obtained at the values of the number of pinion teeth z_1≤8 and gear ratio u≤2, regardless of the radial profile shift coefficient.

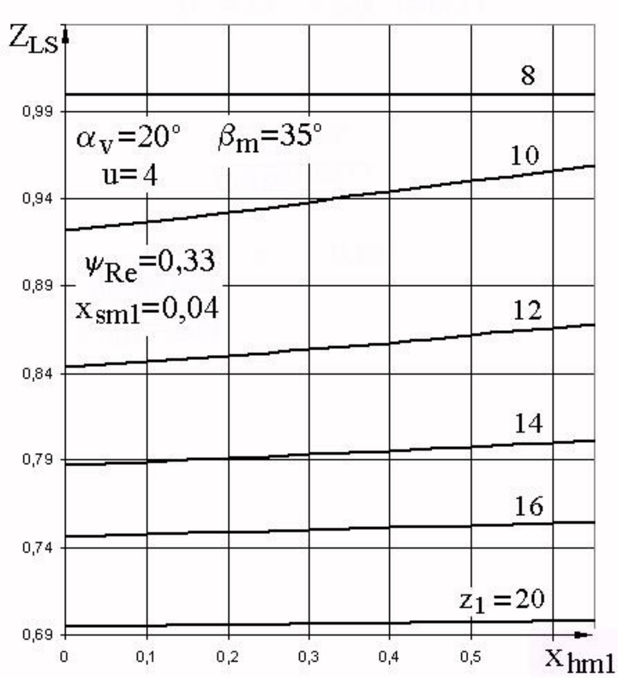

Fig. 3. Z_{LS}=f(x_{hm1}, z_1)

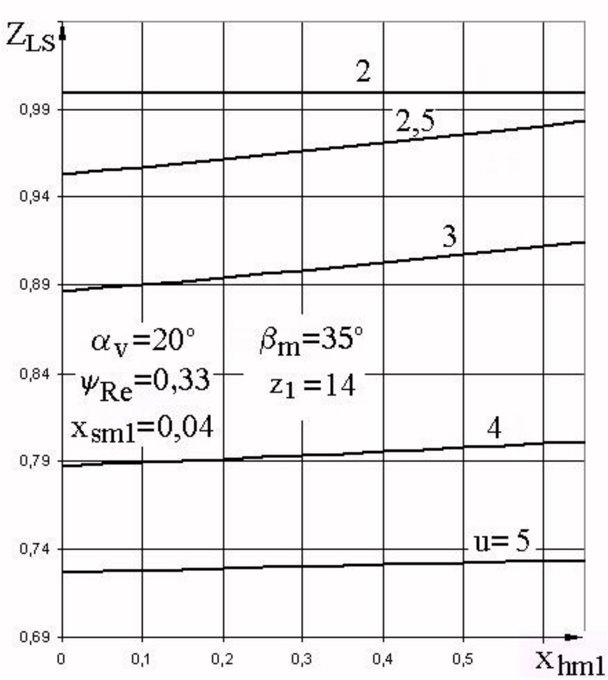

Fig. 4. Z_{LS}=f(x_{hm1}, z_1)

The Z_{LS} factor variation depending by the mean spiral angle of the teeth, is presented in Fig. 5, and according to the tooth width coefficient $\psi_{Rm} = b/R_m$ is presented in Fig. 6. Analysing of these diagrams follows the conclusions presented below.

- Regardless of which parameter varies, the mean spiral angle of the teeth or tooth width ratio, the factor Z_{LS} has a variation similar to that shown in Fig. 3;
- The greater value for factor Z_{LS} occurs where the mean spiral angle of the teeth β_m = 40°, compared to the angle β_m = 30°, respectively, whith the lower value of the tooth width coefficient ψ_{Rm} = 0.28, compared the situation in which ψ_{Rm} = 0.35;
- The maximum value for factor Z_{LS} is obtained at high values of mean spiral angle of the teeth and small values of the tooth width coefficient, regardless of the radial displacement profile coefficient;
- At lower values of the mean spiral angle of the teeth and the higher values of tooth width coefficient, Z_{LS} = 1.0 only at high values of the coefficient of radial displacement profile.

In conclusion, the correct chooosing of geometric parameters of curved toothed bevel gear, for a torque and speed required, results in a lower overall gear.

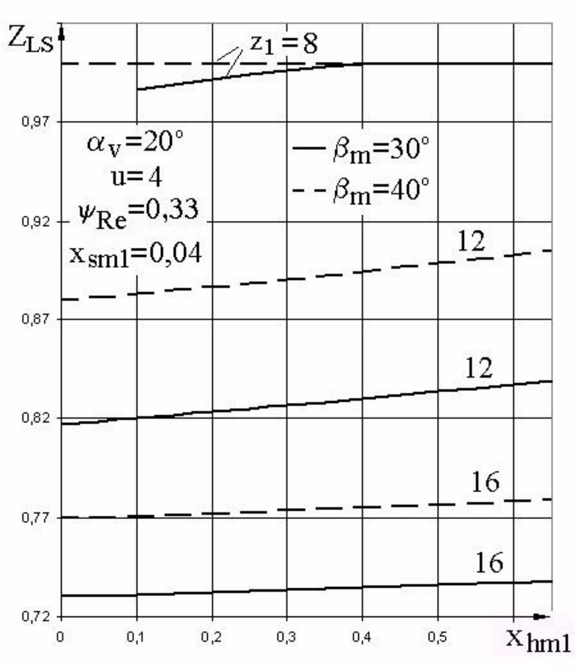

Fig. 5. $Z_{LS}=f(x_{hm1}, z_1, \beta_m)$

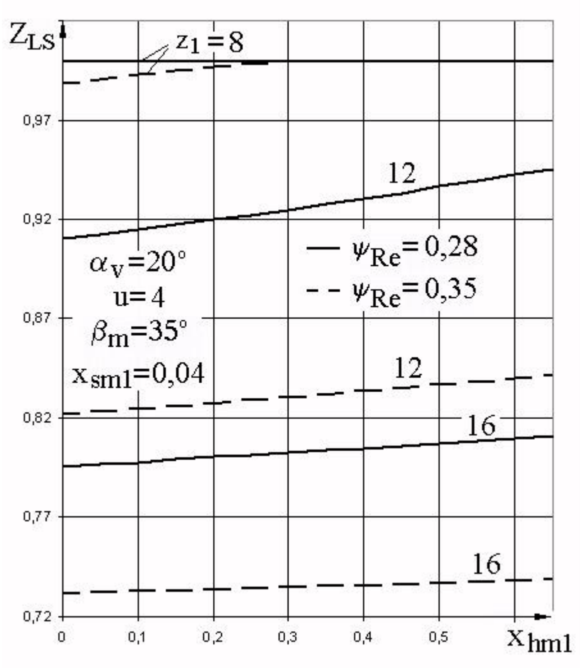

Fig. 6. $Z_{LS}=f(x_{hm1}, z_1, \psi_{Rm})$

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