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CONSIDERATIONS ABOUT THE VIBRATORY BEHAVIOUR OF A LATHE WITH SPEED VARIATOR AND DYNAMIC ABSORBER

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Abstract**.** In this paper it is considered a study of a lathe with speed variator and dynamic absorber. One important criterion of a tool-machine is the dimensional accuracy of the shape and the quality of the processed surface of the lathe work. The dimensional accuracy is determined by the relative displacements, which appear in the cutting space between the cutter and the lathe work. Because the tool-machine is a complex system with a lot of freedom degrees, in its dynamic study, only the vibrations mode which have a significant role in the vibratory process are take into account. In the present paper, a model with seven freedom degrees is used.

1. THE MECHANICAL MODEL

In the figure 1 it is presented the seven freedom degree mechanical model of a lathe with speed variator and dynamic absorber.

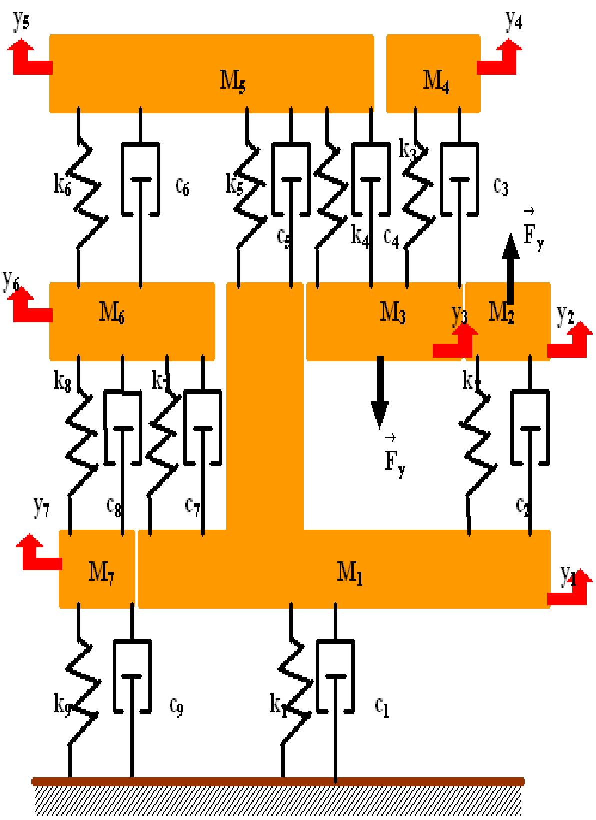

Fig. 1 The mechanical model

These seven degree of freedom have an important influence on the dimensional accuracy and on the wear of the machine members that compose its units.

2. THE MATHEMATICAL MODEL

By using the Lagrange's formalism, the mathematical model is obtained. Because the component elements of the lathe perform oscillating translations on the vertical direction, the kinetic energy of the system is:

$$T = \sum_{j=1}^{7} M_j \vec{v}_{C_j}^2 = \sum_{j=1}^{7} M_j \dot{y}_j^2, \tag{1}$$

where:

- M_1 represents the mass of the frame;
- M_2 represents the mass of the tool holder-slide together the multiple operation tool;
- M_3 represents the mass of the working piece;
- M_4 represents the mass of the dynamic absorber;
- M_5 represents the mass of the main shaft;
- M_6 represents the mass of the speed variator;
- M_7 represents the mass of the driving motor;
- $q_j, j = \overline{1,7}$ represent the Lagrange's coordinates.

The generalized forces are calculated with the relations:

$$Q_j = Q_{j,c} + Q_{j,a} + Q_{j,e}, \quad , j = \overline{1,7} \tag{2}$$

where:

$Q_{j,c}$ are the conservative generalized forces, given by the relations:

$$Q_{j,c} = -\frac{\partial V}{\partial q_j}, \; j = \overline{1,7};$$

$Q_{j,a}$ are the dissipative generalized forces, given by the relations:

$$Q_{j,a} = -\frac{\partial D}{\partial \dot{q}_j}, \; j = \overline{1,7};$$

$Q_{j,e}$ are the generalized forces due to the disturbance during the cutting process, given, in a matric writing, by the relations:

$$\{Q_e\} = \{0 \quad F_y \quad -F_y \quad 0 \quad 0 \quad 0 \quad 0\}^T.$$

In the above relations:

$$V = \frac{1}{2}\Big[K_1 y_1^2 + k_2(y_2 - y_1)^2 + k_3(y_4 - y_3)^2 + k_4(y_5 - y_3)^2 + k_5(y_5 - y_1)^2 + \\ + k_6(y_5 - y_6)^2 + k_7(y_6 - y_1)^2 + k_8(y_6 - y_7)^2 + k_9 y_7^2\Big]$$

$$D = \frac{1}{2}\Big[c_1\dot{y}_1^2 + c_2(\dot{y}_2 - \dot{y}_1)^2 + c_3(\dot{y}_4 - \dot{y}_3)^2 + c_4(\dot{y}_5 - \dot{y}_3)^2 + c_5(\dot{y}_5 - \dot{y}_1)^2 + \\ + c_6(\dot{y}_5 - \dot{y}_6)^2 + c_7(\dot{y}_6 - \dot{y}_1)^2 + c_8(\dot{y}_6 - \dot{y}_7)^2 + c_9 y_7^2\Big]$$

Because, in the analyzed case, the system has total holonomic relations, the Lagrange's equations can be written under the form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j, \quad j = \overline{1,7}. \tag{3}$$

Replacing in the system of equations (3) the relations (1) and (2), it results the mathematical model of the movement, in matric writing, on the following shape:

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = [Q_e], \tag{4}$$

where:

$$[M] = \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_7 \end{bmatrix};$$

$$[K] = \begin{bmatrix} k_1 + k_2 + k_5 + k_7 & -k_2 & 0 & 0 & -k_5 & -k_7 & 0 \\ -k_2 & k_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 + k_4 & -k_3 & -k_4 & 0 & 0 \\ 0 & 0 & -k_3 & k_3 & 0 & 0 & 0 \\ -k_5 & 0 & -k_4 & 0 & k_4 + k_5 + k_6 & -k_6 & 0 \\ -k_7 & 0 & 0 & 0 & -k_6 & k_6 + k_7 + k_8 & -k_8 \\ 0 & 0 & 0 & 0 & 0 & -k_8 & k_8 + k_9 \end{bmatrix};$$

$$[C] = \begin{bmatrix} c_1 + c_2 + c_5 + c_7 & -c_2 & 0 & 0 & -c_5 & -c_7 & 0 \\ -c_2 & c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 + c_4 & -c_3 & -c_4 & 0 & 0 \\ 0 & 0 & -c_3 & c_3 & 0 & 0 & 0 \\ -c_5 & 0 & -c_4 & 0 & c_4 + c_5 + c_6 & -c_6 & 0 \\ -c_7 & 0 & 0 & 0 & -c_6 & c_6 + c_7 + c_8 & -c_8 \\ 0 & 0 & 0 & 0 & 0 & -c_8 & c_8 + c_9 \end{bmatrix};$$

$$\{\ddot{y}\} = \{\ddot{y}_1 \quad \ddot{y}_2 \quad \ddot{y}_3 \quad \ddot{y}_4 \quad \ddot{y}_5 \quad \ddot{y}_6 \quad \ddot{y}_7\}^T;$$

$$\{\dot{y}\} = \{\dot{y}_1 \quad \dot{y}_2 \quad \dot{y}_3 \quad \dot{y}_4 \quad \dot{y}_5 \quad \dot{y}_6 \quad \dot{y}_7\}^T;$$

$$\{y\} = \{y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6 \quad y_7\}^T.$$

It is possible to consider the disturbing force, in the cutting process, having the expression:

$$F_y = F_0 \mu \frac{a}{a_0} \sin(\omega_e t), \tag{5}$$

where:

- F_0 represents the nominal cutting force;
- μ represents the superposition factor between the previous and the present passing of the edged tool;
- $\frac{a}{a_0}$ represents the ratio between the real value and nominal one of the cutting depth;
- $\omega_e = \frac{2\pi v}{l}$ represents the pulsation of the disturbing force,

with:

- v - the cutting speed;
- l - the bend wavelenght of the piece.

3. THE DYNAMIC RESPOSE

In order to determine the dynamic response, it is applied to the matric equation (4) the Laplace unilateral transform in relation to time, in homogeneous initial condition. This way, it results the algebraic system, with the Laplace images $\tilde{y}(s)$ of the displacements like unknowns:

$$[M]\{\tilde{y}(s)\}s^2 + [C]\{\tilde{y}(s)\}s + [K]\{\tilde{y}(s)\} = \frac{\omega_e}{s^2 + \omega_e^2}\left\{0 \quad F_0\mu\frac{a}{a_0} \quad -F_0\mu\frac{a}{a_0} \quad 0 \quad 0 \quad 0 \quad 0\right\}^T \tag{6}$$

or, in a different written:

$$\left([M]s^2 + [C]s + [K]\right)\cdot\{\tilde{y}(s)\} = \frac{\omega_e}{s^2 + \omega_e^2}\left\{0 \quad F_0\mu\frac{a}{a_0} \quad -F_0\mu\frac{a}{a_0} \quad 0 \quad 0 \quad 0 \quad 0\right\}^T. \tag{6'}$$

The algebraic system (6’) has, in a matric written, the solution:

$$\{\tilde{y}(s)\} = \frac{\omega_e}{s^2 + \omega_e^2}\left([M]s^2 + [C]s + [K]\right)^{-1}\cdot\left\{0 \quad F_0\mu\frac{a}{a_0} \quad -F_0\mu\frac{a}{a_0} \quad 0 \quad 0 \quad 0 \quad 0\right\}^T, \tag{7}$$

where:

$$\{\tilde{y}(s)\} = \{\tilde{y}_1(s) \quad \tilde{y}_2(s) \quad \tilde{y}_3(s) \quad \tilde{y}_4(s) \quad \tilde{y}_5(s) \quad \tilde{y}_6(s) \quad \tilde{y}_7(s)\}^T.$$

Applying in (7) the inverse of the Laplace transform, it results, in a symbolical written, the solution of the differential equations system (4), under the form:

$$\{y(t)\} = L^{-1}\left[\frac{\omega_e}{s^2+\omega_e^2}\left([M]s^2+[C]s+[K]\right)^{-1}\cdot\left\{0 \quad F_0\mu\frac{a}{a_0} \quad -F_0\mu\frac{a}{a_0} \quad 0 \quad 0 \quad 0 \quad 0\right\}^T\right]. \tag{7'}$$

The figures below represent the time – displacement curves of some components of the lathe. So, the figure 2 describes the time – displacement curve $y_1 = y_1(t)$ for the frame of the lathe, the figure 3 describes the time – displacement curve $y_2 = y_2(t)$ in case of the tool – holder slide together the lathe tool, the figure 4 shows the time – displacement curve $y_3 = y_3(t)$ of the working piece and, finally, the figure 5 represents the time – displacement curve $y_7 = y_7(t)$ for the electrical driving motor.

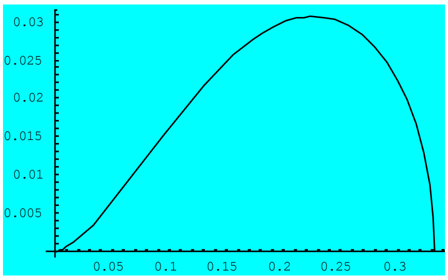

Fig. 2 The variation of the coordinate $y_1 = y_1(t)$

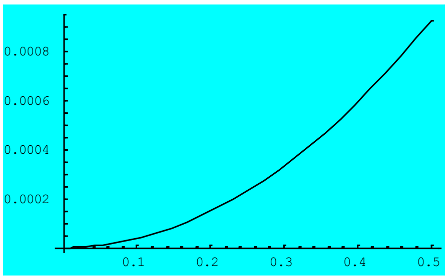

Fig. 3 The variation of the coordinate $y_2 = y_2(t)$

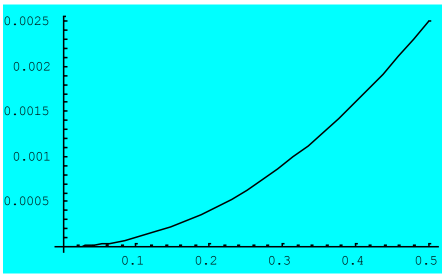

Fig. 4 The variation of the coordinate $y_3 = y_3(t)$

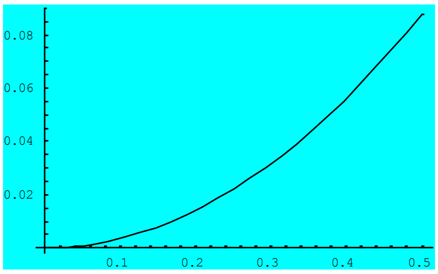

Fig. 5 The variation of the coordinate $y_7 = y_7(t)$

It is clear that the behaviour of the above elements of the lathe is most important as concerns the obtaining of a better dimensional accuracy.

4. CONCLUSIONS

One of the most important attributes of a tool – machine is the dimensional accuracy of the shape and the quality of the working piece surface. The dimensional accuracy is determined by the relative displacements that appear in the cutting zone between the lathe – tool and the working piece. These relative displacements are one direct consequence of a vibrating phenomenon. A way to decrease these displacements is to use a dynamic absorber in the cutting zone.

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