# PARAMETRIC CALCULUS METHOD FOR GENERAL VERTICAL VIBRATIONS OF THE HULL

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**Abstract:** The increase of overall elasticity of a bulk carrier's hull, essentially determined by: a ship's main large dimensions, high power propulsion installations, multiple and different load situations, specific for such types of ships, can highly facilitate local and general vibrations. These can also affect assemblies and structural elements which have an important role in the efficient run, as well as in the resistance of the ship.

Calculus modeling required by the study of own vertical and forced vibrations of the ship's beam under the action of harmonic forces can make use efficiently of modern numeric methods (torsion element, finite differences, transmission matrices etc.).

The paper presents in short some comments upon the use of transmission matrices.

## 1. FREE GENERAL VERTICAL VIBRATION

The overall increase in the bulk carriers hulls elasticity, determined essentially by: the main large dimensions, rigging with high power propulsion installations, multiple and different load cases specific for such types of ships facilitates general and local vibrations to a great extent. They affect assemblies and structural elements, with high functional and resistance role (the aft shell plating, the deck or superstructure floors etc.).

The determination of hull's own frequencies, and structural subassemblies, as vibrations resonators (double compartment in the engine area, propulsion engines, axial lines, superstructures etc.) creates the possibility to estimate the effects of the resonance phenomenon, by comparing the excitatory factors existing on board vessels bulk carriers, or outside them. [1; 3]

Most of the times, the onboard structural sub-assemblies, and even entirely structural beams, may be affected by the interaction of vibrations with the different modes of vibration, characteristic to the same or different items. Therefore, one must have knowledge of dangerous forms of coupling, knowing the fact that the main effect of this phenomenon is the change in natural frequencies for each unit or subassembly taken separately.

Literature in the field [4, 6, 7]mentions the following forms of coupling:

● the general horizontal and torsional vibrations of the hull;
● general vertical vibration of the hull and the local vibration of the superstructure;
● local vibrations of the after peak and of the double bottom of the ship.

Therefore, the general vibrations of the hull must be assigned an important role in observing the dynamic behavior of the beam. The study of different types of vibrations involves acceptance of several hypotheses [1]:

■ **HYPOTHESIS 1.** The general vertical vibrations consider the hull as an elastic beam, with variable section in length, leaned against the elastic medium provided by the aquatic medium in which they appear.

■ **HYPOTHESIS 2.** The hull is divided into $N_e$ elements whose geometrical and mechanical characteristics of which are considered constant on their length.

■ **HYPOTHESIS 3.** Each segment belonging to the hull can be modeled by using Tomoshenko's theory of the beam, taking into account the rotational inertia, and the shearing deformation.

■ **HYPOTHESIS 4.** The hull shapes, additional masses of water, the hydrodynamic damping coefficients are expressed by Lewis formula (neglecting the variation of additional masses of water and the hydrodynamic damping coefficients related with own vibrations frequency).

■ **HYPOTHESIS 5.** Due to the symmetry of the hull in relation to the diametrical plane, general vertical vibrations are considered independent of other types of general vibration of the hull (horizontal, longitudinal and torsional).

The modeling of the calculations required by the study of own and forced vertical vibrations of the ship beam, under harmonic forces action can use with great efficiency, modern numerical methods (finite element, finite differences, transmission matrices, etc.).

This paper makes synthetic comments on the use of transmission matrices for their calculation.

**This paper has contributory role in the above mentioned field.**

Therefore, the transmission matrix method is is easily applicable to the hull, which is modeled using Timoshenko's beam theory, including the rotary inertia and shear deformations [1].

Determination of the hull's own frequencies can be made in conjunction to the presence of water drawn in motion around it, and equivalent to an additional hydrodynamic mass. Under these conditions, the kinetic energy of the hull and of the additional mass of water making vertical motions is:

$$E_c = \frac{(M + M_a)w^2}{2} \quad [\text{kN}] \tag{1}$$

where:

M – the mass of the vessel (distributed along the hull), in t;

MA – additional water mass (distributed along the the hull), in t;

w – velocity of the vessel in her vertical motion;

Also on this basis, after the necessary substitutions and calculations, the differential equation of free vertical vibrations, taking into account the structural and hydrodynamic damping, according to [1], has the following form

$$\begin{aligned} & EI\frac{\partial^4 w}{\partial x^4} + EIb\frac{\partial^5 w}{\partial x^4 \partial t} - \frac{EI}{GA_f} b\overline{m}\frac{\partial^5 w}{\partial x^2 \partial t^3} - \frac{EI}{GA_f}\left(\overline{m} + bc\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} - \bar{j}\frac{\partial^4 w}{\partial x^2 \partial t^2} - \\ & \frac{EI}{GA_f}(c + bk)\frac{\partial^3 w}{\partial x^2 \partial t} - \frac{EI}{GA_f}k\frac{\partial^2 w}{\partial x^2} + \frac{1}{GA_f}\bar{j}\overline{m}\frac{\partial^4 w}{\partial t^4} + \frac{1}{GA_f}\bar{j}c\frac{\partial^3 w}{\partial t^3} + \\ & \left(\overline{m} + \frac{k\bar{j}}{GA_f}\right)\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} + kw = 0 \end{aligned} \tag{2}$$

The Fourier –type solution for equation (2) is:

$$w(x,t) = Re\, w(e)\cdot e^{ipt}, \tag{3}$$

where:

$p$ – the pulsation of own vibrations, in $s^{-1}$.

If we take into account the rotational inertia and the shear deformations, then the dynamic values of the arrows, rotations, bending moments, and shear forces during vibrations are expressed as follow:

$$w(x,t) = (C_1 \cos\beta\xi + C_2 \sin\beta\xi + C_3 ch\alpha\xi + C_4 sh\alpha\xi)\cos(pt - \theta) \tag{4}$$

$$\varphi(x,t) = (-C_1 B\sin\beta\xi + C_2 B\cos\beta\xi + C_3 D sh\alpha\xi + C_4 ch\alpha\xi)\cos(pt - \theta) \tag{5}$$

$$M(x,t)=\frac{EI}{L}(C_1\beta B\cos\beta\xi+C_2B\sin\beta\xi+C_3\alpha Dch\alpha\xi-C_4\alpha sh\alpha\xi)\cos(pt-\theta) \tag{6}$$

$$T(x,t)=\frac{EI}{L^2}(-C_1H\sin\beta\xi+C_2H\cos\beta\xi-C_3Rsh\alpha\xi-C_4Rch\alpha\xi)\cos(pt-\theta) \tag{7}$$

where:

$C_1$, $C_2$, $C_3$, $C_4$ - constants of integration;

Regarding the calculation of own pulsations, based on dynamic transmission matrices [2] some useful details are going to be given below.

Continuous variable section beam vessel turns into a bar with variable-speed sections, with 20 sections. For each section bounded by the vessel - a theoretical and couplings *i-1*, *i*, with $i = \overline{1,n}$, calculate the geometric and mechanical characteristics of the different situations of vessel loading.

The parameters: $\overline{w_i}$, $\overline{\ {}_i}$, $\overline{M_i}$, $\overline{T_i}$ of each section to pass from one section to another, is the column vector $\left[\overline{z_i}\right]$, $i=\overline{0,n}$.

Further, the calculation can be done in two stages.

- *In a first stage*, the relative successive relation of recurrence associated with arrows, rotations, bending moments and shear forces, for all nodes in the beam, is applied, thus resulting in the matrix relation:

$$\left[\overline{z_n}\right]=\prod_{i=1}^{n}\left[\overline{A_i}\right]\cdot\left[\overline{z_i}\right]. \tag{8}$$

In the case of the beam, the null components of vectors $\left[\overline{z_0}\right]$ and $\left[\overline{z_n}\right]$ are zero efforts $M_0$, $T_0$, $M_n$, $T_n$ and, by extending (8) and taking into account the conditions mentioned, we obtain the homogeneous system:

$$\begin{bmatrix} A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix}\cdot\begin{Bmatrix} w_0 \\ {}_0 \end{Bmatrix}=0 \tag{9}$$

The existence of vibrations supposes the cancellation of the characteristic determinant, so:

$$\Delta(K)=\begin{bmatrix} A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix}=0 \tag{10}$$

which is a transcendental equation in parameter K, called the equation of own pulsations. Its roots as determined by the method of the rests, lead to pulsations own values, and their own frequencies respectively.

- *In the second stage* the own forms of vibrations are determined, by calculating quantities $\overline{w_i}$, $\overline{\varphi_i}$, with the matrix relation:

$$[z_i]=[A_i]\cdot[z_{i-1}] \tag{11}$$

applied for each their own pulsations.

Own forms of vibrations are dependant on a parameter, usually can be considered one.

The above briefly described algorithm can be transferred on the computer, by using the Transfer Matrix Programme.

## 2. GENERAL VERTICAL VIBRATIONS MAINTAINED BY SHORT – LENGTH WAVES

In short – length wave navigation, the hydrodynamic forces associated with them, with harmonic pulsatory development, disrupt the elastic balance of the beam, causing maintained vibrations.

An important issue, solved with difficulty in the study of the hull vibrations, generated and maintained by waves, refer to the determination of hydrodynamic forces and excitatory moments that appear to the immersed part of the hull. It ultimately involves the phenomenon of diffraction.

Solving the diffraction problem has been a constant concern for the scientists in the field. [1] Thus: Korvin-Kroukovski (1955), Ogilvie and Tuck (1966) used a two-dimensional theory sections for the study of the ship motions without taking into account the effect of the speed of movement of the extreme part of the bow on a free surface; Magee and Beck (1988), Lin and *Yue* (1990) use the singularity distribution method for the study of the ship motion in time, where the effects of velocity are taken into account only by considering by the border condition of the body, using the theory section*; Nakos* and *Sclavounos* (1990) apply the method of *Rankine* plate in order to solve the motion of the ship.

■ In order to determine the equations that describe the motion of the liquid (water) we have to consider it as ideal, incompressible and irrotational. Taking into account the speed's potential $\Phi$ which describes the motion of the liquid around a body, we can then write the potential of the incident wave, in deep water [1] (figure 1) i.e.

$$\varphi_1 = c\bar{h}_A \cos(kx - \omega_a t)e^{kz} \tag{12}$$

where:

c - the apparent speed of the wave ;

$\bar{h}_A$ the amplitude of the regular wave;

$\omega_A$ the pulsation of apparent ship-wave meeting;

V - the ship's speed.

Further, we can write:

• the *Laplace* equation for 3D space:

$$\frac{\partial\varphi}{\partial x^2} + \frac{\partial\varphi}{\partial y^2} + \frac{\partial\varphi}{\partial z^2} = 0; \tag{13}$$

• *kinematic limit* condition on the free surface of the liquid:

$$\frac{\partial\zeta}{\partial t} + \left(v + \frac{\partial\varphi}{\partial x}\right)\frac{\partial\zeta}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial\zeta}{\partial y} = \frac{\partial\varphi}{\partial z}, \qquad \text{for: } z = \varsigma; \tag{14}$$

• the *dynamic limit* on the free surface of the liquid

$$\frac{\partial\varphi}{\partial t} + v\frac{\partial\varphi}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right] + g\zeta = 0, \qquad \text{for } z = \varsigma. \tag{15}$$

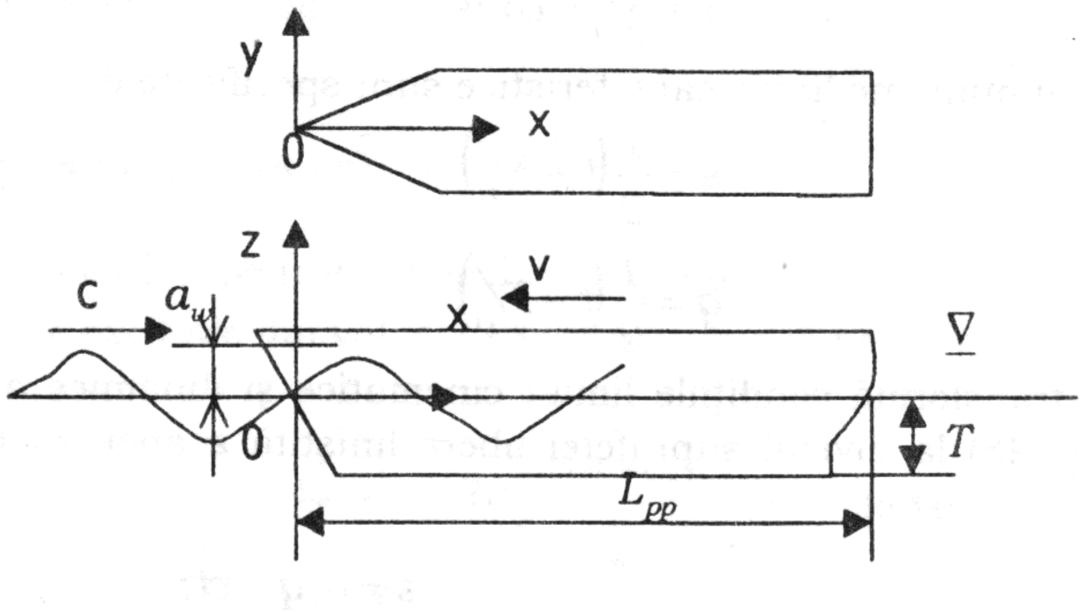


*Figure 1.*

Since the hull is slender, the Laplace equation can be reduced to the form:

$$\frac{\partial \varphi}{\partial x^2} + \frac{\partial \varphi}{\partial z^2} = 0 . \tag{16}$$

Also, since the amplitudes of incident wave and the ship motions are small, the infinite terms at high exponents are neglected, and the problem becomes linear.

The conditions of the linearized cinematic and dynamic free surface of the liquid are:

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\zeta = \frac{\partial \varphi}{\partial z}, \qquad \text{for } z = 0; \tag{17}$$

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\varphi + g\zeta = 0, \qquad \text{for } z = 0. \tag{18}$$

The equations correspondent to the speed potential $\Phi$ (describing the movement of fluid around the body) as well as to the potential of the wave in deep water $\varphi_1$, have in plane ( *x/v*, *t* ) the form of characteristic lines (figure 2.), and are defined with:

$$t - x/v = \text{constant}; \qquad t + x/v = \text{constant}. \tag{19}$$

The coordinates of a point belonging to the characteristic lines are:

$$s = (t + x/v); \quad q = (t - x/v) \tag{20}$$

These variables transform the conditions (17) and (18) at the level of free calm water (i.e. at z = 0) along the characteristic line, as follows:

$$\frac{\partial \zeta}{\partial s} = \frac{\partial \varphi}{\partial z}, \qquad \text{for } z = 0,\ q = ct; \tag{21}$$

$$\frac{\partial \varphi}{\partial s} + g\zeta = 0, \qquad \text{for } z = 0,\ q = ct \tag{22}$$

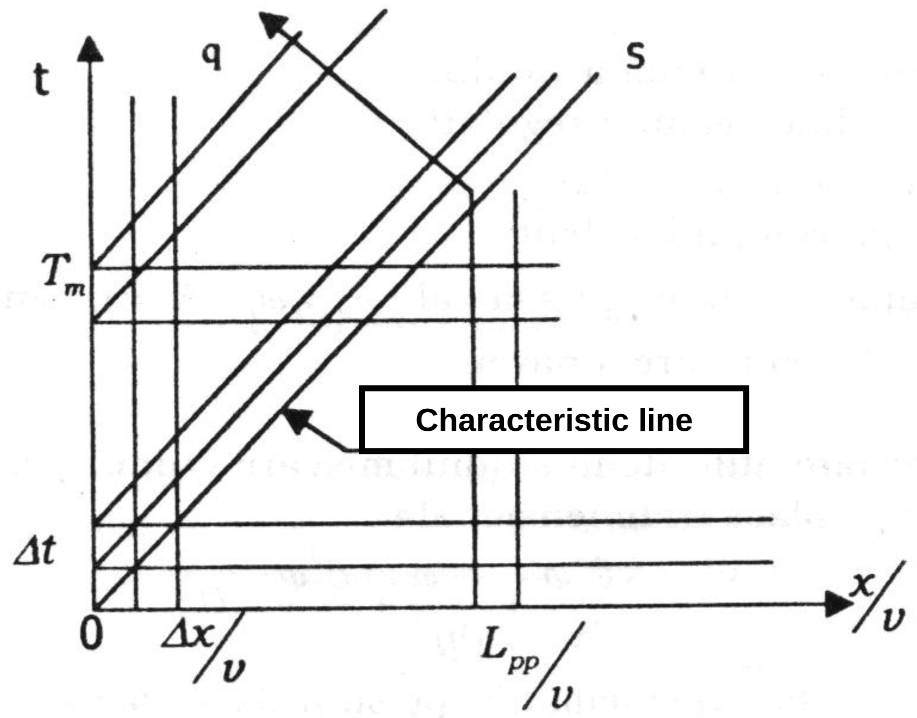


***Figure 2.***

The condition of open border along the characteristic line is:

$$\frac{\partial \varphi}{\partial s} + c\frac{\partial \varphi}{\partial l} = E \text{ , for } q = ct, \tag{23}$$

where:

c - the apparent wave velocity,

l – the normal line at the open border,

E - the error for the radiation condition.

For small amplitudes of the ship motions, the rigid border condition is:

$$H(x, y, z) = 0, \tag{24}$$

while the limit condition on the surface of the body, along the characteristic line becomes

$$\frac{\partial(\varphi+\varphi_i)}{\partial y}\frac{\partial H}{\partial y} + \frac{\partial(\varphi+\varphi_i)}{\partial z}\frac{\partial H}{\partial z} = -\frac{\partial H}{\partial s}, \qquad \text{on } H(s, q, y, z) = 0 \tag{25}$$

Also, the speed potential can be written as

$$\varphi = \varphi_u + \varphi_d \tag{26}$$

where:

$\varphi_u$ – the potential of the longitudinal speed;

$\varphi_d$ – the potential of the diffraction speed.

Thus, the rigid border condition appears as:

$$\frac{\partial \varphi_u}{\partial y}\frac{\partial H}{\partial y} + \frac{\partial \varphi_u}{\partial z}\frac{\partial H}{\partial z} = -\frac{\partial H}{\partial s}; \tag{27}$$

$$\frac{\partial \varphi_d}{\partial y}\frac{\partial H}{\partial y} + \frac{\partial \varphi_d}{\partial z}\frac{\partial H}{\partial z} = -\frac{\partial \varphi_{i1}}{\partial y}\frac{\partial H}{\partial y} - \frac{\partial \varphi_{id}}{\partial z}\frac{\partial H}{\partial z}. \tag{28}$$

If the case of a bow wave, which we are interested in here, the potential is symmetrical in regard to PD, while the limit condition under the hull, in a horizontal plane becomes:

$$\frac{\partial \varphi}{\partial y} = 0 \tag{29}$$

With the incident wave given by (12) and the limit conditions given by (23), (27), (28) and (29), the diffraction problem of the ship motion, moving at a constant speed, can be solved by using the equations (16), (21) and (22) along the characteristic lines.

■ Knowing the speed potential φ on the borders along the characteristic lines, the vertical force of excitation in the wave can be determined, and can be modified on an element along the body, with the following relation:

$$f(s,q) = -2p\int_K^W \frac{\partial(\varphi + \varphi_i)}{\partial s} \cdot n_z ds \tag{30}$$

in which:
K - keel point of the vessel (figure 3),
W - the point on the hull by the water line;
$n(n_y, n_z)$ – the normal unit vector at the surface of the hull.

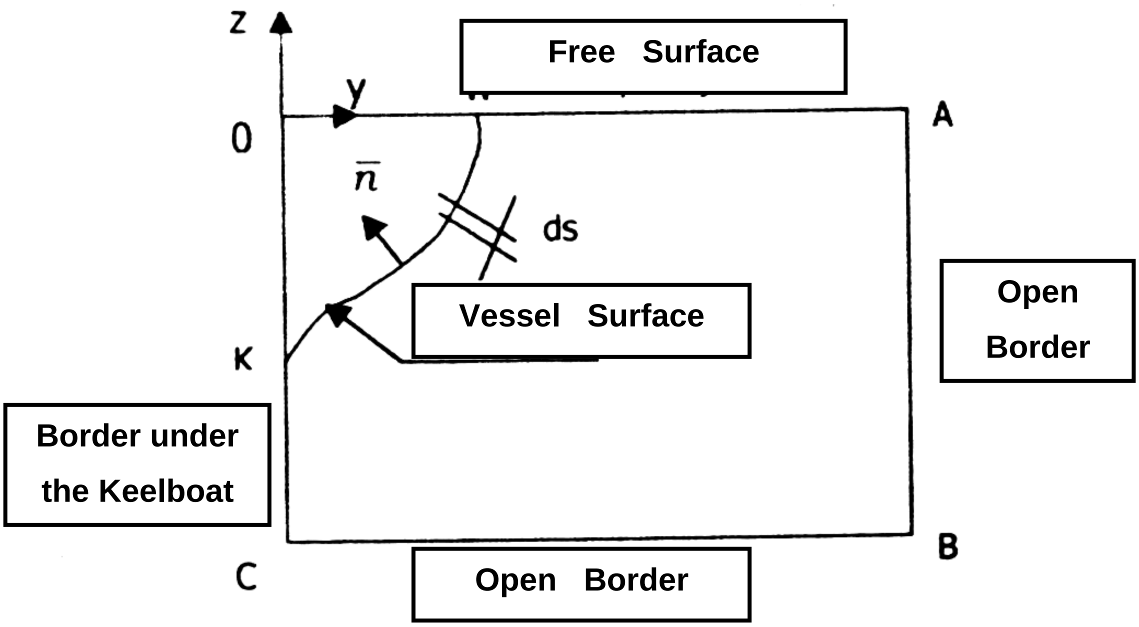


***Figure 3.***

*The total hydrodynamic excitation force of the regular wave*, the *excitation moment respectively,* that occurs in the hull's middle length are obtained by the help of integral relations:

$$\overline{F_V} = \int_0^{L_{pp}} f(s,q) dx \tag{31}$$

$$\overline{M_\theta} = \left(\frac{L_{pp}}{2}\right)\overline{F_V} - \int_0^{L_{pp}} f(s,q) x dx \tag{32}$$

## 3. CONCLUSIONS

General vibrations of bulk carriers hulls are assigned an important role in studying the dynamic behavior of the ship's beam girder.

Amongst the wide range of existing vibrations, the general vertical vibrations were of much interest as they are considered independent of other types of general hull vibrations (such as horizontal, longitudinal or torsional vibrations).

The modeling of calculations of own forced vertical vibrations of the beam under harmonic forces action can use the methods of: finite element, finite differences, or transmission matrices. The method of transmission matrices adjusts very well to the hull, which will be modeled using Timoshenko beam theory, including the rotary inertia and shear deformations [1].

Determination of own frequencies of the hull is made in regard to the presence of water around it, driven in motion, which is equivalent to an additional hydrodynamic mass. After the calculus and necessary replacements are done there resulted the differential of free vertical vibrations, taking into account the structural and hydrodynamic damping.

. The dynamic values of arrows, rotations, bending moments and shear forces for all nodes of the beams were expressed during vibrations, while the rotary inertia and shear deformations arising from the load were taken into account. After this, the own vibration forms were determined by calculating the following quantities $\overline{w_i}, \overline{\varphi_i}$, with the matrix relation $[z_i]=[A_i]\cdot[z_{i-1}]$ applied to each pulsation.

The second part of the paper deals with maintained vibrations due to the hydrodynamic forces with harmonic pulsatory action which disrupts the elastic balance of the ship beam, occurring in short waves.

The study of hull vibrations generated and maintained by waves are materialized through the determination of excitation hydraulic forces and moments manifested on the surface of submerged part of the ship, which, in its turn, implies t the phenomenon of diffraction. Solving the problem of diffraction is a constant concern for the scientific environment [1].

This paper has contributory role in the above mentioned field.

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