OPTIMUM SHAPE DESIGN OF ROTOR SHAFT IN STATIC RANGE

Dumitru Nicoara
Transilvania University of Brasov, tnicoara@unitbv.ro

Keywords: rotor system, optimization, finite element.

Abstract: The paper deals with the optimization of the rotor–bearing systems in static range. The goal of this study is to find out the position of the bearings and the diameters of the shaft (different diameters for several segments of the shaft) in order to minimize static stiffness. Some constraints are imposed: the maximum stress, the minimum diameter, distances between bearings and constant volume (weight) of the shaft

1. INTRODUCTION

The rotor-bearing systems play an important role in many fields of engineering, such as the machine tool, the gas and steam turbines, turbogenerators etc. On account of the ever-increasing demands for high power, high speed, and light weight which are the main reasons of failure in performances and fatigue in structures of the rotor-bearing systems, the designer needs to have some new techniques for the prediction of critical speeds, unbalanced responses, and threshold speeds of instability for synchronous and nonsynchronous whirling. Thus, the machining performance can be raised remarkably by improving dynamic stiffness and static stiffness of the rotor-bearing systems. Forced vibration analysis of a rotating equipment subject to synchronous or asynchronous harmonic excitation becomes essential to identify the vibration source or to ensure the proper design in considering vibration problems.

In this paper we use finite element analysis and optimization principles to determine the optimal shape of the rotor shaft bearing system. Cost functional is static stiffness. The design parameters are the geometric parameters of the physical model. To do this we use a computer program developed by the author.

2. FINITE ELEMENT MODEL OF ROTOR-BEARING SYSTEMS

The model consists of a rotor treated as a continuous elastic shaft with several rigid disk, supported on an anisotropic elastic bearings. Consider that the dynamic equilibrium configuration of the rotor-bearing system the undeformed shaft is along the *x*- direction of an inertial *x, y, z* coordinate system. In the study of the lateral motion of the rotor, the displcement of any point is defined by two translations (v,w) and two rotations (φ_y,φ_z). In the following, only axisymmetric rotors are considered. The model could use one of the following three beam finite element types [1]:

- Beam C^1 finite element type based on Euler-Bernoulli beam model;
- Beam C^1 finite element type based on Timoshenko beam model;
- Beam C^0 izoparametric finite element type based on Timoshenko beam model;

The beam finite element has two nodes. In the case of the dynamic analysis four degrees of freedom (DOF) per node are considered: two displacements and two slopes measured in two perpendicular planes containing the beam. We do a comparative study of the three proposed models and on its basis we adopt the optimal model of the goal. Timoshenko beam model is finally adopted as the beam might be short and therefore the effect of the shear force must be considered. The gyroscopic effect and damping in bearings may be taken into account. The liniarized bearing are commonly modeled as four spring coefficients and four damping coefficients.

3. OPTIMIZATION MODEL

The optimization problems were defined is a static problem for which the objective function is the static flexibility, that is the displacement **u** under the force **F** applied on the shaft, divided by the force. The design parameters are the distances between the bearings, $\mathbf{s_i}$, and the diameters of the different portions of the shaft, $\mathbf{d_i}$. The problem is subjected to some constraints: the maximum and minimum limits of the distances $\mathbf{s_i}$, limits for the diameters $\mathbf{d_i}$, the bending stress must be smaller that an available value $\boldsymbol{\sigma}_a$ and the volume **V** of the shaft must be constant. This may be written, for a beam on three bearings shaft, Fig. 1, as:

$$\min_{s_1,s_2,d_1,d_2,d_3} u/F$$

$$\begin{cases} s_1^l \le s_1 \le s_1^u \\ s_2^l \le s_2 \le s_2^u \end{cases} \quad \begin{cases} d_1^l \le d_1 \le d_1^u \\ d_2^l \le d_2 \le d_2^u \\ d_3^l \le d_3 \le d_3^u \end{cases} \quad \begin{cases} \sigma \le \sigma_a \\ V = const. \end{cases}$$

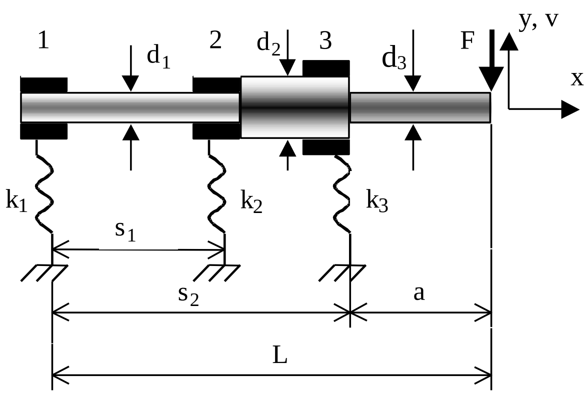


Figure 1 Beam on three bearings

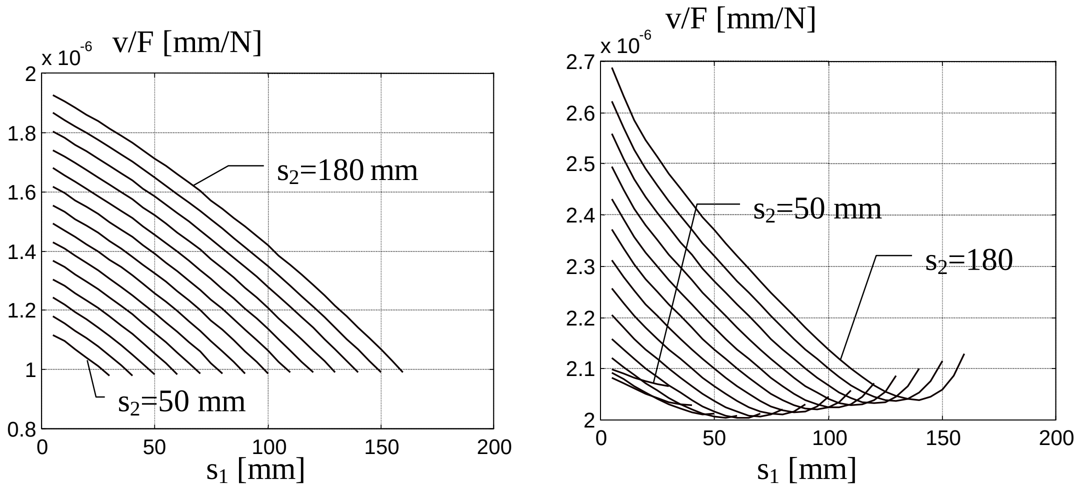
4. NUMERICAL EXAMPLE

The example illustrates the static optimization of one three bearings shaft, Fig. 1. The initial data are: $\mathbf{d_1}$=$\mathbf{d_2}$=$\mathbf{d_3}$=80 mm, **a**=100 mm, **F**=60 000 N. mm, **a**=100 mm, **F**=60 000 N. Figures 2 and 3 show the variation of the vertical displacement **v** under force **F**, with the distances $\mathbf{s_1}$ and $\mathbf{s_2}$, for several cases: rigid bearings and elastic bearings (**k**=10^7 N/m), for Euler-Bernoulli beam model, or Timoshenko beam model. It can be noticed that the bearing stiffness strongly influences the optimum values of the two design parameters $\mathbf{s_1}$ and $\mathbf{s_2}$. Timoshenko beam model is suitable as the distances between bearings may become small and the influence of the shear force may not be neglected.

Table 1 contains some results of the optimization of the rotor-bearing system represented in the figure 1, for different values of the shaft volume. The design parameters are $\mathbf{s_1}$, $\mathbf{s_2}$, $\mathbf{d_1}$, $\mathbf{d_2}$ and $\mathbf{d_3}$. The system is subjected to the following constraints: $\mathbf{d_i}$ < 40 mm, **σ**□< 100 Mpa and the volume of the shaft is constant. Timoshenko C^1 beam element was used.

Table 1

Shaft volume [mm^3]	s_1,s_2 [mm]	d_1, d_2, d_3 [mm]	σ_1, σ_2, σ_3 [Mpa]	v_{max} / F
6.2832e+005	11.2 / 31.7	53.1 / 79.9 / 79.9	100 / 100 / 100	5.34e-6
7.5398e+005	12.1 / 58.4	47.4 / 79.9 / 79.9	60.6 / 100 / 100	3.17e-6
8.7965e+005	13,4 / 72.7	51.4 / 87.0 / 82.1	44.9 / 77.29 / 92	2.64e-6
1.0053e+006	14.5 / 72.7	53.8 / 91.4 / 86.7	39 / 66.6 / 78.1	2.27e-6
1.1310e+006	15.5 / 77.4	56.1 / 95.4 / 90.9	35.2 / 58.6 / 67.7	2.01e-6
1.2566e+006	16.5 / 82.0	58.2 / 99.1 / 94.9	31.8 / 52.3 / 59.6	1.80e-6



a) Euler-Bernoulli model ***b) Timoshenko model***

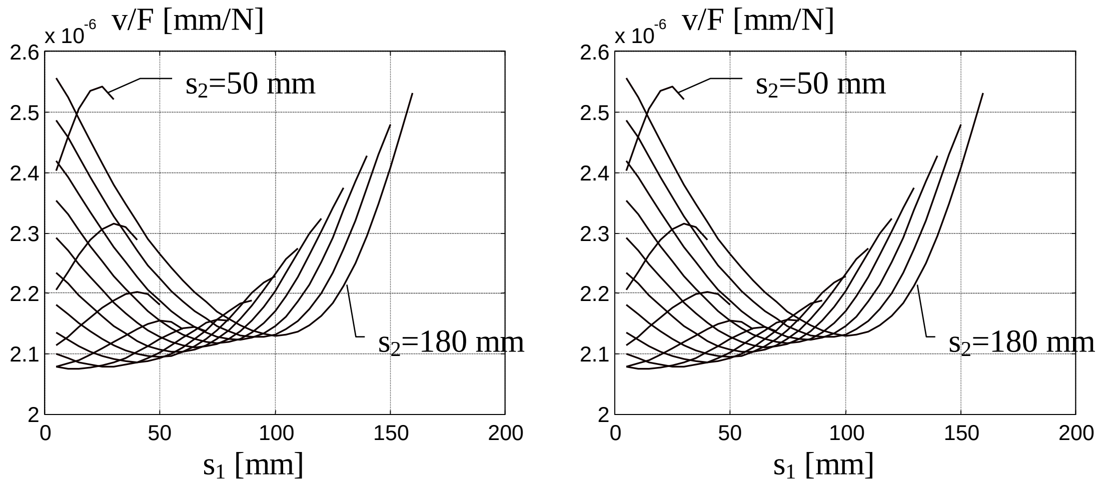
Figure 2 Rigid bearings

4. CONCLUSIONS AND RECOMMENDATIONS

Figures 2 and 3 show the influence of the two distances $\mathbf{s_1}$ and $\mathbf{s_2}$, Fig. 1, on the flexibility of the shaft, the vertical displacement under the force F divided by the force.

The results are strongly influenced by the bearing stiffness. Two cases were considered: rigid bearing and bearing having a stiffness of $k=10^7$ N/mm. The beam model is very important, too.

Correct results are obtained using Timoshenko beam model as the distance between bearings may become small and the influence of the shear force may not be neglected. This aspect is more important for statically indeterminate systems because in this case the values of the reactions, and therefore the values of efforts and displacements, depends on the stiffness of the beam. For instance, from figure 2a it results that the displacement under force **F** becomes smaller as $\mathbf{s_1}$ becomes close to $\mathbf{s_2}$, that is the distance 2-3 tends to zero. This is completely false if we consider that when the



a) Euler-Bernoulli model ***b) Timoshenko model***

Figure 3 Elastic bearings, k=10^7 N/m

distance 2-3 is very short, even if bending moment is small, the effect of shear force becomes very important, [2], [3].

For a given volume of the shaft, the maximization of the dynamic stiffness was done, considering two cases: (I) the diameters $\mathbf{d_1}$,$\mathbf{d_2}$, $\mathbf{d_3}$ given and the design parameter is **s**, the distance between the bearings, and (II), the design parameters are in the same time the distance between the bearings as well as the diameters $\mathbf{d_1}$, $\mathbf{d_2}$, $\mathbf{d_3}$.

In the second case, by distributing the diameters by optimization program, the dynamic stiffness increased by 75 % comparing to the first case.

References

[1] Akella, S., (1986). Modification to a Timoshenko Beam-shaft Finite Element to Include Internal Disk and Change in Cross- section, Journal of Sound and Vibration, 106, p.227-239.
[2] Bathe, K.J. Finite Element Procedures in Engineering Analysis, Prentice-Hall, Englewood Cliffs, 1995.
[3] Brandon, J.A., (1992). AL-Sharreef, K.J.H., Optimization Strategies for Machine Tool Spindle-Bearing Systems: A Critical Review, ASME Journal of Engineering for Industry, Vol.114, p. 244-253.
[4] Childs, D., (1993). Turbomachinery Rotordynamics, John Wiley & Sons, New York.
[5] Gill, P.E., Murray, W., Wright, M.M., (1991) Numerical Linear Algebra and Optimization, Addison Wesley.
[6] Hughes, J.R.T., (1987) The Finite Element Method, Prentice –Hall Inc..
[7] Krämer, E., (1993) Dynamics of Rotors and Foundations. Springer Verlag, Berlin.
[8] MATLAB: Reference Guide, The MathWorks Inc., Natick, Mass., USA, 1993.
[9] Nicoara, D.D**.**, Munteanu, M. Gh., (1999) Contributions on the Optimization of Rotor –Bearing Systems, Anspruch und Tendenzen in der experimentellen Strukturmekanik, VDI Verlag – Düsseldorf, 1999, pp. 369-375.
[10] Nicoara, D.D**.**, Munteanu, M. Gh., (2000) *Using of optimization principle for dynamic analysis of rotor-bearing systems*, Proceedings (CD) of the VIII th International Conference Numerical Methods in Continuum Mechanics, Liptovsky Jan Slovak Republic, 2000, pp. 199-200.
[11] Vanderplaats, S.V., Numerical Optimization Tehniques for Engineering Designe: with Applications, McGraw-Hill, New York.