ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume X (XX), 2011, NR2

A NUMERICAL SOLUTION TO THE CATTANEO-MINDLIN PROBLEM PART II: PROGRAM VALIDATION AND LOADING HISTORY SIMULATION

Sergiu Spinu, Dorin Gradinaru
"Stefan cel Mare" University of Suceava, Romania
sergiu.spinu@fim.usv.ro, gradinaru@fim.usv.ro

Keywords: numerical simulation, elastic frictional contact, slip-stick, loading history

Abstract: Numerical predictions of a numerical program advanced in a companion paper are validated against existing analytical solution for the Cattaneo-Mindlin problem. A sphere pressed with a normal constant force against an elastic half-space undergoes simultaneous tangential loading. Shear tractions profiles, and variations of stick radius and of rigid-body tangential displacement with the loading level, match well the analytical model. In order to simulate a loading-unloading cycle, the algorithm is updated to allow for incremental loading application. Different strategies are required on the loading and on the unloading path, respectively. In all cases, validation against closed-form relations is considered satisfactory.

1. INTRODUCTION

The numerical program proposed in the first part of this paper for resolution of Cattaneo-Mindlin problem is validated with the existing closed-form solution. A spherical contact is submitted to a normal constant force, and an oscillating tangential force acting simultaneously. The solution of the normal contact problem, namely contact area and pressure distribution, is assumed to be known from Hertz theory, [3], or can be assessed, for any kind of contact geometry, using the well-known algorithm for the rough normal contact by Polonsky and Keer, [6]. As shown extensively by Gallego, [2], uncoupling of normal and tangential effects is not possible unless materials of contacting bodies have similar elastic properties. In other words, solution of normal contact cannot be obtained independently of shear tractions.

A second assumption considers that the shear tractions in the other tangential direction (the one normal to tangential force support) have little or no effect on problem solution in the considered tangential direction.

2. REVIEW OF CLOSED-FORM SOLUTION

Cattaneo, [1], and Mindlin, [5], advanced a closed-form solution for the contact between two elastic bodies undergoing normal and tangential loading, W and T respectively, when the limiting surfaces can sustain shear tractions. They proved that, for Hertz contact geometries, the solution of the fully adherent contact exhibits a singularity on the boundary of the contact area, where shear tractions reach infinity:

$$q(r) = \frac{T}{2\pi a_H^2 \sqrt{1 - r^2 / a_H^2}}, \tag{1}$$

where a_H denotes Hertz contact radius and r is the radial coordinate. This solution verifies neither Coulomb's law of friction, nor Linear Elasticity Theory, which requires continuity of stresses. Consequently, the problem cannot be solved in the frame of Linear Elasticity unless a slip region is assumed. The solution advanced in [1] postulates that, when the tangential force increases from zero to a limiting value, $T_{\lim} = \mu W$, with μ the frictional coefficient (assumed constant over all contact area, and equal for static or kinetic friction), an annulus of slip penetrates from the edge of the contact area, until the stick region reduces to a single point and gross-slip is imminent. Consequently, the contact area consists of reunion of two domains:

1. a stick central disc, of radius a_S, where shear tractions yield from relation:

$$q(r) = (1 - a_S/a_H)\,\mu p_H \sqrt{1 - r^2/a_H^2}\,,\; r \le a_S; \tag{2}$$

2. a peripheral annulus of slip, where shear stresses obey the kinetic law of friction:

$$q(r) = \mu p(r) = \mu p_H \sqrt{1 - r^2/a_H^2}\,,\; a_S < r \le a_H\,. \tag{3}$$

Stick radius a_S yields from the following equation:

$$a_S = a_H \sqrt[3]{1 - T/(\mu W)}\,, \tag{4}$$

and rigid-body tangential translation can be expressed as:

$$\delta = \frac{3\mu W}{16 a_H}\left(\frac{2-\nu_1}{G_1} + \frac{2-\nu_2}{G_2}\right)\left[1 - \left(1 - \frac{T}{\mu W}\right)^{2/3}\right]. \tag{5}$$

where ν_i and G_i, $i = 1{,}2$, are the Poisson's ratio and the shear modulus of contacting bodies materials.

In an extension of Cattaneo's results, Johnson, [4], advances the solution for the slip-stick spherical contact, undergoing constant normal loading and a variable tangential force, oscillating between two limiting values $\pm T^*$, where $T^* \le T_{\lim}$. When T increases from zero to T^*, the loading curve is accurately described by the set of equations (2) - (5). However, on the unloading curve, when T decreases from T^* to $-T^*$, these equations no longer apply. As stated in [4], during unloading, which is equivalent to application of a negative increment in T, a region of reversed slip, where $q(r) = -\mu p(r)$, penetrates from periphery to a radius a_S, and no reversed slip is present for $r < a_S$. If a_S^* and δ^* denotes the stick radius and the rigid-body translation at the maximum loading level on the loading curve, $T = T^*$, then the following relations hold on the unloading curve:

$$(a_S/a_H)^3 = \frac{1}{2}\left[1 + (a_S^*/a_H)^3\right], \tag{6}$$

$$q(r) = \begin{cases} -\dfrac{3\mu W}{2\pi a_H^3}\sqrt{a_H^2 - r^2}, & a_S \le r \le a_H; \\ -\dfrac{3\mu W}{2\pi a_H^3}\left(\sqrt{a_H^2 - r^2} - 2\sqrt{a_S^2 - r^2}\right), & a_S^* \le r \le a_S; \\ -\dfrac{3\mu W}{2\pi a_H^3}\left(\sqrt{a_H^2 - r^2} - 2\sqrt{a_S^2 - r^2} + \sqrt{\left(a_S^*\right)^2 - r^2}\right), & r \le a_S^*, \end{cases} \tag{7}$$

$$\delta = \frac{3\mu W}{16a_H}\left(\frac{2-\nu_1}{G_1}+\frac{2-\nu_2}{G_2}\right)\left[2\left(1-\frac{T^*-T}{2\mu W}\right)^{2/3}-\left(1-\frac{T^*}{\mu W}\right)^{2/3}-1\right]. \quad (8)$$

When $T=-T^*$, the original slip was covered by the reversed slip, and the state achieved is a complete reversal of that at $T=T^*$. Further loading leads to a series of states similar to unloading from $T=T^*$, but of opposite sign, as depicted in Fig. 1.

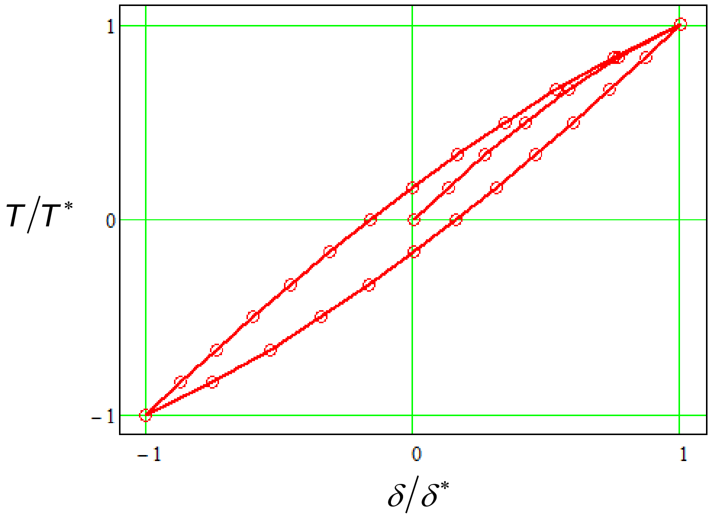

Figure 1. A loading-unloading cycle

3. PROGRAM VALIDATION

Predictions of the newly advanced numerical program are matched against aforementioned closed-form solution for elastic spherical contact under slip-stick regime. At this point, the materials of the contacting bodies are assumed to have similar elastic properties: Young modulus $E=210GPa$, Poisson's ratio $\nu=0.3$, and the frictional coefficient is assumed constant over contact area, $\mu=0.1$. This leads to uncoupling of normal contact problem from the tangential one, and, consequently, the solution of the normal contact problem can be adopted from Hertz formalism.

In order to validate the algorithm in case of the Cattaneo-Mindlin problem, a sphere of radius $R=18mm$ is pressed against an elastic half-space with a constant normal load $W=1kN$ and an increasing tangential force T acting along direction of $\vec{x}$, ranging from zero to the limiting value which induce a gross-slip regime. At this point, the load is applied in one step.

Distributions of dimensionless shear tractions, normalized by Hertz contact pressure, $\bar{q}=q/p_H$, in the plane $y=0$, are depicted in Fig. 2. Dimensionless radial coordinates are defined as ratio to Hertz contact radius a_H, $\bar{r}=r/a_H$.

In Fig. 3, program validation is extended to dimensionless stick radius, defined as ratio to Hertz contact radius, $\bar{a}_S=a_S/a_H$, and to dimensionless rigid-body tangential translation, $\bar{\delta}=\delta/\delta_0$, normalized by $\delta_0=3\mu W(2-\nu)(1+\nu)/(4a_H E)$. The tangential load ranges from zero to $T_{\lim}$: $\bar{T}=T/T_{\lim}$. The distribution of relative slip distances when $\bar{T}=0.9$, normalized by the rigid-body tangential translation, $\bar{s}=s/\delta$, is depicted in Fig. 4.

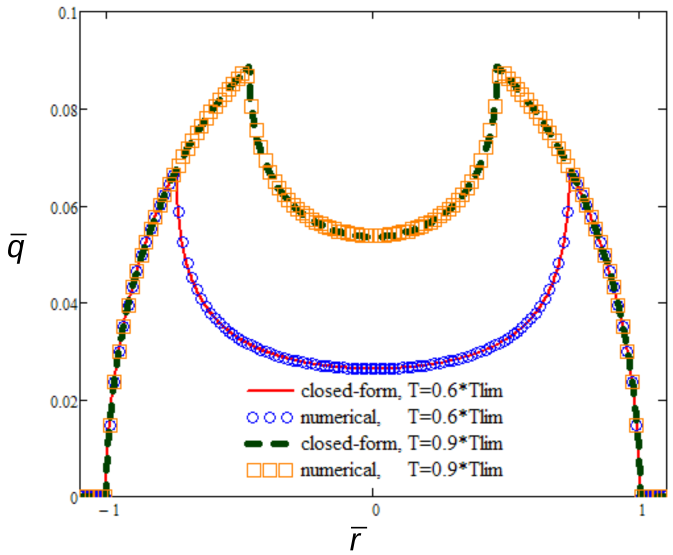

Figure 2. Dimensionless shear stress profiles in the plane $y = 0$

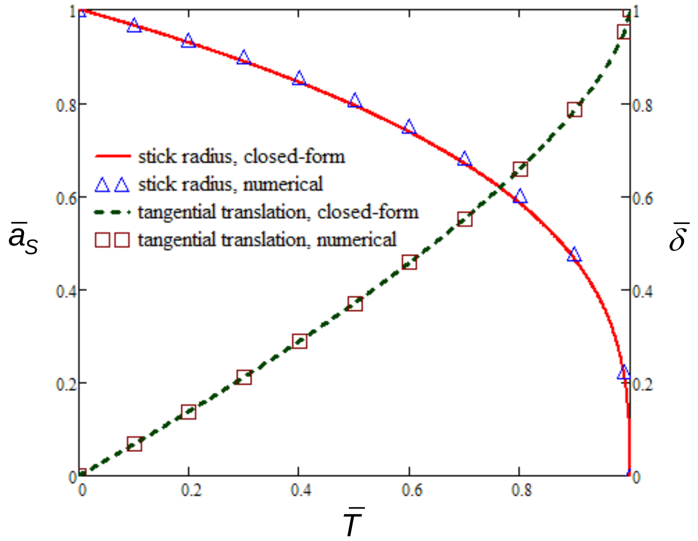

Figure 3. Dimensionless stick radius and tangential displacement versus loading level

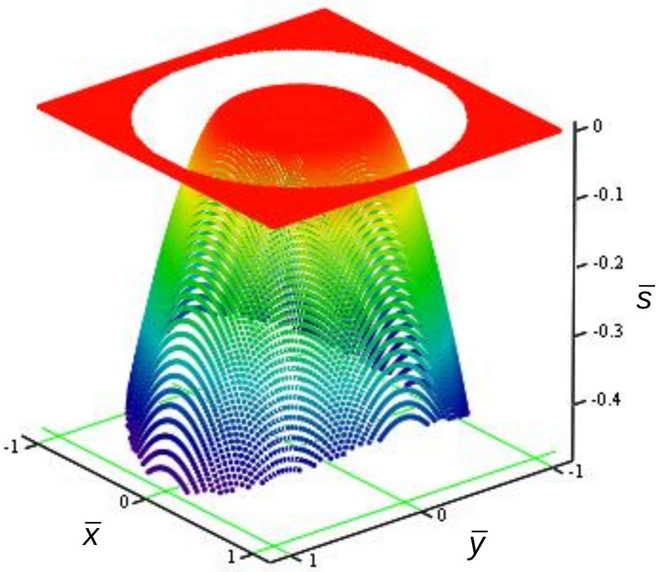

Figure 4. Dimensionless slip distances, $\bar{T} = 0.9$

4. LOADING HISTORY

The newly advanced algorithm can be easily modified to account for an incremental load application, thus allowing simulation of contact scenario for oscillating tangential force.

On the loading curve, when the $(k+1)^{\text{th}}$ increment of T is applied, regions already in slip preserve their status, and new cells from the stick area enters slip. Therefore, it is convenient to use the proposed algorithm with an initial guess for the stick area matching the existing stick area, instead of contact area, which does not vary as normal loading is constant: $A_{S\,init}^{(k+1)} = A_S^{(k)}$. At the end of the loading curve, the stick area results from solving the state corresponding to application of last loading increment, and the slip area results as reunion of all slip regions achieved at each loading increment. It should be noted that, on the loading curve, the results obtained when the load is applied incrementally or in one step match.

A different approach is needed on the unloading curve. The slip s^* achieved at the end of the loading curve, depicted in Fig. 4, enters as an initial state in the new condition of deformation:

$$s(i,j) = -s^*(i,j) + u(i,j) - \delta, (i,j) \in A_C; \qquad (9)$$

The negative sign of s^* appears as slip on the unloading curve is opposed to the one on the loading curve. The resulting model can be solved using the algorithm presented in the first part of this paper.

The contact used in previous simulations was submitted this time to an increasing tangential force T reaching its maximum $T_{\max} = 0.9T_{\lim}$, which then decreased to the maximum value but with a negative sign, $T = -T_{\max}$. The numerically predicted shear tractions profiles on the unloading curve are compared in Fig. 5 with the analytical curves computed with Eq. (7) and the agreement is considered satisfactory. Due to its averaging technique, the numerical approach fails to accurately predict the slope discontinuities at a_S^*. The gap to the analytical profile is expected to narrow with increasing resolution.

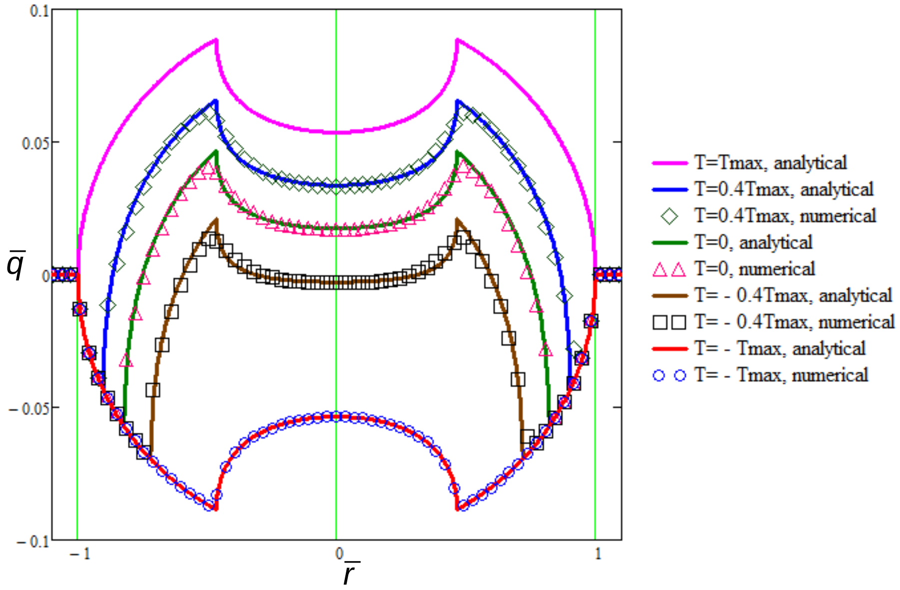

Figure 5. Shear tractions profiles on the unloading path

5. CONCLUSIONS

A numerical program advanced in a companion paper is validated against existing closed-form solutions for the Cattaneo-Mindlin problem. A tangential force, less than the limiting value inducing gross-slip, is applied, in one step or incrementally, in addition to normal loading in a spherical contact, in which pressure distribution and contact area are assumed to be known from Hertz Theory.

Incremental loading application allows simulating a loading cycle, in which the contact undergoes constant normal load and oscillating tangential force. Program predictions match well existing analytical models.

The newly advanced computer code will allow simulating the elastic contact between elastically dissimilar bodies without neglecting the friction at the interface. Therefore, some of the assumptions plaguing the existing models will be removed, leading to a better understanding of phenomena involved and, consequently, to a better prediction of load-carrying capacity of the mechanical contact.

ACKNOWLEDGEMENTS

This paper was supported by the project “Progress and development through post-doctoral research and innovation in engineering and applied sciences– PRiDE - Contract no. POSDRU/89/1.5/S/57083”, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

References

[1] Cattaneo, C., (1938), Sul contatto di due corpi elastici: distribuzione locale degli sforzi, Accademia Nazionale Lincei, Rendiconti, Ser. 6, vol. XXVII, pp. 342–348, 434–436, 474–478.
[2] Gallego, L., (2005), Fretting et Usure des Contacts Mécaniques: Modélisation Numérique, Ph.D. Thesis, INSA Lyon, France.
[3] Hertz, H., (1895), Uber die Beruhrung fester elasticher Korper, Gesammelte Werke, Bd. 1, Leipzig, 155-173.
[4] Johnson, K. L., (1985), Contact Mechanics, Cambridge University Press.
[5] Mindlin, R. D., (1949), Compliance of Elastic Bodies in Contact. ASME J. Appl. Mech., vol. 16, pp. 259–268.
[6] Polonsky, I. A., and Keer, L. M., (1999), A Numerical Method for Solving Rough Contact Problems Based on the Multi-Level Multi-Summation and Conjugate Gradient Techniques, Wear, 231(2), pp. 206–219.