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MOBILE COUPLING KINEMATICS, AS MULTIBODY SYSTEM

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Abstract: Mechanical systems definition as multibody systems is a modern way of modeling aiming to the real time simulation of complex product's dynamic behavior. The paper presents aspects regarding the kinematics of the mobile transversal coupling as multibody system. The mobile couplings are used in a lot of power transmissions at cars, road trucks, railway trucks, ships and other industrial systems. Previously, based on an adequate graph and specific conditions, like the degree of freedom, number of bodies and the type of geometrical constraints between the bodies, the synthesis of all possible graphs for these transversal couplings as multibody systems were developed. First it is presented the structural scheme of the analyzed mobile coupling. Then are defined: the parts of the multibody system associated to the mobile coupling (input and output semicouplings, intermediary element and basis); the body reference frames and also the general reference frame; the geometrical and cinematic constraints. Finally, there are determined the cinematic equations, useful to found the optimal geometrical configuration of the coupling, depending by the initial requests of a mobile coupling (cinematic and constructive conditions) and also to obtain some new constructive variants, presented in the final part of the paper.

1. INTRODUCTION

The mobile couplings are used in a lot of power transmissions at cars, road trucks, railway trucks, ships and other industrial systems, in movement and torque transmission between two shafts with parallel axis. The shafts are connected by a kinematical linkage with the possibility to have translations in the transversal plane. These translations are named transversal movements. A transversal coupling is homokinetic if the shafts angular speeds are identical. If the shafts angular speeds are almost identical, the mobile coupling is quasihomokinetic [1].

If the shafts are connected to the basis, it will result the associated mechanism, which is a plane mechanism [1]. The transversal coupling multibody model has, as bodies, the input and output shafts and also, some of the intermediary bodies which have, usually, more than two connections [3, 5, 6, 7]. The studied coupling is Buchli [1, 8], with the main particularity to have a gear sector between the intermediary elements.

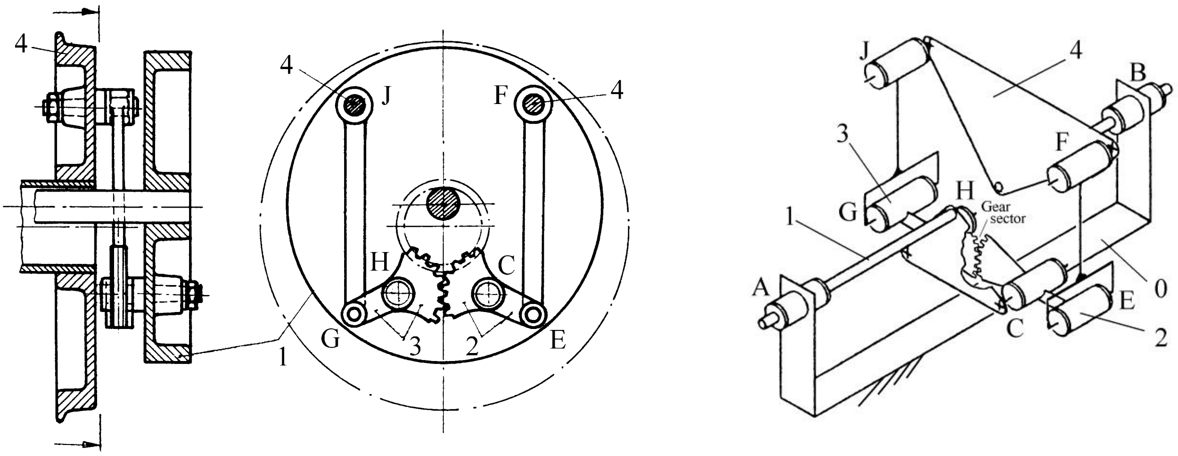
2. THEORETICAL BASIS

The analysed mobile coupling is composed by two semicouplings and between them there are the intermediary elements (fig. 1) [1].

The bodies of the multibody system are: input semicoupling, 1, intermediary bodies 2 and 3, output semicoupling 4 and the basis 0. Between the bodies there are the restrictions detailed in table 1 and figure 1. The mobility of the associated mechanism multibody system is [3, 6, 7]

$$M = 3(n_b - 1) - \sum g_c = 3(5-1) - 11 = 1, \tag{1}$$

where n_b=5 is the number of the associated mechanism bodies (including the basis) and Σg_c=11 is the number of the geometrical constraints between the bodies.



a. Coupling constructive scheme.

b. Structural scheme of the associated mechanism.

Figure 1. The Buchli mobile coupling

Table 1: Restrictions

Body i – Body j	Geometrical constraint	Joint	Number of restrictions
0 – 1	R	A	2
0 – 3	R	B	2
1 – 2	R	C	2
1 – 3	R	H	2
2 – 3	Gear sector	Gear sector	1
2 – 4	RR	EF	1
2 – 4	RR	GJ	1

For the mobile transversal coupling shown in figure 1, the geometrical and kinematical multibody model is presented in figure 2. The interest point’s coordinates are presented in table 2.

To define the geometrical restrictions, is necessary to know the point coordinates relative to the fixed coordinates system. The general relation of these coordinates is [7]:

$$\begin{bmatrix} x_{M_i} \\ y_{M_i} \end{bmatrix} = \begin{bmatrix} x_{0i} \\ y_{0i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_i & -\sin\varphi_i \\ \sin\varphi_i & \cos\varphi_i \end{bmatrix} \begin{bmatrix} r_{ix} \\ r_{iy} \end{bmatrix}, \qquad (2)$$

where i=1, 2, 3, 4, for mobile bodies.

Geometrical restrictions equations are obtained from the geometrical restrictions between the bodies and are presented as follow.

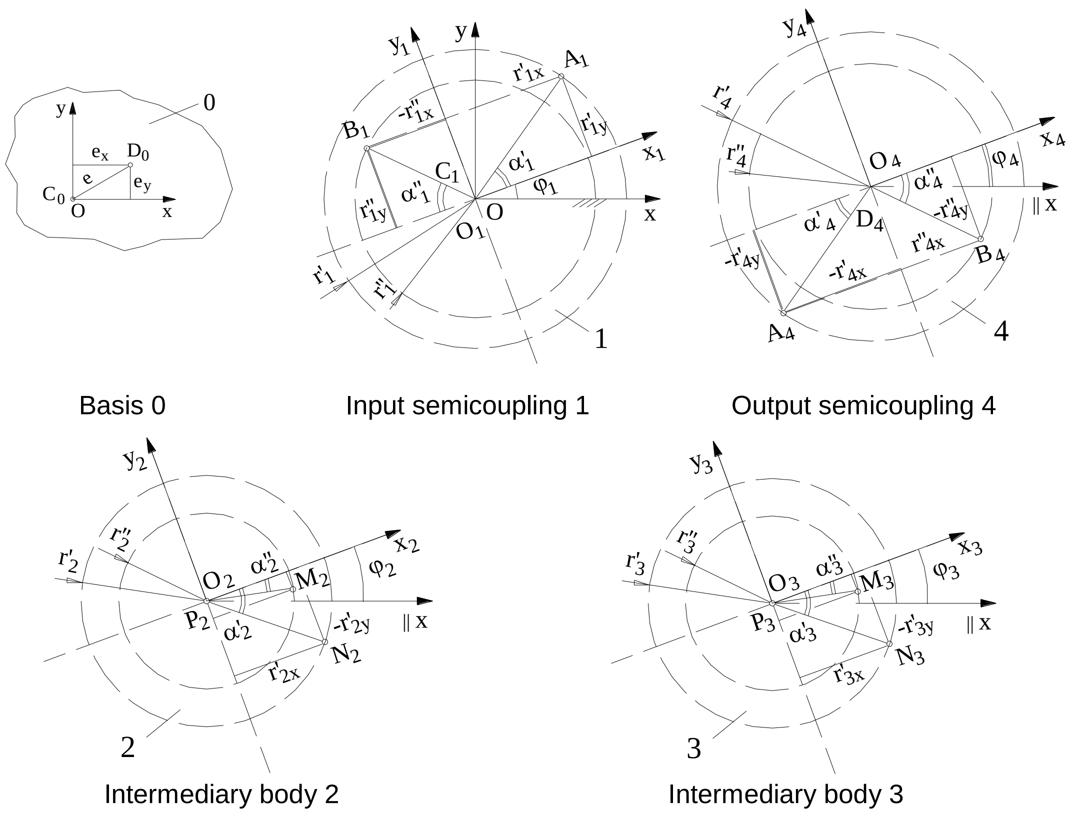


Fig. 2. The geometrical and kinematical multibody model (continued)

Table 2: Points coordinates relative to the local coordinate system.

Body	Points coordinates
Basis 0	$C_0(0, 0)$; $D_0(e_x, e_y)$.
Input semicoupling 1	$A_1(r'_{1x}, r'_{1y})$, $B_1(-r''_{1x}, r''_{1y})$, $C_1(0, 0)$, where $\begin{cases} r'_{1x} = r'_1 \cos\alpha'_1 \\ r'_{1y} = r'_1 \sin\alpha'_1 \end{cases}$ and $\begin{cases} r''_{1x} = r''_1 \cos\alpha''_1 \\ r''_{1y} = r''_1 \sin\alpha''_1 \end{cases}$
Intermediary body 2	$M_2(r''_{2x} - r''_{2y})$, $N_2(r'_{2x} - r'_{2y})$, $P_2(0,0)$ where $\begin{cases} r''_{2x} = r''_2 \cos\alpha''_2 \\ r''_{2y} = r''_2 \sin\alpha''_2 \end{cases}$ and $\begin{cases} r'_{2x} = r'_2 \cos\alpha'_2 \\ r'_{2y} = r'_2 \sin\alpha'_2 \end{cases}$
Intermediary body 3	$M_3(r''_{3x_1} - r''_{3y})$, $N_3(r'_{3x_1} - r'_{3y})$, $P_3(0,0)$ where $\begin{cases} r''_{3x} = r''_3 \cos\alpha''_3 \\ r''_{3y} = r''_3 \sin\alpha''_3 \end{cases}$ and $\begin{cases} r'_{3x} = r'_3 \cos\alpha'_3 \\ r'_{3y} = r'_3 \sin\alpha'_3 \end{cases}$
Output semicoupling 4	$A_4(-r'_{4x_1} - r'_{4y})$, $B_4(r''_{4x_1}, -r''_{4y})$, $D_4(0, 0)$ unde $\begin{cases} r'_{4x} = r'_4 \cos\alpha'_4 \\ r'_{4y} = r'_4 \sin\alpha'_4 \end{cases}$ unde $\begin{cases} r''_{4x} = r''_4 \cos\alpha''_4 \\ r''_{4y} = r''_4 \sin\alpha''_4 \end{cases}$

For the studied mobile transversal coupling multibody model, the generally geometrical restrictions equations are [2, 3, 6, 7]:

- for rotation $$P_i \equiv P_j \ , \tag{3}$$

- for rotation-rotation $$(x_{Pj} - x_{Pi})^2 + (y_{Pj} - y_{Pi})^2 = l^2 , \tag{4}$$

- for gearing $$v_{Pi} = v_{Pj} . \tag{5}$$

The kinematic restriction equation is

$$\rho_1 - f(t) = 0 . \tag{6}$$

Based on the relations (3)…(6), it is obtained an equation system with 12 equations and 12 unknowns: x_{01}, y_{01}, φ_1, x_{02}, y_{02}, φ_2, x_{03}, y_{03}, φ_3, x_{04}, y_{04}, φ_4. It results

$$x_{O1} = 0 \ ; \ y_{O1} = 0, \tag{7}$$

$$x_{O4} = e_x \ ; \ y_{O4} = e_y, \tag{8}$$

$$x_{P2} - x_{A1} = 0; \qquad y_{P2} - y_{A1} = 0 \tag{9}$$

$$x_{P3} - x_{B1} = 0; \qquad y_{P3} - y_{B1} = 0 \tag{10}$$

$$(x_{M_2} - x_{B_4})^2 + (y_{M_2} - y_{B_4})^2 = l_1'^2 \tag{11}$$

$$(x_{M_3} - x_{A_4})^2 + (y_{M_3} - y_{A_4})^2 = l_1''^2 \tag{12}$$

$$v_{N2} = v_{N3} \tag{13}$$

To resolve the equation system, it is necessary to take account by the constructive conditions, as follow:

- at the couplings of this type, the construction of semicouplings is symetric: $r_1' = r_1''$, respectively $r_4' = r_4''$ and $\alpha_1' = \alpha_1''$, respectively $\alpha_4' = \alpha_4''$.
- the linkages between the bodies 2 and 4 respectively 3 and 4 have the same length l_1' = l_1'' = l_1;
- the intermediary bodies 2 and 3 have the same dimensions $r_2'' = r_3''$ and $\alpha_2'' = \alpha_3''$, respectively $r_2' = r_3' = r_w$ (for gearing sector) and $\alpha_2' = \alpha_3'$;
- also, is necessary to be mentioned the particularity $M_2M_3 \parallel A_1B_1 \parallel A_4B_4$ and the length of this is $M_2M_3 = A_4B_4 = l_2$, for coupling eccentricity e=0.

Will result

$$\begin{aligned}&2x_{04}r_1'\left[\cos\left(\alpha_4'+\varphi_4\right)-\cos\left(\alpha_1'+\varphi_1\right)\right]+2y_{04}r_1'\left[\sin\left(\alpha_4'+\varphi_4\right)-\sin\left(\alpha_1'+\varphi_1\right)\right]+2x_{04}r_1''\left[\cos\left(\alpha_4''-\varphi_4\right)-\cos\left(\alpha_1''-\varphi_1\right)\right]+\\&+2y_{04}r_1''\left[\sin\left(\alpha_1''-\varphi_1\right)-\sin\left(\alpha_4''-\varphi_4\right)\right]-2r_2''r_4''\left[\cos\left(\varphi_2-\varphi_4-\alpha_2''+\alpha_4''\right)-\cos\left(\varphi_1-\varphi_3-\alpha_1''+\alpha_3''\right)\right]-\\&-2r_3''r_4'\left[\cos\left(\varphi_3-\varphi_4-\alpha_3''-\alpha_4'\right)-\cos\left(\varphi_1-\varphi_2+\alpha_1'+\alpha_2''\right)\right]-2r_1'r_4''\left[\cos\left(\varphi_1-\varphi_4+\alpha_1'+\alpha_4''\right)-\cos\left(\varphi_1-\varphi_4-\alpha_1''-\alpha_4'\right)\right]=0.\end{aligned} \quad (14)$$

For gearing sector between the intermediary bodies, relation (13) led to the gearing ratio [2, 4]

$$i=\frac{\omega_2}{\omega_3}=\frac{r_{w3}}{r_{w2}}=\frac{z_3}{z_2}=1, \quad (15)$$

for the considered constructive conditions $r_{w2}=r_{w3}=r_w$ (and also $z_2=z_3$).

Integrating, $\dot{\varphi}=\omega$, will result $\varphi_2=\varphi_3+C$. For C=0, will obtain

$$\varphi_2=\varphi_3 \quad (16)$$

Theoretically, relations (14) and (16) led to the coupling homokinetism condition

$$\varphi_4=\varphi_1. \quad (17)$$

But, in the real case, because of the gearing clearance,

$$\varphi_2\approx\varphi_3 \quad (18)$$

which led to the coupling quasihomokinetism condition

$$\varphi_4\approx\varphi_1. \quad (19)$$

3. RESULTS

For intermediary elements 2 and 3, with gear sector, taking account by the relation (15), and constructive particularities $r_2''=r_3''$ and $\alpha_2''=\alpha_3''$, respectively $\alpha_2'=\alpha_3'$, result some theoretical cases, as follow:

- case 1, figure 1, b, Buchli:

$$\alpha_2'\in\left(\frac{\pi}{2},\pi\right);\ \alpha_2''\in\left(\frac{3\pi}{2},2\pi\right)\ \text{and}\ \alpha_3'\in\left(\frac{\pi}{2},\pi\right);\ \alpha_3''\in\left(\frac{3\pi}{2},2\pi\right);$$

- case 2, figure 3, new constructive variant:

$$\alpha_2'\in\left(\frac{\pi}{2},\pi\right);\ \alpha_2''\in\left(\frac{\pi}{2},\pi\right)\ \text{and}\ \alpha_3'\in\left(\frac{\pi}{2},\pi\right);\ \alpha_3''\in\left(\frac{\pi}{2},\pi\right).$$

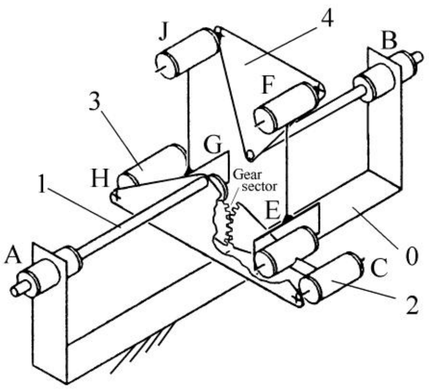


Figure 3. The mobile coupling new scheme

5. CONCLUSIONS

In the case of the studied mobile coupling as multibody system with four mobile bodies, the homokinetism is given by $\varphi_1 = \varphi_4$, in the condition $\varphi_2 = \varphi_3$, according to the constructive particularities relative to the geometry of the bodies. Specifying the coupling constructive condition is important here, because is useful in coupling's dynamic behavior and also in manufacturing process [1, 7].

Because of the gearing clearance, the real coupling became quasihomokinetic, and $\Delta\varphi = \varphi_2 - \varphi_3 \approx \text{const}$.

From this coupling type, as constructive variants, result the known Buchli coupling and a new constructive solution.

The method can be applied also for other known mobile couplings, as multibody systems with four or five mobile bodies and for other new solutions of mobile couplings, identified previously by the authors in structural analysis.

In the future researches, the authors intend to analyze these new particular cases and also to find their optimal shape configuration, in the design process.

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