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CONSIDERATIONS ON SHAVERS TEETH DEFORMATION

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Abstract: The precision of a gear tooth processed by shaving depends greatly on the dynamic stress of the shaver teeth and of the gear that are to be worked on. Following the shaver stress modelling, for example, its tooth deformation is determined by using the following approximations: *the tooth is considered as a beam embedded in the shaver body; the tooth section is variable by an arc and an involute, the involute being approximated by a parabolic spline function.*

The size that characterizes the profile deviation of the shaver tooth (the variable amount of distortions, extreme value), given the complex stress of its body and targeted purpose, is calculated with:

$$Y = (Y_S + Y_r)_{x=x_r} - (Y_S + Y_r)_{x=x_s} \tag{1}$$

Considering the shaver tooth as a beam embedded in its body (see Figure 1), with variable section (the section is variable by an arc and an involute), the equation of the deformed average fibre is given by:

$$\ddot{y} = -\frac{M_x}{E \cdot I_x} \tag{2}$$

where:

M_x –bending moment in section *x*,
E – elastic modulus,
I_x – moment of inertia in section *x*,

One can see that:

$$M_x = -F_t \cdot (h - x) \quad \text{şi} \quad I_x = \frac{B \cdot g^3(x)}{12} \tag{3}$$

where:

B – tooth width,
g(x) – tooth thickness at distance x from the base,

therefore equation (2) becomes:

$$\ddot{y}(x) = \frac{12 \cdot F_t}{E \cdot B} \cdot \frac{h - x}{g^3(x)} \tag{4}$$

Given the geometric model of a shaver tooth, we will attach to the differential equation (4), on each of the intervals: $[0, x_u]$, $[x_u, t]$, $[t, x_e]$ of the approximate profile of the tooth, an approximate differential equation which, later, will be integrated. The functions and constants index marks the considered interval for the differential equation (4).

Since the curve at the base of the shaver tooth consists of an arc, having the following equation:

$$(x - \rho)^2 + (y - t)^2 = \rho^2, \tag{5}$$

the tooth thickness at distance *x* from the base, $x \in [0, x_u]$, has the expression:

$$g(x) = 2 \cdot \left[b - \sqrt{x \cdot (2 \cdot \rho - x)}\right] \tag{6}$$

where *b* is half the distance between the centres of two neighbouring arcs.

For $x \in [0, x_u]$, the differential equation of the deformed average fibre is:

$$\ddot{y}_1(x) = K \cdot \frac{h - x}{\left[b - \sqrt{x \cdot (2 \cdot \rho - x)}\right]^2} \tag{7}$$

where $$K = \frac{3 \cdot F_t}{2 \cdot E \cdot B}$$

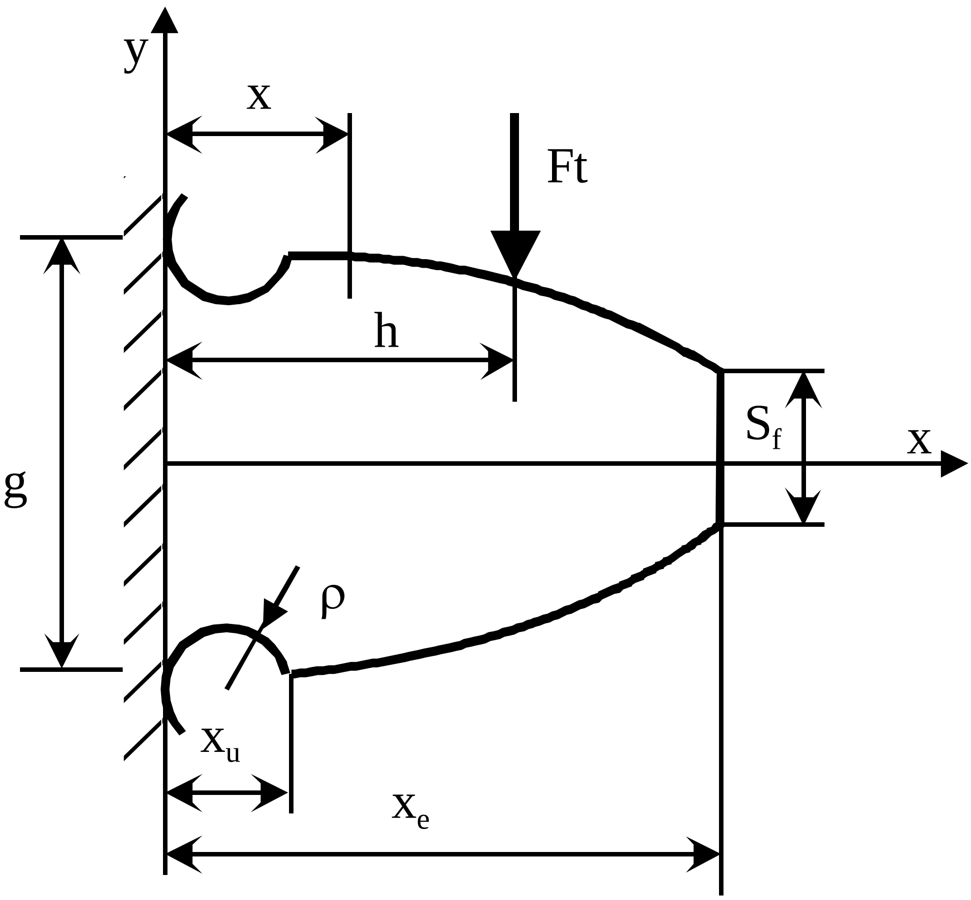

Figure 1. The approximation of a shaver tooth with an embedded beam

When integrating both sides of the equation (7) we get:

$$\dot{y}_1(x,h) = K \cdot \Phi_1(x,h) + k \cdot A_1(h) \tag{8}$$

where:

$$\Phi_1(x,h) = \frac{2\cdot\rho - b}{b^2}\cdot\frac{2\cdot\rho - x}{b-\sqrt{x\cdot(2\cdot\rho - x)}} + \frac{2\cdot\rho\cdot(2\cdot\rho - h) - b^2}{2\cdot b^2}\cdot$$
$$\cdot\left[\frac{2\cdot\rho - x}{b-\sqrt{x\cdot(2\cdot\rho - x)}}\right]^2 + \frac{3\cdot\rho\cdot(h-\rho)}{2\cdot(b^2-\rho^2)^2}\cdot\frac{b\cdot\sqrt{x\cdot(2\cdot\rho - x)} - \rho\cdot(2\cdot\rho - x)}{b-\sqrt{x\cdot(2\cdot\rho - x)}} +$$
$$+\frac{[b^2\cdot(3\cdot\rho - h) - 2\cdot\rho^2\cdot(2\cdot\rho - h)]\cdot(2\cdot\rho - x)}{2\cdot b^3\cdot(b^2-\rho^2)}\cdot\frac{b\sqrt{x\cdot(2\cdot\rho - x)} - \rho\cdot(2\cdot\rho - x)}{\left[b-\sqrt{x\cdot(2\rho - x)}\right]^2} +$$
$$+\frac{3\cdot b\cdot\rho^3\cdot(h-\rho)}{(b^2-\rho^2)^2\cdot\sqrt{b^2-\rho^2}}\cdot arctg\,\frac{b\cdot\sqrt{x\cdot(2\cdot\rho - x)} - \rho\cdot(2\cdot\rho - x)}{(2\cdot\rho - x)\cdot\sqrt{(b^2-\rho^2)}}.$$

By integrating relation (8), we get:

$$y_1(x,h) = 4\cdot\rho^2\cdot K\cdot\Phi_1(x,h) + K\cdot A_1(h) + 4\cdot\rho^2\cdot K\cdot B_1(h) \tag{9}$$

where:

$$\Phi_1(x,h)=\frac{1}{8\cdot\rho\cdot b\cdot\left(b^2-\rho^2\right)}\cdot\left[\frac{b^2-2\cdot\rho\cdot(2\cdot\rho-h)}{\rho}-\frac{b^2\cdot(3\cdot\rho-h)-2\cdot\rho^2\cdot(2\cdot\rho-h)}{\left(b^2-\rho^2\right)^2}\right]\cdot$$

$$\cdot\frac{\rho\cdot\sqrt{x\cdot(2\cdot\rho-x)}-b\cdot(2\cdot\rho-x)}{b-\sqrt{x\cdot(2\cdot\rho-x)}}+\frac{1}{4\cdot h\cdot\rho\cdot b^2\cdot\left(b^2-\rho^2\right)}\cdot$$

$$\cdot\left\{\frac{\rho\cdot\left[b^2\cdot(3\rho-h)-2\cdot\rho^2\cdot(2\rho-h)\right]}{\left(b^2-\rho^2\right)^2}-\left[b^2-2\cdot\rho\cdot(2\cdot\rho-h)\right]\right\}\cdot\frac{b\cdot\sqrt{x\cdot(2\cdot\rho-x)}-\rho\cdot(2\cdot\rho-x)}{b-\sqrt{x\cdot(2\cdot\rho-x)}}+$$

$$+\frac{\left[b^2\cdot(3\cdot\rho-h)-2\cdot\rho^2\cdot(2\cdot\rho-h)\right]\cdot\left[\rho^2+\left(b^2-\rho^2\right)\right]}{4\cdot\rho^2\cdot b^2\cdot\left(b^2-\rho^2\right)^2}\cdot\ln\frac{2\cdot\rho-x}{2\cdot\rho\cdot\left[b-\sqrt{x\cdot(2\cdot\rho-x)}\right]}+\frac{1}{\sqrt{b^2-\rho^2}}\cdot$$

$$\cdot\left\{\frac{2\cdot\rho-h}{\rho\cdot b^2}+\frac{3\cdot b\cdot(h-\rho)\cdot x}{4\cdot\left(b^2-\rho^2\right)^2}-\right.$$

$$-\frac{\left[b^2\cdot(3\cdot\rho-h)-2\cdot\rho^2\cdot(2\cdot\rho-h)\right]\cdot\left[2\cdot\left(b^2-\rho^2\right)^2-\rho^2\cdot\left(3\cdot b^2-2\cdot\rho^2\right)\right]}{4\cdot\rho^2\cdot b^2\cdot\left(b^2-\rho^2\right)^2}-$$

$$\left.-\frac{\left[b^2-2\cdot\rho\cdot(2\cdot\rho-h)\right]\cdot\left(b^2-2\cdot\rho^2\right)}{4\cdot\rho^2\cdot b^2}\right\}\cdot arctg\frac{b\cdot\sqrt{x\cdot(2\cdot\rho-x)}-\rho\cdot(2\cdot\rho-x)}{(2\cdot\rho-x)\cdot\sqrt{b^2-\rho^2}}+$$

$$+\frac{1}{4\cdot\rho}\cdot\left[\frac{3\cdot\rho\cdot(h-\rho)}{2\cdot\left(b^2-\rho^2\right)^2}-\frac{2\cdot\rho-h}{b^2\cdot\rho}\right]\cdot\sqrt{x\cdot(2\cdot\rho-x)}+\frac{1}{8\cdot\rho^2}\cdot\left[\frac{3\cdot b\cdot\rho\cdot(h-\rho)}{\left(b^2-\rho^2\right)^2}-\right.$$

$$\left.-\frac{\rho\cdot\left[b^2\cdot(3\cdot\rho-h)-2\cdot\rho^2\cdot(2\cdot\rho-h)\right]}{b^3\left(b^2-\rho^2\right)^2}-\frac{b^2-2\cdot\rho\cdot(2\cdot\rho-h)}{b^3}\right]\cdot(2\cdot\rho-x)-$$

$$-\frac{1}{4\cdot b}\left[\frac{2\cdot\rho-h}{\rho^2}+\frac{b^2-2\cdot\rho\cdot(2\cdot\rho-h)}{b^2\cdot\rho}+\frac{b^2\cdot(3\cdot\rho-h)-2\cdot\rho^2\cdot(2\cdot\rho-h)}{b^2\cdot\left(b^2-\rho^2\right)^2}\right]\cdot\ln\frac{2\cdot\rho-x}{2\cdot\rho}+$$

$$+\frac{1}{2\cdot\rho}\cdot\left[\frac{1}{\rho}-\frac{3\cdot(2\cdot\rho-h)}{b^2}-\frac{3\cdot(h-\rho)}{2\cdot\left(b^2-\rho^2\right)}\right]\cdot arctg\sqrt{\frac{x}{2\cdot\rho-x}}$$

The constants of integration $A_1(h)$ and $B_1(h)$ are determined from the boundary conditions:

$$y_1(0,h)=0,\quad \dot{y}_1(0,h)=0 \tag{10}$$

obtaining:

$$A_1(h)=-\psi_1(0,h); \tag{11}$$

$$B_1(h)=-\Phi_1(0,h) \tag{12}$$

If the involute profile of the shaver tooth is approximated by a *parabolic spline function*, i.e. on each of the intervals $[x_u,t]$ and $[t,x_e]$ the thickness of each tooth at a distance *x* from the base has the expression $g(x)=2\cdot\left(a\cdot x^2+b\cdot x+c\right)$ where the coefficients *a, b, c* are polynomial coefficients of $B_2(l_1)$ for $x\in[x_u,t]$, respectively polynomial coefficients of $B_2(l_2)$ for $x\in[t,x_e]$.

Therefore, for $x\in[x_u,t]$ and also for $x\in[t,x_e]$, the differential equation for the deformed average fibre will be:

$$\ddot{y}(x) = K \cdot \frac{h - x}{\left(a \cdot x^2 + b \cdot x + c\right)^3} \tag{13}$$

By integrating the differential equation (13) we get:

$$\dot{y}(x, h) = K \cdot \psi(x, h) + A(h) \tag{14}$$

where, using the notation $D = 4 \cdot a \cdot c - b^2$,

$$\psi(x, h) = \frac{1}{D} \cdot \left[\frac{(2 \cdot a \cdot h + b) \cdot x + b \cdot h + 2 \cdot c}{2 \cdot \left(a \cdot x^2 + b \cdot x + c\right)^2} + \frac{3 \cdot (2 \cdot a \cdot h + b) \cdot (2 \cdot a \cdot x + b)}{2 \cdot D \cdot \left(a \cdot x^2 + b \cdot x + c\right)} + \right.$$

$$\left. + \frac{6 \cdot a \cdot (2 \cdot a \cdot h + b)}{D} \cdot \begin{cases} \dfrac{1}{\sqrt{D}} \cdot arctg \dfrac{2 \cdot a \cdot x + b}{\sqrt{D}}, & if \ D > 0 \\ \dfrac{1}{\sqrt{-D}} \cdot A \cdot r \cdot t \cdot h \cdot \dfrac{2 \cdot a \cdot x + b}{\sqrt{-D}}, & if \ D < 0 \end{cases} \right]$$

respectively:

$$\psi(x, h) = \frac{4 \cdot a}{(2 \cdot a \cdot x + b)^4} \cdot \left[1 - \frac{4 \cdot (2 \cdot a \cdot h + b)}{5 \cdot (2 \cdot a \cdot x + b)} \right], \qquad if \ D = 0.$$

then:

$$y(x, h) = K \cdot \Phi(x, h) + A(h) \cdot x + B(h) \tag{15}$$

where:

$$\Phi(x, h) = \frac{1}{2 \cdot D^2} \cdot \left[\frac{D \cdot (x - h)}{a \cdot x^2 + b \cdot x + c} + 3 \cdot (2 \cdot a \cdot h + b) \cdot \ln\left(a \cdot x^2 + b \cdot x + c\right) - \right.$$

$$- 3 \cdot (2 \cdot a \cdot h + b) \cdot \ln\left[1 + \frac{(2 \cdot a \cdot x + b)^2}{D} \right] +$$

$$\left. + 2 \cdot \begin{cases} \sqrt{D} + \dfrac{3 \cdot (2 \cdot a \cdot h + b) \cdot (2 \cdot a \cdot x + b)}{\sqrt{D}} \cdot arctg \dfrac{2 \cdot a \cdot x + b}{\sqrt{D}}, & if \ D > 0 \\ -\sqrt{-D} + \dfrac{3 \cdot (2 \cdot a \cdot h + b) \cdot (2 \cdot a \cdot x + b)}{\sqrt{-D}} \cdot A \cdot r \cdot t \cdot h \cdot \dfrac{2 \cdot a \cdot x + b}{\sqrt{-D}}, & if \ D < 0 \end{cases} \right]$$

respectively:

$$\Phi(x, h) = \frac{2}{(2 \cdot a \cdot x + b)^3} \cdot \left[\frac{2 \cdot a \cdot h + b}{5 \cdot (2 \cdot a \cdot x + b)} - 1 \right], \qquad if \ D = 0$$

Using now the indexed functions and constants of integration for $x \in [x_u, t]$, it follows:

$$\dot{y}_2(x, h) = K \cdot \psi_2(x, h) + A_2(h) \tag{16}$$

and

$$y_2(x, h) = K \cdot \Phi_2(x, h) + A_2(h) \cdot x + B_2(h) \tag{17}$$

and for $x \in [t, x_e]$, it follows:

$$\dot{y}_3(x, h) = K \cdot \psi_3(x, h) + A_3(h) \tag{18}$$

and

$$y_3(x, h) = K \cdot \Phi_3(x, h) + A_3(h) \cdot x + B_3(h) \tag{19}$$

The constants of integration $A_3(h)$ and $B_3(h)$ will be obtained from the conditions:

$$\dot{y}_2(t,h)=\dot{y}_3(t,h); \quad y_2(t,h)=y_3(t,h) \tag{20}$$

getting:

$$A_3(h)=K\cdot\{[\psi_2(t,h)-\psi_3(t,h)]+[\psi_1(x_u,h)-\psi_2(x_u,h)]-\psi_1(0,h)\} \tag{21}$$

respectively,

$$\begin{aligned}B_3(h)=\{&\Phi_2(t,h)-\Phi_3(t,h)+h\cdot\rho^2\cdot\Phi_1(x_u,h)-\Phi_2(x_u,h)-\\&-t\cdot[\psi_2(t,h)-\psi_3(t,h)]-x_u\cdot[\psi_1(x_u,h)-\psi_2(x_u,h)]-4\cdot\rho^2\Phi_1(0,h)\}\end{aligned} \tag{22}$$

Since in the application point of the force $x=h$, from relations (17) and (19), for computing the deformation resulted from shaver tooth deflection as a consequence of nominal shear force at the distance $h\in(x_u, x_e)$ from the tooth base, we get:

$$y(h)=\begin{cases}K\cdot\Phi_2(h,h)+A_2(h)\cdot h+B_2(h), & h\in(x_u,t)\\ K\cdot\Phi_3(h,h)+A_3(h)\cdot h+B_3(h), & h\in(t,x_e)\end{cases} \tag{23}$$

To take into account the effect of cutting channels by altering the section, on the interval $[x_u, x_e]$ from the tooth thickness, we subtract the depth of the cutting channels (cutting channels are on the both sides of the same tooth).

As shown, the nominal normal force that determines the strength of the cutting force stresses the shaver tooth with shock at zero height. As such, the values of the average deformed fibre, obtained from relation (23) will be multiplied by a factor of 2.

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