ANNALS of the ORADEA UNIVERSITY.
Fascicle of Management and Technological Engineering, Volume X (XX), 2011, NR2

THE ANALYSIS OF THE UNDULATIONS INFLUENCE OVER THE DEFORMATION STATE OF A LOADED ELASTO-PLASTIC POINT CONTACT

Alexandru Potorac, Dorel Prodan, Luminiţa Irimescu, Florina Ciornei
"Stefan cel Mare" University of Suceava

Keywords: undulations, stress state, elastic deformations, plastic deformations, point contact

Abstract: The materials behaviour in real loaded contacts is elasto-plastic. At repeated loading and unloading the phenomenon of cold-hardening appears. The contact surfaces aren't smooth, having a microtopography that can be modelled, in a first stage, by undulations of a certain length and of a certain depth.

All these were taken into consideration when realizing a numerical calculus program for the determination of the deformation state of a normally loaded elasto-plastic point contact .

1.General Considerations

The elasto-plastic behaviour is characterized by the apparition of irreversible deformations that start from a certain level of stress. During the traction-compression stress there can be observed the phenomena illustrated in Figure 1, [1]:

-initially, the behaviour is elastic as long as the stress is situated between the limit of elasticity to traction σ_t and the elasticity limit to compression σ_c;

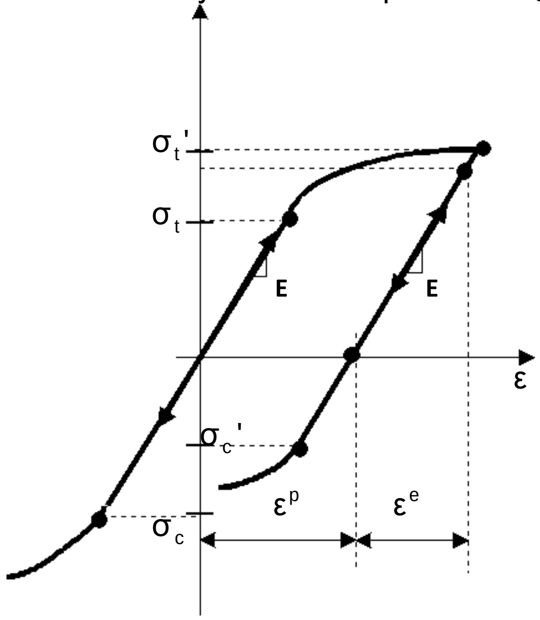

Figure 1: Phenomenological aspects of the plasticity during a traction-compression strain

- when the stress exceeds these limits, the behaviour follows a different law of evolution;
- if the test bar is loaded to a value $\sigma_t' > \sigma_t$ and it's discharged, the return is produced purely elastic, with the initial elastic properties, in rest state registering a permanent (plastic) deformation ε_p;
- if, in this new state of rest, a new elongation stress is applied, the elastic domain will expand until the value σ_t', the maximum value of the charge from the anterior cycle. This modification of the elasticity limit is called cold-hardening.
- if a new stress is of compression, the new limit of elasticity to compression is smaller than its initial value $\sigma_c' < \sigma_c$. This phenomenon bears the name of Bauschinger effect.

-In the new elastic field, $\sigma'_c < \sigma < \sigma'_t$, the independence of the elastic properties towards the plastic behaviour justifies the division of the total deformation ε^t in an elastic part ε^e and in a plastic part ε^p:

$$\varepsilon^t = \varepsilon^e + \varepsilon^p. \quad (1)$$

For an uniaxial stress, the relationship between the stress and the deformation has the following form:

$$\sigma = E\,(\varepsilon^t - \varepsilon^p). \quad (2)$$

These phenomena, especially the cold-hardening, aren't general for all the materials. There are elastic-perfectly plastic materials that don't present the cold-hardening, Figure 2.

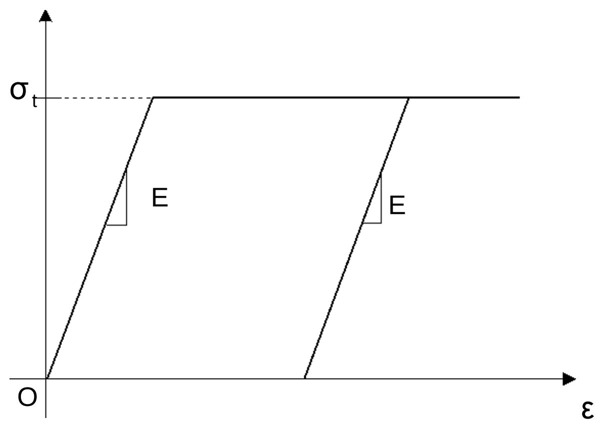

Figure 2: The elastic-perfectly plastic behaviour.

The elasticity limit isn't changed and the deformation grows even for $\sigma = \sigma_t$ = ct. The plasticity is, in fact, a special representation of the non linear behaviour of the materials. It is a limit case of the viscoelastic behaviour, where the permanent deformations depend on time and, hence, on the stress speed [1].

2.The numerical determination of the pressure distribution, in elasto-plastic point contacts, between undulated surfaces

The pressure distribution on the contact area directly influences the apparition and the development of the plastically deformed spaces inside the bodies. On their turn, the plastic deformations affect the pressure and determine smaller values compared to the case in which the bodies are considered perfectly elastic.

Tabor considers that there are plastic contact deformations when the maximum contact pressure exceeds three times the flowing limit at the uniaxial stress of pure traction σ_y:

$$\Sigma_{max} = 3\,\sigma_y \approx 6k, \quad (3)$$

where k represents the flowing limit at stress of pure shearing.

For the totally hardened steal bearing, the flow limit at the uniaxial stress of pure traction is located in the field 1850- 2200 MPa, according to the drawback temperature, that corresponds to a maximum contact pressure σ_{max} = 5550 ÷ 6600 MPa. Under these conditions, the elastic model for the calculus of the distribution of the contact pressure, previously developed, must be modified correspondingly.

The limitation of the maximum contact pressure σ_0 at the value $\sigma_0 = 3\sigma_y$ is a solution which is frequently adopted in these type of problems. This simplifying hypothesis corresponds to the perfectly elasto-plastic materials.

But, because of the cold-hardening phenomenon it is obvious that after the exceed of the limit $3\sigma_y$, called flow pressure on the contact surface, the pressure cannot stay constant (only in the case of the elastic-perfectly plastic bodies, [2]) and varies accordingly to the deformable body model used.

Mayeur, studied the behaviour of some steel bearing test bars in the elasto-plastic field by using a linear cold-hardening model and the Huber-Mises-Hencky plasticity criteria. He estimates the real contact pressure by a linear function of Hertzian pressure σ_0:

$$\sigma_{real} = \sigma_{max} + K\,(\sigma_0 \;\; \sigma_{max}), \qquad (4)$$

where K is a coefficient that depends on the plasticity modulus h of the material.

For steel bearings equivalent to RUL 1, the plasticity modulus used by researchers has the value of 188 MPa that corresponds to a value $K \approx 0{,}80$.

The paper is based on personal programs used for the evaluation of the pressure distribution for two models, [3]:
-elastic-perfectly plastic;
-elasto-plastic.

The results obtained will be used for the determination of the stress state and of the deformations inside the bodies.

The method of analysis of the pressure distribution and of the real contact surface is based on the elastic semispace method, previously presented.

If the maximum contact pressure is limited according to the relation (3) for the elastic-perfectly plastic bodies, respectively according to the relation (4), for elasto-plastic bodies with linear or non linear cold-hardening, there are two possibilities of determining the contact pressure:
- the modification of the original undeformed geometry of the contact bodies, until the pressures are smaller or equal to the flow pressure;
- the consideration of the pressure to be equal to the flow pressure and the recalculation of the pressure distribution until all the pressures satisfy the contact conditions:

$$p_{ij} \le \sigma_{max} \text{, inside the contact area;} \qquad (5)$$
$$p_{ij} = 0\text{, outside the contact area.}$$

The numerical method developed hereinafter uses a combination of the two mentioned possibilities. Thus, it will be obtained the pressure distribution and the contact area without being necessary other hypothesis referring to the modification of the bodies' geometry due to the plasticity phenomena.

The comparison between the obtained results is presented in Figure 3.

MPa

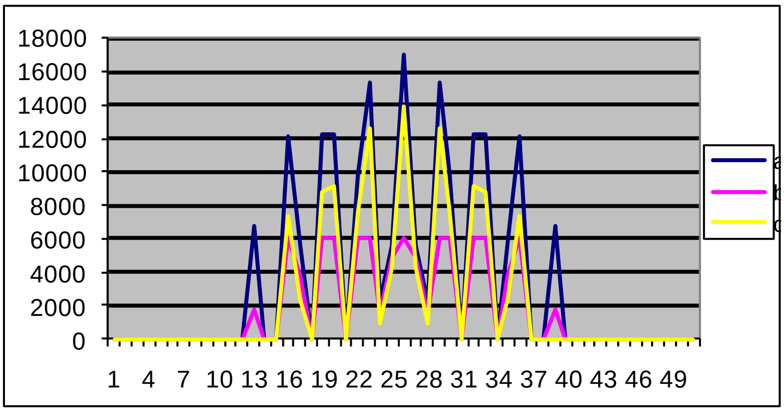

Figure 3. The comparison of the pressure distribution:
a. elastic, b. elastic-perfectly plastic, c. Elasto-plastic with cold-hardening

3. The numerical determination of the pressure distribution, in elasto-plastic point contacts, between undulated surfaces

The metallic materials submitted to a charge that exceeds the elasticity limit doesn't present anymore a linear relation between the stress and the deformations. The schematic representations can use, in this case, either two straight-lines (for elasto-plastic bodies with linear cold-hardening) or a non linear relationship, Ramberg-Osgood type, Figure 4:

$$\varepsilon = \frac{\sigma_E}{\varepsilon} + \frac{\sigma^N}{B}, \qquad (6)$$

where: σ_E – elasticity limit;
σ – the strain equivalent after the Von Mises criteria;
B, N - material constants.

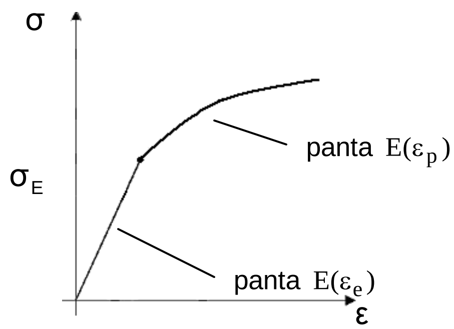

Figure 4: The strain-deformation relation for elasto-plastic bodies with non linear cold-hardening

The consideration of the variable rigidity of the materials, in case of a load which exceeds the elasticity limit, can be made by two methods:
- by using an iterative solution, where starting from the initial rigidity of the material, that is from the elasticity modulus that characterizes the linear portion of the stress-deformation curve, it can finally obtain to the values of the stresses and deformations given by the real mode of behaviour of the material in the plastic field;
- by using some linear relations between the stress increments and the deformation increments - this method is called the incremental theory of the plasticity. It allows the

extension of the linear elasticity theory in the plastic field and will be used hereinafter because of the relative easiness in making the calculations.

Based on the specially designed calculation program, it was obtained the remanent deformation for the case of the elastic-perfectly plastic bodies and the elasto-plastic ones with linear cold-hardening, respectively non linear one. [3].

The comparison between the obtained results is presented in Figure 5.

um

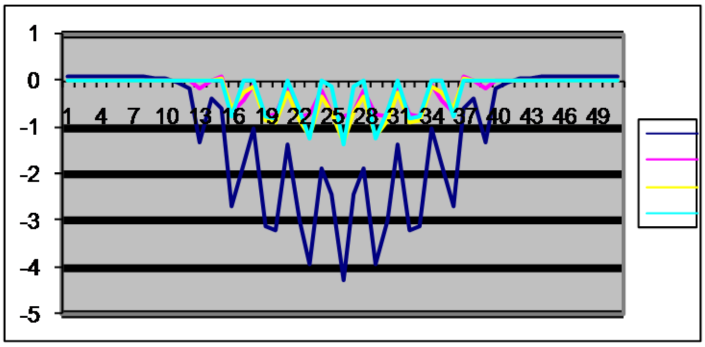

Figure 5: The Comparison between the axial profile of the remanent deformations and the elastic deformation:
a-elastic deformation; b-remanent elastic-perfectly plastic deformation; c-remanent deformation for the linear cold-hardening case; d. the remanent deformation for the non linear cold-hardening.

4.Conclusions

- the presence of undulations on contact surfaces creates local maximum pressures; On their turn, these create intense stresses under the contact surface that produce plastic deformation even at low Hertzian pressures.
- the axial profiles of the remanent deformations have a complementary rate of curve with the axial profiles of the corresponding pressure distribution;
- the values of the elastic deformations exceed a few times the values of the remanent elasto-plastic deformations with linear, respectively non linear cold-hardening;
- even if the pressure distribution was considered the same, there is a difference between the elasto-plastic remanent deformations with linear cold-hardening, respectively non-linear in what regards the form and their values.

The programs realized inside this paper will be used for realizing a complex soft for the calculation of the pressure distribution and of the stress and deformations state in a three-dimensional circular contact between undulated or rough surfaces, statically loaded in elasto-plastic domain. It will be used, in the first time, for realizing a detailed study of the influence of the undulations, of the cold-hardening type of the materials and of the random component upon the contact. After that, it will be developed and used to analyze some contacts between real surfaces in order to study the influence of the microtopography upon the contact's elements.

Bibliography

[1] MAYEUR, C., - Modelisation du contact rugueux elastoplatique. Thèse de docteur- ingénieur (The modeling of the elasto-plastic rugose contact), INSA Lyon, 1995, p.179
[2] CHAABA, A., BOUSSHINE, L., SAXCE, G., - Kinematic Limit Analysis of Nonassociated Perfectly Plastic Material by the Potential Approach and finite Element Method, Transactions of the ASME, Journal of Applied Mechanics, May 2010
[3] PRODAN, D., POTORAC A., - Numerical Solution in 3D for the Normal Elliptic Contact of the Undulated Surfaces, VAREHD 9, Suceava, June 1988
[4] PRODAN, D., DIACONESCU, E.- The Influence of the Undulations and of the Material Cold-Hardening on the Technical Dry Circular Contact in the Elasto-Plastic Field, VAREHD 11, April 2003.