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APPROACH TO THE TEST OF FLEXURAL STRENGTH OF THE TEETH OF GEARS WITH ASYMETRIC TEETH

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Key words: gears with asymmetric teeth, gear teeth flexural strength test, test system, restraint section of teeth of gears with asymmetric teeth.

Abstract: When sizing the gears with asymmetric teeth, the tooth restraint section must be set, and a correction factor that takes into account the asymmetry must be inserted into the standard calculation formula for the resistance to flexural strength of the gear tooth. This factor must be determined in practice.

In order to find the restraint section and the correction factors, there has been designed a system which simulates the rack gearing on a pull-and-compression test machine. This is how we will study the tooth charge and its breakage.

1. Introduction. Presentation of gears with asymmetric teeth

In order to improve performance of cylindrical gears, there are successful attempts for using gears with asymmetric teeth, with different properties for each flank, depending on the rotation direction.

Improving the gearing conditions of one of the flanks is done in relation to the opposite flank, which can be made with most of the gears, because gears generally rotate inside industrial machinery in one direction only, and there are reversing gears for obtaining a reverse rotation.

For example, gears in the gearboxes of the causeway equipment, internal combustion engines, water or gas turbines, wind turbine gearboxes etc. have only one active rotational direction. Following this direction, one flank of the gear is active and only this flank is enough for ensuring an optimized quality, with respect of gearing. One of the most important aspects is that a tooth with a high mechanic resistance can be imposed in order to be able to optimize the transmission as much as possible.

Gears with asymmetric teeth are involute flanks with different base circles, which confer the gear the specificity of having two different pressure angles depending on the direction of rotation.

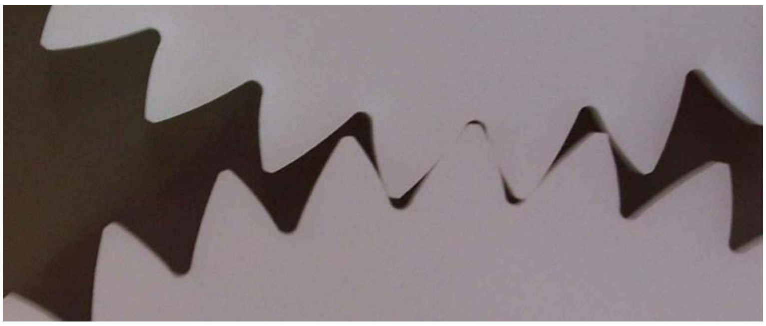


Fig. 1. Gears with asymmetric teeth

All these being gears and gearings still under research, a uniquely accepted and standardized symbol notation does not exist yet. In terms of notation used for gear flanks, Kapelevich [3] uses the terms ‘*coast involute profile’ and ‘drive involute profile*’, but which of the flanks will be the active one is not specified.

From the point of view of determining the optimal direction, the gearing will be obtained with a *'coast involute profile'* or a *'drive involute profile'*.

Depending on the angle of the reference rack we agree with the following conventions:

- The normal flank, without indicative, is the specific flank for conventional gears with symmetric teeth, where the reference gear rack angle and the generator gear rack angle are equal for the two flanks, usually in correlation with the standardized values.
- The modified flank, noted with indicative m, is the specific flank for a gear with asymmetric teeth, where the value of the angles of the base/reference flank and of the generating flank is a different value as compared to the value specific to the normal/common flank, with standardized values. The flank m+ is modified positively (increasing) and the flank m- is modified negatively (decreasing) as compared to standardized values.

The asymmetric teeth of the gear are defined by two involutes generated on two different basic circles (Fig.2). The operating pitch circle which, in case of tooting with no addendum modification coincides with the pitch circle, is the same for both flanks. Therefore, the pressure angles on both tooth flanks of the tooth will be different. In the case of gearing with no addendum modification, the relation between the basic circles, the pitch circle and the pressure angle is as follows:

$$d_d = \frac{d_b}{\cos\alpha} \quad \text{or} \quad d_b = d_d \cdot \cos\alpha \qquad (1.)$$

d_d - Reference diameter

d_b - Base circle diameter

α - Pressure angle

The reference diameter is equal for the two involutes, so we can write:

$$d_{b\_m+} = d_d \cdot \cos\alpha_{m+} \qquad (2.1.)$$
$$d_{b\_m-} = d_d \cdot \cos\alpha_{m-}$$
$$d_d = \frac{d_{b\_m+}}{\cos\alpha_{m+}} = \frac{d_{b\_m-}}{\cos\alpha_{m-}} \qquad (2.2.)$$

d_d - reference diameter

d_{b_m+} - base circle diameter for the flank with pressure angle m+

d_{b_m-} - base circle diameter for the flank with pressure angle m-

α_{m+} - pressure angle m+

α_{m-} - pressure angle m-

The asymmetry coefficient k is introduced, as the gear's invariant:

$$k_{\frac{m-}{m+}} = \frac{\cos\alpha_{m-}}{\cos\alpha_{m+}} = \frac{d_{b\_m-}}{d_{b\_m+}} \quad \text{or} \quad k_{\frac{m+}{m-}} = \frac{\cos\alpha_{m+}}{\cos\alpha_{m-}} = \frac{d_{b\_m+}}{d_{b\_m-}} \qquad (3.)$$

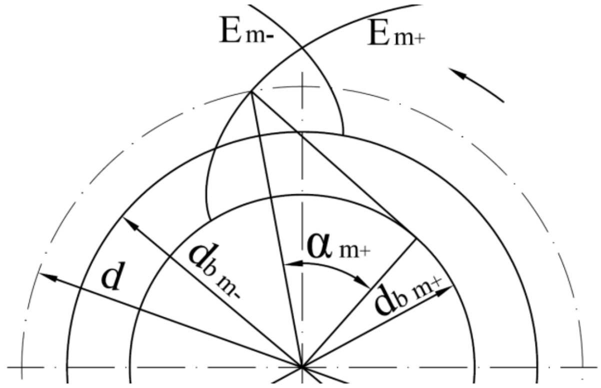


Fig. 2. Characteristic elements of the gear with asymmetric teeth

2. Stress calculation for gears with asymmetric teeth

Bending stress calculation for the gears with symmetric teeth is standardized and is presented in detail in STAS 12268, DIN 3990, and in the international standard ISO 6336.

- The above mentioned standards perform the stress calculation by taking into consideration only the bending of a subsequent tooth making corrections by using the contact ratio coefficient.

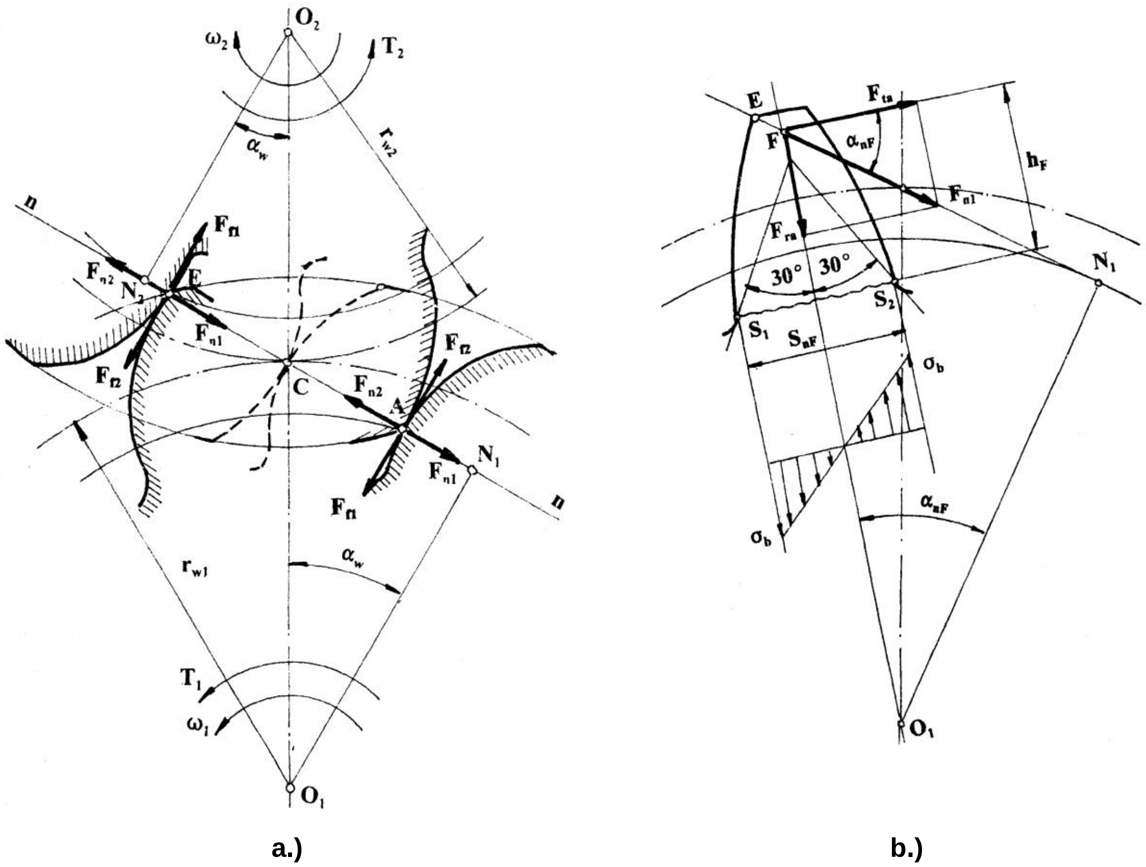


Fig.3. Hypotheses regarding stress calculation for standardized gears (with symmetric teeth)

The calculation is based on the fact that the gear tooth is bent at maximum values in the moment when the point of contact between teeth reaches the point "E" (Fig.3) at recess, or point “A” on approach, that is when the nominal force “F_n” acts on the tooth top [1].

The tooth is seen as a shaped recessed beam clamped in the gear body and charged with normal force “F_n”. During calculation process, the following assumptions are made: the force is applied to the tooth top and is taken over by one tooth only, and the restraint section ”s_F”, the dangerous section is where the lines are laid out under an angle of 30 ° in relation to the tooth axis are tangent to the contact profile, i.e. the "S_1S_2” area [1,2].

Highlighting the loading of the tooth, the normal force "Fn" moves on its working line to the "F" point and splits into the radial force ”F_{ra}”, whose effect is ignored, and in the tangential force ”F_{ta}”, which will bend the tooth root.

The flexural unit stress is:

$$\sigma_b = \frac{F_{ta} \cdot h_F}{\frac{S_{nF}^2 \cdot b}{6}} \tag{3.1.}$$

Or, in the case of gears with asymmetric teeth, the (3.1) formula changes into:

$$\sigma_b = k_\sigma \cdot \frac{F_{ta} \cdot h_F}{\frac{S_{nF}^2 \cdot b}{6}} \tag{3.2.}$$

where: k_σ – correction coefficient of the asymmetric tooth

Due to the asymmetric shape of the teeth of gears with asymmetric teeth, the sizing (testing/checking) relation – presented in (rel.3.1.) and used for the gears with symmetric teeth for being applicable to the gears with symmetrical and asymmetrical teeth – must be completed with some correction coefficients which take into account the asymmetric shape of the toothing when performing the stress calculation of the tooth and when checking the resistance to contact pressure (Hertzian pressure).

For determining the correction coefficients, the restraint section of the asymmetric tooth must be set.

For this study, we have developed a test method and device with which you can analyze the behavior of the tooth and the maximum load that makes the tooth break.

3. Method and device for testing the flexural strength of the gear teeth

Application

Study of the breakage zone of the teeth belonging to cylindrical gears with symmetric or asymmetric profile.

Operating principle and components of the testing device

The testing device presented in Fig.4. is contrived and designed to be used on the presses that perform tests under compression.

The device consists of the motherboard "1" which has abutments located on it:

- sample abutment "2";
- rack abutment "5"

Sample “4” is placed in the sample abutment and secured with a vice grip type device, named fixation frame “6”. In the rack abutment “2” the single toothed rack (in this case) slips over. If considered and needed, multiple toothed racks can be used, but the test sample must be properly designed. The deformation head of the press will press down the top of the rack. By using the monitoring system of the press, the behavior of the tooth being bent can be watched in accordance with the tangential component variation of the engagement force, then the diagram is drawn up.

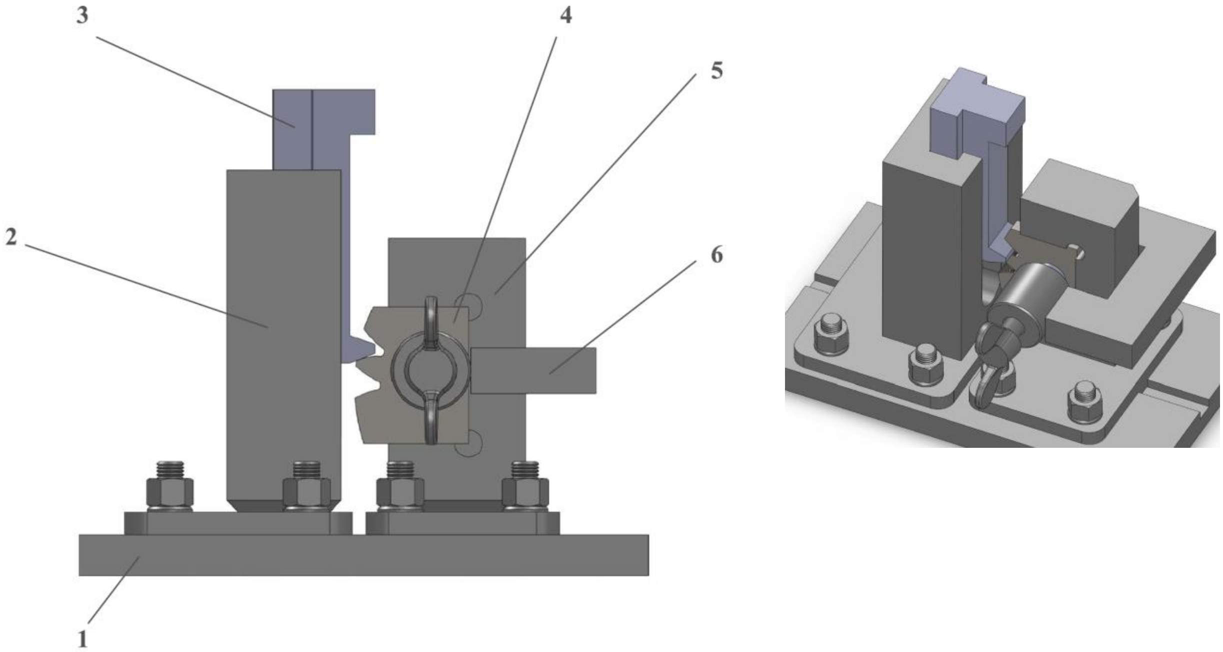


Fig. 4. Device for testing the flexural strength of the gear teeth

The functioning of the testing device is based on the gearing between a gear and a rack. During its functioning, the apparatus blocks the gear, and the rack - being activated and designed to last - will break the gear tooth by its movements. This is the way stress is shaped until the tooth breaks.

For an easy stiffening and blocking of the tested gear toothing, a single tooth is tested, as presented in chapter "Test sample shape" and Fig. 5.

Tooth loading speeds are standardized and used for materials tensile tests or speeds used in the adjustment range of the pull-and-compression test machines.

4. Test sample shape

The shape of the test sample used within the presented test method is obtained keeping in mind that the tooth of the gear is maximally bent when the place of contact reaches the “E” position. In this point, the nominal force ”F_n” actuates the tooth top (fig.3., fig.5.).

The test sample objectifies an entire tooth, namely: one at a time of a preceding tooth flank and one at a time of a subsequent tooth flank.

In the point "E", the tangent to the basic circle of the involute defining the tooth flank is perpendicular to the rack side and the tangent passes through the pitch point "C".

The gauge dimensions of the test sample “A_E” and “B_E” are set according to the dimensions of the testing device. These may be modified in conformity with the test characteristics and with the test machine.

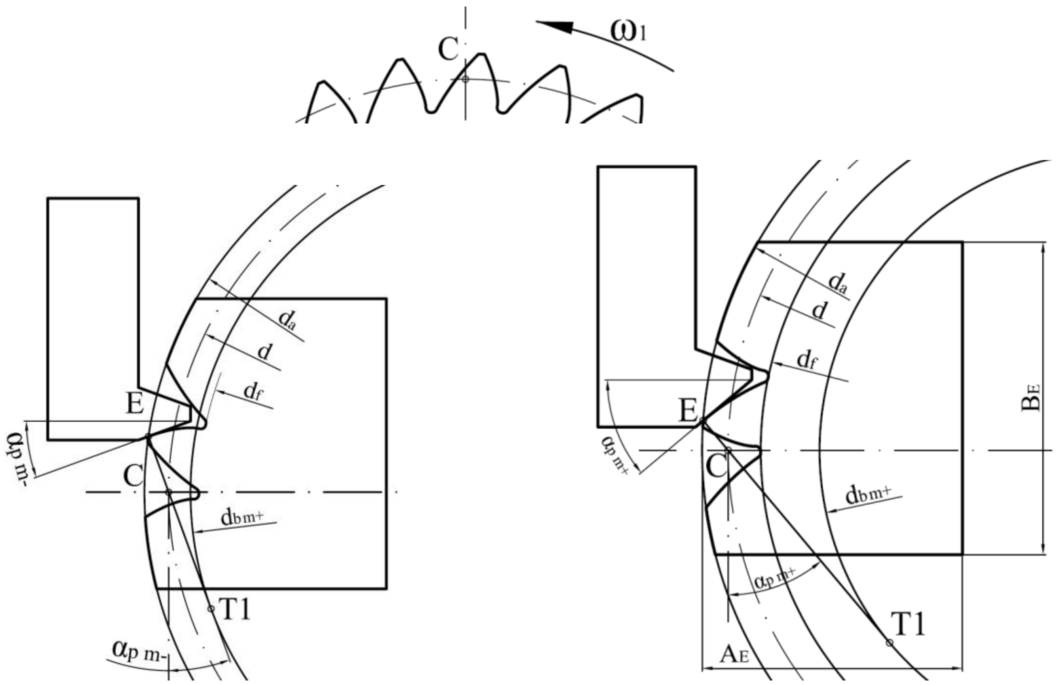


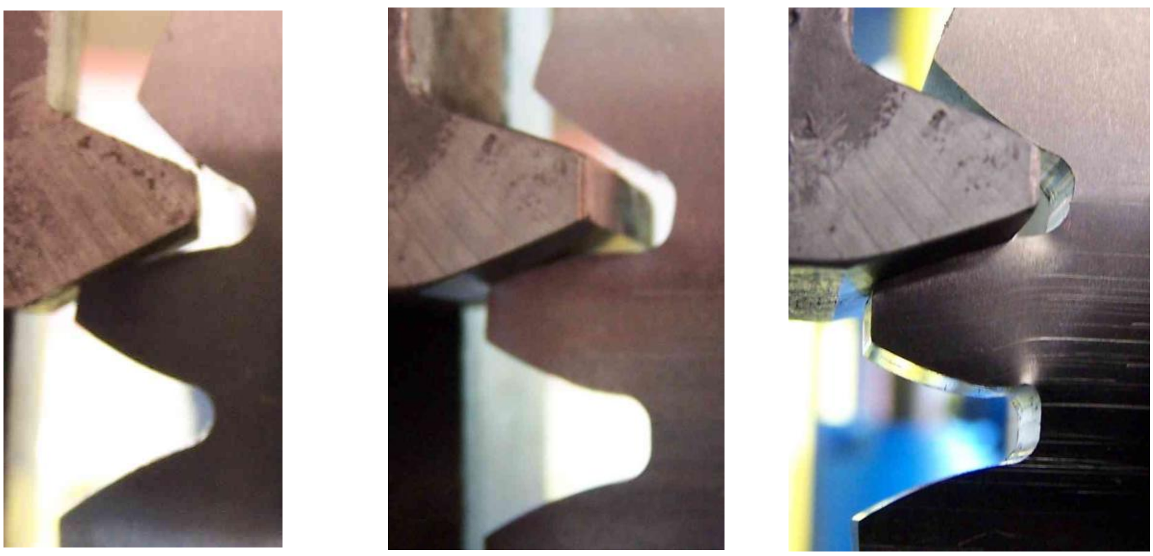
Fig. 5. The test sample shape for the flexural tests

5. Breaking kinematics of the tooth in the straight-toothed gears with symmetrical and asymmetrical teeth.

During the test, the following breaking areas have been identified in the fillet diagram:

1. – area of elastic deformation of the tooth;
2. – area of occurrence of first plastic deformations of the tooth in the fillet area;
3. – area of occurrence of plastic deformations on the tooth flank;
4. - tooth brocks / cut.

According to carried out tests, the tooth shearing occurs at the top of the tooth rack.



Stage 1. **Stage 2.** **Stage 3.**

Fig. 6. Stages of tooth damaging

Stage 1 - natural behavior of the tooth during its gearing. The tooth charge can be seen in the diagram presenting the variation of pressure force on the rack (the tangential component of the gearing force). In this area, the tooth works inside the gear by passing the force further. Area 1 ends before reaching the moment denoted by point "A" (Fig. 7).

When the tangential force value exceeds the intensity of point "A", plastic deformation of the tooth occurs at its root. After that, the tooth cannot be used anymore. We consider this value is the maximal value that a tooth can sent. This value will be taken into account in determining the correction coefficient required to be used for relation no. 3.

Stage 2 and 3 - the plastic deformation of the tooth already happens due to load ratings and looks like a bending followed by shearing, which already represents the moment of teeth being damaged and the gear no longer meeting its functional part. (the end of life of the gear).

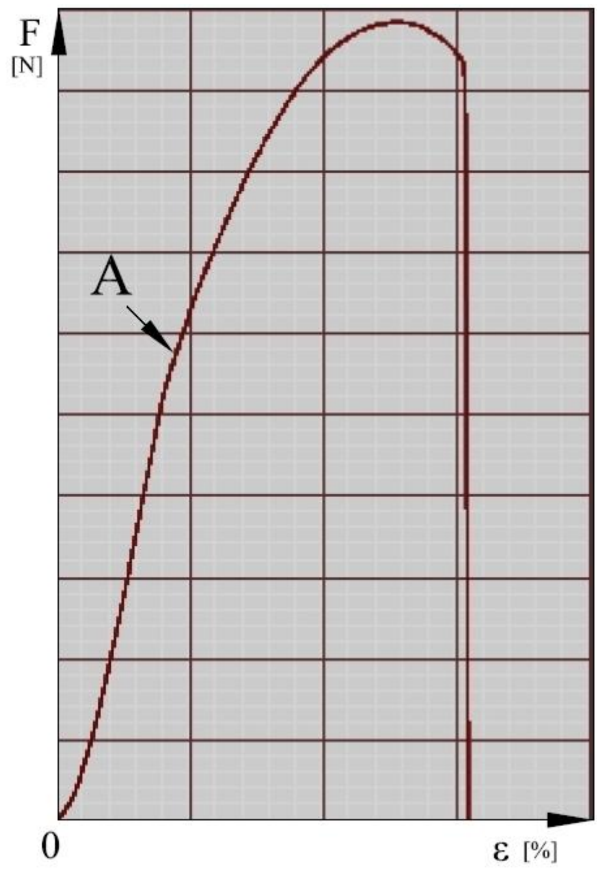


Fig. 7. Point "A", the beginning of plastic deformation of the tooth.

6. Conclusions

The present paper proposes a system and a solution to finding the restraint section, and the maximal stress of a tooth of a gear with asymmetric and symmetric teeth.

The system we have presented has both advantages and disadvantages, like:

1.) - Advantages

1.1. - simple test system based on running a gear with a rack.

1.2. - the test is performed by pull-and-compression test machines, standard machines for automated data acquisition.

2.) - Disadvantages

2.1. - The tooth testing is performed statically

2.2. - The test samples specific to the proposed test system are rather expensive.

3.) - The problem which rises is determining the k_σ correction coefficient of the asymmetric gear dimensioning.

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