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WAYS TO INCREASE THE CONTACT RATIO FOR SPUR GEARS

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Abstract: Many times, in techniques, a low sound level in the running of enclosed gears is imposed. The researches on international field have shown that one of the main causes which lead to the noisy running of the spur gears is the periodical variation of the mesh stiffness, as a consequence of a contact ratio ε_α<2. In this article, the ways to obtain spur gears which can reach a contact ratio $\varepsilon_\alpha \in [1.95, 2.0]$, to assure a silent gear running, is analyzed.

1. INTRODUCTION

The conditions imposed to a spur gear can be synthesized as following: reduced overall size; resistance at contact stress; resistance at bending stress; high efficiency; low running sound; low errors of motion transmission.

The main causes which influence the vibrations and the sound produced by gears are [10, 11]: the periodical variation of mesh stiffness, the tooth errors as a manufacturing result and the impulse given by the change of friction forces direction in the engagement pole. The most important factors which influence the sound produced by spur gears are [12, 13]: the teeth profile, the pressure angle, the module, the gear ratio, the teeth loading, the speed on the pitch circle, the accuracy class in which the gears are manufactured, the gear contact ratio.

From the point of view of the transverse contact ratio value, the spur gear can be [2] with Low Contact Ratio LCR, ε_α<2.0 or with High Contact Ratio HCR, $\varepsilon_\alpha \geq 2.0$. The advantages of HCR gears in comparison to the LCR gears are materialized in the high carrying capacity, due to the greater meshing teeth number, a higher uniformity of the transmitted torque and a lower sound level during running [2]. From the disadvantages of the HCR gears, the following can be mentioned [2, 3]: the thickness decrease on the tooth tip, lower tribological conditions, the decrease of the scoring resistance and higher needs regarding the manufacturing accuracy. In the case of spur gears, heavily loaded and manufactured with high accuracy, the effect of the manufacturing errors is neglect able, while the periodical change of the teeth stiffness becomes the main case of the vibrations and of the sound which appear during the engagement [1, 11].

The gears with high contact ratio can be used to decrease or to exclude the variation of teeth stiffness [10]. In Figure 1 [9], the variation of the teeth stiffness for a gearing LCR (Figure 1, a), respectively for a gearing HCR, with ε_α=2.0 (Figure 1, b) is presented. It was experimentally proved that the dynamic loading of the gear diminishes with the increase of the transverse contact ratio [6] and that the minimal value of the dynamic factor appears at spur gears with a transverse contact ratio smaller than 2.0 (ε_α=1,95) [14]. Similar results were obtained also by other researchers, on experimental [5], but also on theoretical bases [7].

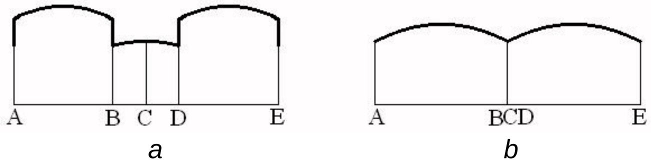


Fig. 1. Variation of the teeth stiffness

Due to increase of the machines running speed, a great importance is awarded to sound and vibrations of the gears of machine transmissions, there are norms and standards which impose specifications and rules regarding the sound produced by them. An example for this purpose is contained in the standard ANSI/AGMA 6025-D98 [15], where a diagram (Figure 2 [13, 15]) is shown, with the maximum sound level in decibels, for enclosed spur or bevel gears, depending on their rotational speed.

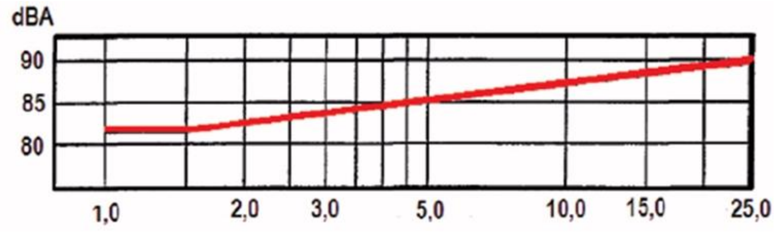


Fig. 2. Maximum sound level

In this article, some ways to obtain spur gears with a transverse contact ratio enclosed in the domain $\varepsilon_\alpha \in [1.95, 2.0]$ are presented.

2. THEORETICAL CONSIDERATIONS

The contact ratio of the spur gear is determined with the relation [8]

$$\varepsilon_\alpha = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2} - 2a_w \sin\alpha_w}{2\pi m \cos\alpha}\,, \tag{1}$$

and depends on the base and tip diameters of the gears, on the centre distance and the profile shifts of the two gears.

Compared to the situation when the gears are realized without profile shifts (x_1=0, x_2=0 and x_s=x_1+x_2=0), the tip diameters increase in the case of gears plus (x_s>0) or minus (x_s<0) modified and remain unchanged in the case of zero gears (x_2=-x_1 and x_s=0). The gears teeth can be shortened to maintain the normal backlash on the teeth tip, or can be left with the depth resulting after the profile modification. The shortening coefficient is determined with the relation [4]

$$k = (x_1 + x_2) - \left(\frac{z_1 + z_2}{2}\right)\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right) = x_s - \frac{z_s}{2}\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right), \tag{2}$$

in the case of teeth shortening, respectively k=0, if the teeth shortening is not wanted.

To generalize the conclusions, the relation (1) can be written not depending on the module, resulting

$$\varepsilon_\alpha = \varepsilon_{\alpha z1} + \varepsilon_{\alpha z2}\,, \tag{3}$$

where the two part contact ratios are determined with:

$$\varepsilon_{\alpha z1} = \frac{1}{2\pi\cos\alpha}\left[\sqrt{\left[z_1 + 2\left(h_a^* + x_1 - k\right)\right]^2 - \left(z_1\cos\alpha\right)^2} - z_1\cos\alpha\,\mathrm{tg}\,\alpha_w\right]; \tag{4}$$

$$\varepsilon_{\alpha z2} = \frac{1}{2\pi\cos\alpha}\left[\sqrt{\left[z_2 + 2\left(h_a^* + x_2 - k\right)\right]^2 - \left(z_2\cos\alpha\right)^2} - z_2\cos\alpha\,\mathrm{tg}\,\alpha_w\right]. \tag{5}$$

From the relations (4) and (5) results that the contact ratio of a spur gearing depends on: the pinion number of teeth z_1; the gear ratio u ($z_2 = uz_1$); the parameters of the reference rack (α. h_a^*, c^*); the shortening coefficient k; the profile shift coefficients x_1 and x_s (x_2= x_s - x_1). To determine the real pressure angle α_w the following relations are used:

$$inv\alpha_w = inv\alpha + 2\frac{x_s}{z_s}tg\alpha\,;\; inv\alpha_w = tg\alpha_w - \alpha_w[rad]\,. \quad (6)$$

The restrictions imposed to the teething of the two gears are:

- The avoidance of teeth interference in engagement

$$x_{1,2} \geq \left(z'_{\min} - z_{1,2}\right)/z_{\min}, \text{ where } z_{\min} = 2h_a^*/\sin^2\alpha\,; \quad (7)$$

for α=20º; it was considered $z'_{\min} = 14$, and for $\alpha \neq$20º, $z'_{\min} = z_{\min}$;

- The avoidance of sharpening for the teeth of the two gears on the tip circle

$$\frac{s_{a1,2}}{m} = \left[\left(inv\alpha - inv\alpha_{a1,2}\right)z_{1,2} + \frac{s_{1,2}}{m}\right]\frac{\cos\alpha}{\cos\alpha_{a1,2}} \geq 0.3\,, \quad (8)$$

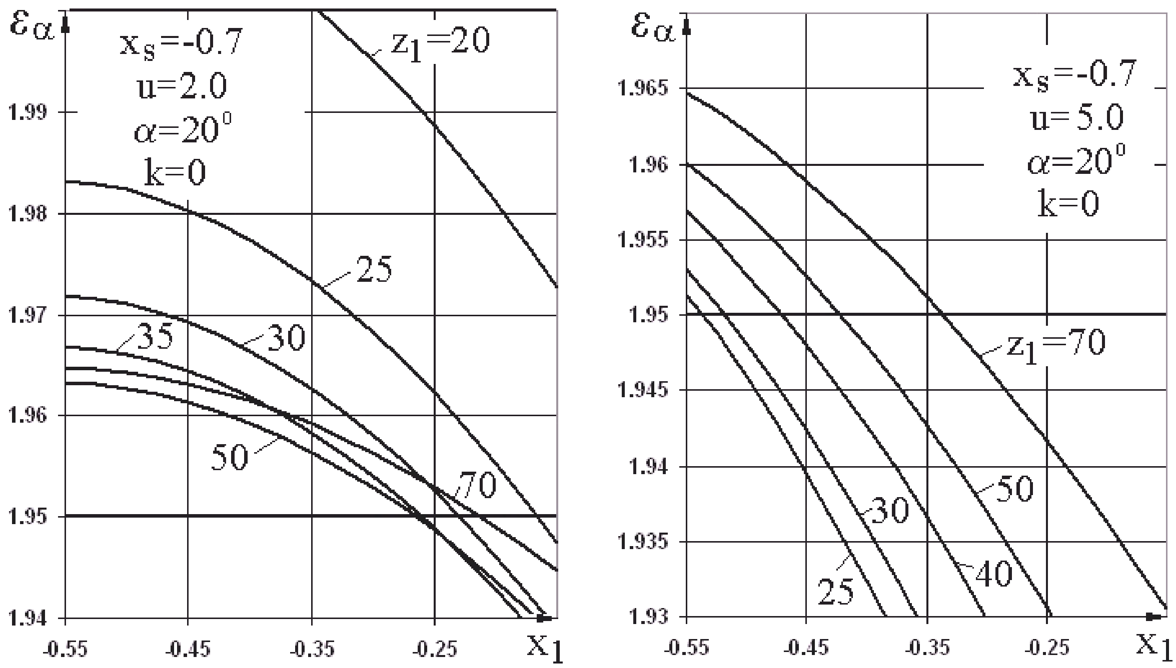
where: $s_{1,2}/m = \pi/2 + 2x_{1,2}tg\alpha$ represents the tooth thickness on the pitch circle; $\alpha_{a1,2} = \cos^{-1}\left[\frac{z_{1,2}\cos\alpha}{z_{1,2} + 2\left(h_a^* + x_{1,2} - k\right)}\right]$ - the pressure angle on the tip circle;

- The avoidance of the backlash diminishing on the tooth tip, $c^* \geq 0.2$.

3. CALCULUS AND CONCLUSIONS

Based on the above presented relations, a computational program was elaborated and many diagrams of the variation of the transverse contact ratio were laid out, where the domain for which the condition $\varepsilon_\alpha \in [1.95, 2.0]$ is met was determined. The parameters of the studied involute gears were: α=20º and α=17.5º; h_a^*=1.0; c^*=0.25, for the gears without teeth shortening (k=0).

In Figure 3, the variation of the transverse contact ratio for a spur gear with α=20º, without teeth shortening, is presented.



a *b*

Fig. 3. ***ε_α=f(x_1, z_1, u, α=20º, x_s=-0.7, k=0)***

The analysis of the diagrams presented in Figure 3 allows the following conclusions:

- for pressure angle α=20º, the transverse contact ratio ε_α exceeds the minimal value of the imposed range only at high negative values of the sum of the profile shifts (x_s<=-0.7) and with no shortened gears teeth (k=0);
- in this situation, at low gear ratios ($u\leq3$, Figure 3, a), the contact ratio is higher at a small teeth number of the pinion. For high gear ratios (Figure 3, b), the contact ratio ε_α is higher when the teeth number of the pinion is great;
- in the case of teeth shortening, to maintain a normal backlash at the teeth tip, for all the calculated situations ($z_1\leq70$, u≤5 and $x_1\geq$-0.6), the result is a maximum value ε_α =1,9488≈1.95.

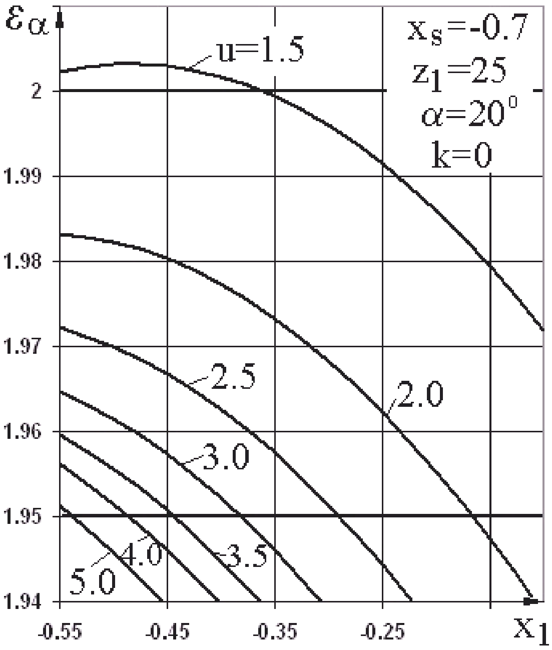
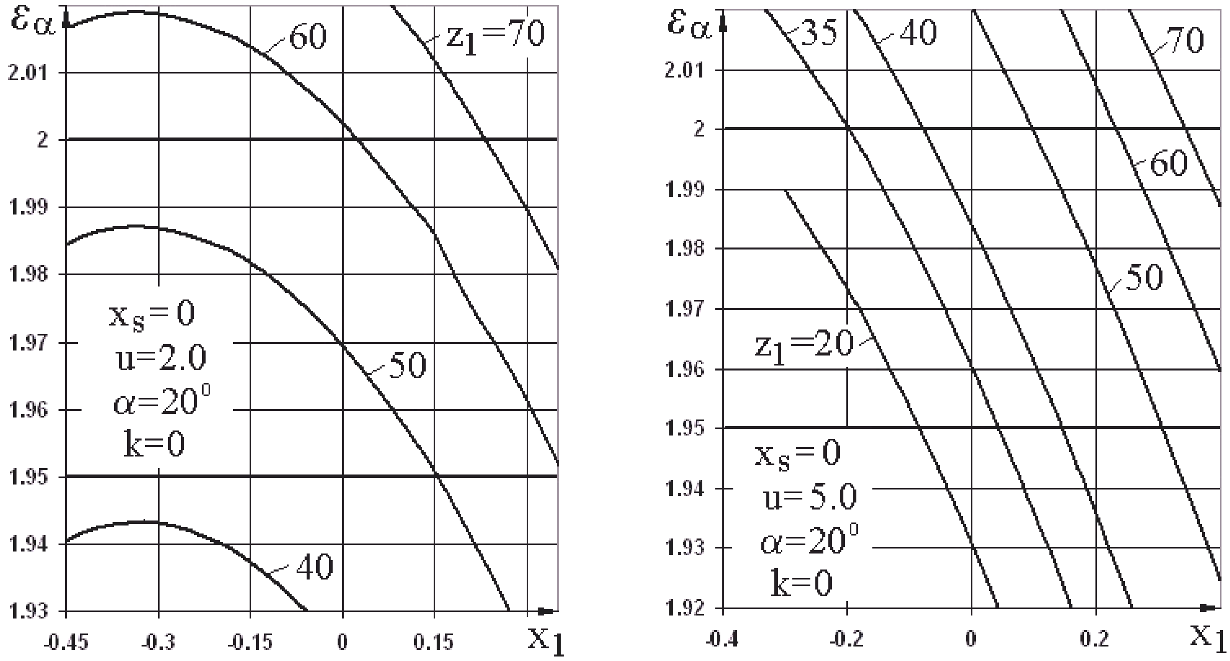


Fig. 4. ε_α*=f(*x_1*, u,* z_1*=25,* α*=20º,* x_s*= -0.7, k=0)*

In Figure 4, the variation of the contact ratio ε_α depending on the gear ratio u is presented. From this emerges all that was affirmed above, but there is the possibility to find the situation for which ε_α >1.95, also for high gear ratios (u=5).

In the case of zero spur gears, the shortening coefficient k=0, situation in which the bordering of the contact ratio ε_α in the optimal domain can be realized with positive, but also with negative shifts of the pinion teeth. Thus, as shown in Figure 5, at low gear ratios (see Figure 5, a) the bordering in the optimal domain is obtained only for relatively high teeth number of the pinion (z_1>40); at higher gear ratios (see Figure 5, b), the bordering in the optimal domain is obtained also at smaller teeth numbers of pinion ($z_1\geq30$).



a *b*

Fig. 5. ε_α*=f(*x_1*, ,* z_1 *u=2,*α*=20º,* x_s*=0, k=0)*

In the Figures 6 and 7, there is presented the variation of the contact ratio ε_α for the case when the pressure α=17.5º, a situation met in techniques. The conclusions can be formulated as following:

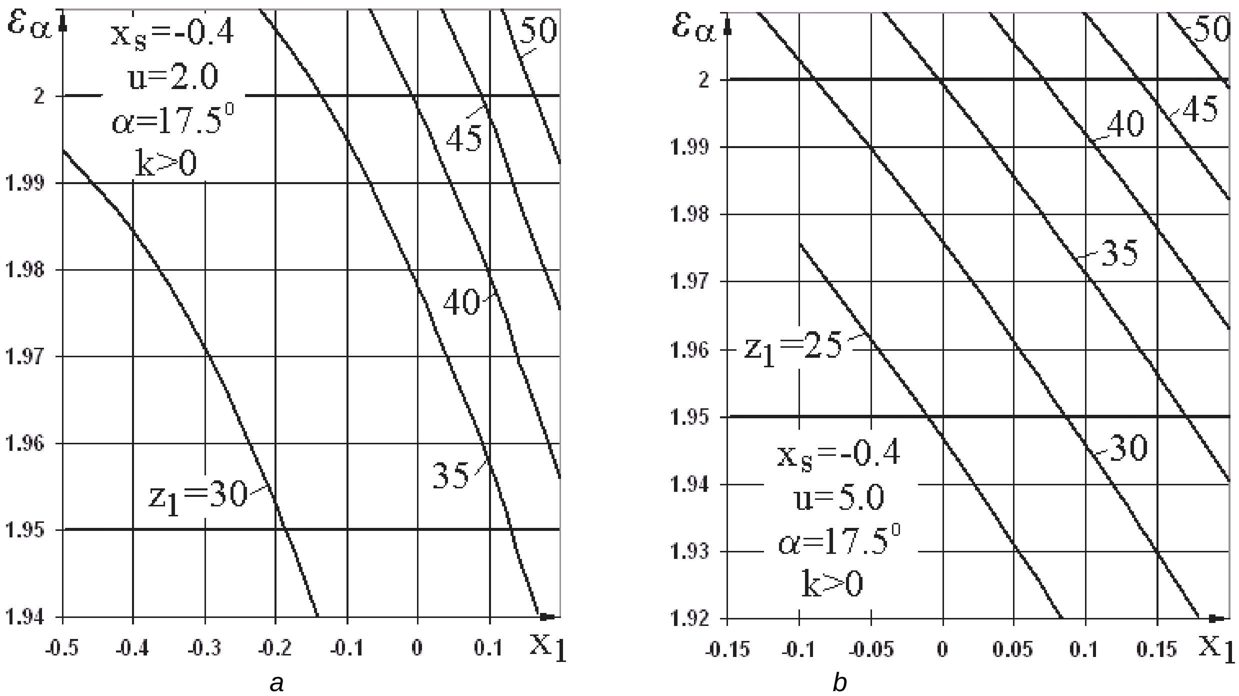


***Fig. 6.** ε_α=f(x_1, z_1, u, α=17.5º, x_s=-0.4, k=0)*

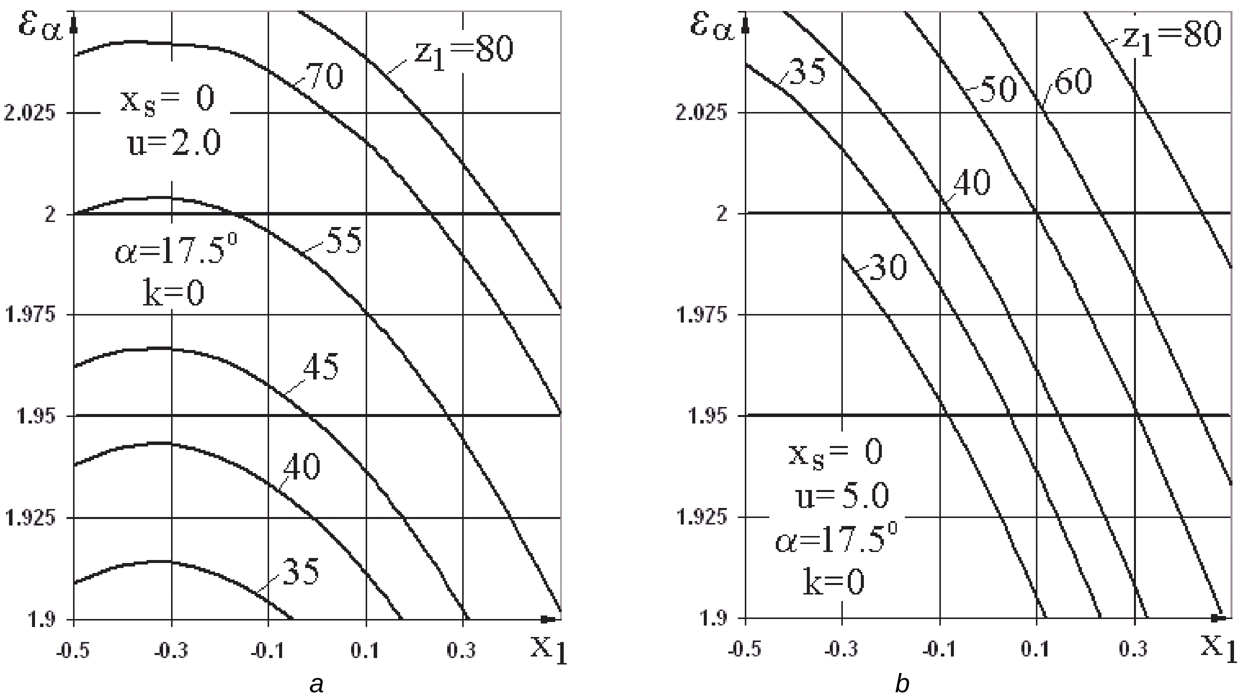


***Fig. 7.** ε_α=f(x_1, z_1, u, α=17.5º, x_s=0, k=0)*

- for these gears, the optimal domain is obtained only at values of the sum of profile shift coefficients $x_s \leq -0.4$;
- even through teeth shortening, to maintain a normal backlash at their tip, there are obtained spur gears which correspond to the optimal domain for which the teeth number of the pinion is relatively reduced, $z_1 \geq 30$, for u=2 and $z_1 \geq 25$, for u=5;

- there are also positive values for the profile shift coefficient of the pinion x_1, which lead to the bordering of the gear in the optimal domain of the contact ratio ε_α;
- if the gear is a zero one (x_s=0 and x_2=-x_1, respectively k=0) the range of the pinion teeth number increases ($z_1 \geq 30$) for high values of the gear ratio u, in comparison to the situation when the gear ratio is reduced, $z_1 \geq 40$, for u=2.

The general conclusion of this analysis can be resumed as following: because of the diminishing the base of the dedendum tooth and the decrease of its bending resistance through the reducing of the pressure angle α, there can be obtained spur gears which run with low sound, through the proper selection of the pinion teeth number and of the sum of the profile shifts, respectively of the profile shift coefficient of the pinion.

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