

TRANSVERSAL MOBILE LEVER COUPLING, KINEMATICS AS MULTIBODY SYSTEM

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Abstract: Mechanical systems definition as multibody systems is a modern way of modeling aiming to the real time simulation of complex product's dynamic behavior. The paper presents aspects regarding the kinematics of the mobile transversal coupling as multibody system. The mobile couplings are used in many power transmissions at cars, road trucks, railway trucks, ships and other industrial systems. Previously, based on an adequate graph and specific conditions, like the degree of freedom, number of bodies and the type of geometrical constraints between the bodies, the synthesis of all possible graphs for these transversal couplings as multibody systems were developed. First it is presented the structural scheme of the analyzed mobile coupling. Then are defined: the parts of the multibody system associated to the mobile coupling (input and output semicouplings, intermediary element and basis); the body reference frames and also the general reference frame; the geometrical and cinematic constraints. Finally, there are determined the cinematic equations, useful to found the optimal geometrical configuration of the coupling, depending by the initial requests of a mobile coupling (cinematic and constructive conditions) and also to obtain some new constructive variants, presented in the final part of the paper.

1. INTRODUCTION

The mobile couplings are used in many power transmissions at cars, road trucks, railway trucks, ships and other industrial systems, in movement and torque transmission between two shafts with parallel axis. The shafts are connected by a kinematical linkage with the possibility to have translations in the transversal plane. These translations are named transversal movements. A transversal coupling is homokinetic if the shafts angular speeds are identical. If the shafts angular speeds are almost identical, the mobile coupling is quasihomokinetic [1].

If the shafts are connected to the basis, it will result the associated mechanism, which is a plane mechanism [1]. The associated mechanism can be represented simplified, using some graphic modules, previously introduced [5]. The transversal coupling multibody model has, as bodies, the input and output shafts and also, some of the intermediary bodies which have, usually, more than two connections [2...6].

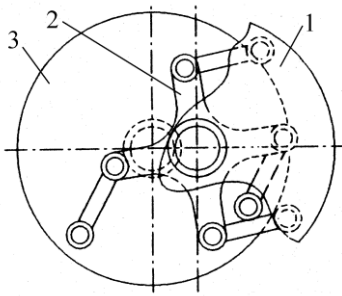
2. THEORETICAL BASIS

The analysed transversal mobile coupling (Karger type) is composed by two semicouplings and between them there are the intermediary elements (fig. 1) [1].

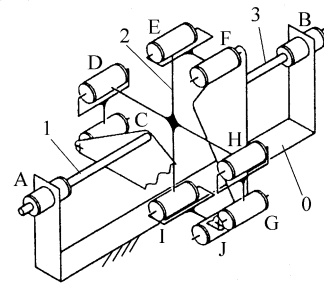
The bodies of the multibody system are: input semicoupling, 1, intermediary body 2, output semicoupling 3 and the basis 0. Between the bodies there are the restrictions detailed in table 1. Between the bodies there are the restrictions detailed in table 1 and figure 2. The mobility of the associated mechanism multibody system is [6]

$$M = 3(n_b - 1) - \sum g_c = 3(4 - 1) - 8 = 1, \quad (1)$$

where $n_b=4$ is the number of the associated mechanism bodies (including the basis) and $\sum g_c=8$ is the number of the geometrical constraints between the bodies.



a. Coupling constructive scheme.



b. Structural scheme of the associated mechanism.

Figure 1. The transversal mobile coupling, type Karger

Table 1: Restrictions

Body i – Body j	Geometrical constraint	Joint	Number of restrictions
0 – 1	R	A	2
0 – 3	R	B	2
1 – 2	RR+RR	CD, GH	1+1
2 – 3	RR+RR	EF, IJ	1+1

For the mobile transversal coupling shown in figure 1, the geometrical and kinematical multibody model is presented in figure 2. The interest point's coordinates are presented in table 2.

To define the geometrical restrictions, is necessary to know the point coordinates relative to the fixed coordinates system. The general relation of these coordinates is [5]:

$$\begin{bmatrix} x_{M_i} \\ y_{M_i} \end{bmatrix} = \begin{bmatrix} x_{0i} \\ y_{0i} \end{bmatrix} + \begin{bmatrix} \cos \varphi_i & -\sin \varphi_i \\ \sin \varphi_i & \cos \varphi_i \end{bmatrix} \begin{bmatrix} r_{ix} \\ r_{iy} \end{bmatrix}, \quad (2)$$

where $i=1, 2, 3$, for mobile bodies.

Geometrical restrictions equations are obtained from the geometrical restrictions between the bodies and are presented as follow.

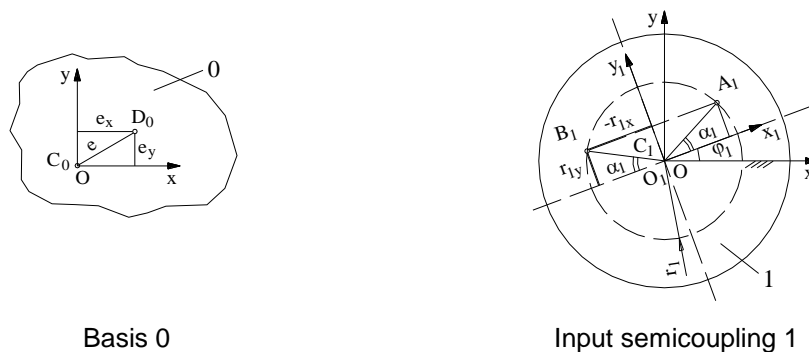


Fig. 2. The geometrical and kinematical multibody model

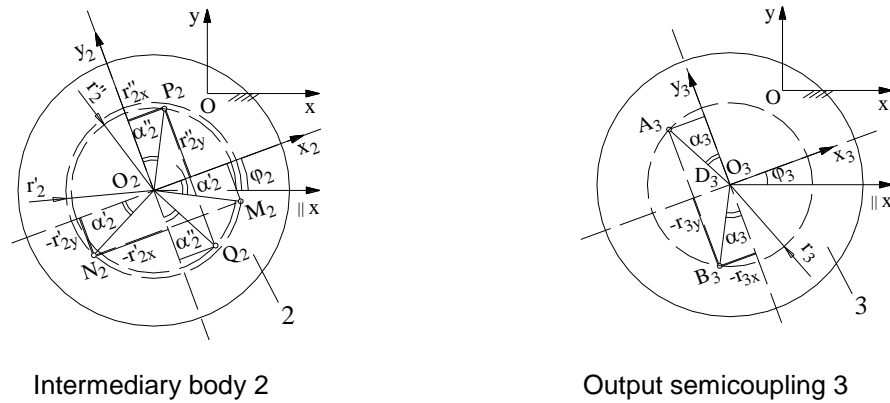


Fig. 2. The geometrical and kinematical multibody model (continued)

Table 2: Points coordinates relative to the local coordinate system.

Body	Points coordinates
Basis 0	$C_0(0, 0); D_0(e_x, e_y).$
Input semicoupling 1	$A_1(r_{1x}, r_{1y}); B_1(-r_{1x}, r_{1y}); C_1(0, 0),$ where: $r_{1x} = r_1 \cos \alpha_1$ and $r_{1y} = r_1 \sin \alpha_1$
Intermediary body 2	$M_2(r'_2, 0); N_2(-r'_2, 0); P_2(r''_{2x}, r''_{2y}), Q_2(r''_{2x}, -r''_{2y}),$ where: $r''_{2x} = r''_2 \cos \alpha''_2$ and $r''_{2y} = r''_2 \sin \alpha''_2$
Output semicoupling 3	$A_3(-r_{3x}, r_{3y}); B_3(-r_{3x}, -r_{3y}); D_3(0, 0);$ where: $r_{3x} = r_3 \cos \alpha_3$ and $r_{3y} = r_3 \sin \alpha_3$

For the studied mobile transversal coupling multibody model, the generally geometrical restrictions equations are:

$$P_i \equiv P_j, \tag{3}$$

and

$$(x_{P_j} - x_{P_i})^2 + (y_{P_j} - y_{P_i})^2 = l^2. \tag{4}$$

The kinematic restriction equation is

$$\rho_1 - f(t) = 0. \tag{5}$$

Based on the relations (3)... (5), it is obtained an equation system with 9 equations and 9 unknowns: $x_{01}, y_{01}, \varphi_1, x_{02}, y_{02}, \varphi_2, x_{03}, y_{03}, \varphi_3$. It results

$$x_{01} = 0; y_{01} = 0, \tag{6}$$

$$x_{03} = e_x; y_{03} = e_y, \tag{7}$$

$$(x_{M_2} - x_{A_1})^2 + (y_{M_2} - y_{A_1})^2 = l_1'^2 \text{ and } (x_{N_2} - x_{B_1})^2 + (y_{N_2} - y_{B_1})^2 = l_1''^2 \quad (8)$$

$$(x_{P_2} - x_{A_3})^2 + (y_{P_2} - y_{A_3})^2 = l_2'^2 \text{ and } (x_{Q_2} - x_{B_3})^2 + (y_{Q_2} - y_{B_3})^2 = l_2''^2 \quad (9)$$

If the lever links between the bodies 1 and 3 are equal, then $l_1' = l_1'' = l_1$, and from relations (8) result

$$\varphi_1 = \varphi_2 \text{ ,} \quad (10)$$

when

$$r_2' \cos \alpha_2' = r_1 \cos \alpha_1 \text{ .} \quad (11)$$

If the lever links between the bodies 2 and 3 are equal, then $l_2' = l_2'' = l_2$, and from relations (9) result

$$\varphi_2 = \varphi_3 \text{ ,} \quad (12)$$

when

$$r_2'' \cos \alpha_2'' = r_3 \cos \alpha_3 \text{ .} \quad (13)$$

Taking account the relations (10) and (12), it is obtained

$$\varphi_1 = \varphi_3 \text{ .} \quad (14)$$

The relation (14) is the homokinetism condition of the transversal mobile coupling.

3. RESULTS

The homokinetism condition of the transversal mobile coupling is available only if the equalities (11) and (13) are respected. If we consider α_1 and r_1 respectively α_3 and r_3 known, it is necessary to obtain α_2' and r_2' , respectively α_2'' and r_2'' . There are obtained:

$$\cos \alpha_2' = \frac{r_1}{r_2'} \cos \alpha_1, \text{ for } \alpha_2' \in \mathbb{R} \text{ and } \frac{r_1}{r_2'} \cos \alpha_1 \in \mathbb{R}, \quad (15)$$

respectively

$$\cos \alpha_2'' = \frac{r_3}{r_2''} \cos \alpha_3, \text{ for } \alpha_2'' \in \mathbb{R} \text{ and } \frac{r_3}{r_2''} \cos \alpha_3 \in \mathbb{R}. \quad (16)$$

Taking account of the cosinus function properties will result the possible theoretical cases presented as follow. For every coupling known and used in practice [1, 2] there are presented the structural and constructive scheme. For every new theoretical coupling variant it is presented just the structural scheme.

- Semiflex coupling (fig. 3, a), with

$$\alpha_2' = \alpha_1 + 2K\pi; \alpha_2'' = \alpha_3 + 2K\pi, \text{ for } r_1 = r_2'; r_3 = r_2''.$$

- Schmidt coupling coupling (fig. 3, b), with

$$\alpha'_2 = -\alpha_1 + 2K\pi; \alpha''_2 = -\alpha_3 + 2K\pi, \text{ for } r_1 = r'_2; r_3 = r''_2.$$

- Kärger coupling (fig. 1, a), with

$$\alpha'_2 = 0; \alpha''_2 = 0, \text{ for } r'_2 < r_3; r'_2 < r_1.$$

- John coupling (fig. 3, c), with

$$\alpha'_2 = \pm \arccos\left(\frac{r_1}{r'_2} \cos \alpha_1\right) + 2K\pi; \alpha''_2 = \pm \arccos\left(\frac{r_3}{r''_2} \cos \alpha_3\right) + 2K\pi, \text{ for } \frac{r'_2}{r_1} > 1; \frac{r''_2}{r_3} > 1.$$

- new constructive variant (fig. 3, d), with

$$\alpha'_2 = \arccos\left(\frac{r_1}{r'_2}\right) + 2K\pi; \alpha''_2 = \arccos\left(\frac{r_3}{r''_2}\right) + 2K\pi, \text{ for } \frac{r_1}{r'_2} < 1, \frac{r_3}{r''_2} < 1 \text{ and } \alpha_1 = 0, \alpha_3 = 0.$$

- new constructive variant (fig. 3, e), with

$$\alpha'_2 = -\arccos\left(\frac{r_1}{r'_2}\right) + 2K\pi; \alpha''_2 = -\arccos\left(\frac{r_3}{r''_2}\right) + 2K\pi, \text{ for } \frac{r_1}{r'_2} < 1, \frac{r_3}{r''_2} < 1 \text{ and } \alpha_1 = 0, \alpha_3 = 0.$$

- new constructive variant (fig. 3, f), with

$$\alpha'_2 = \alpha_1 = 0; \alpha''_2 = \alpha_3 = 0, \text{ and } r_1 = r'_2; r_3 = r''_2.$$

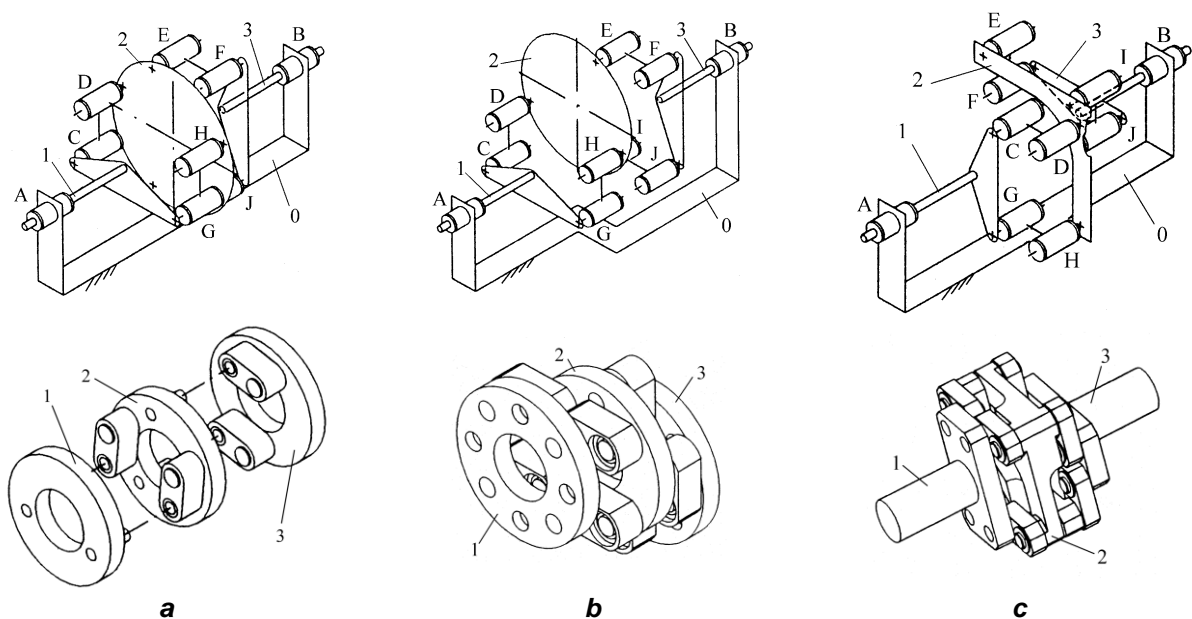


Figure 3. The homokinetic transversal mobile couplings

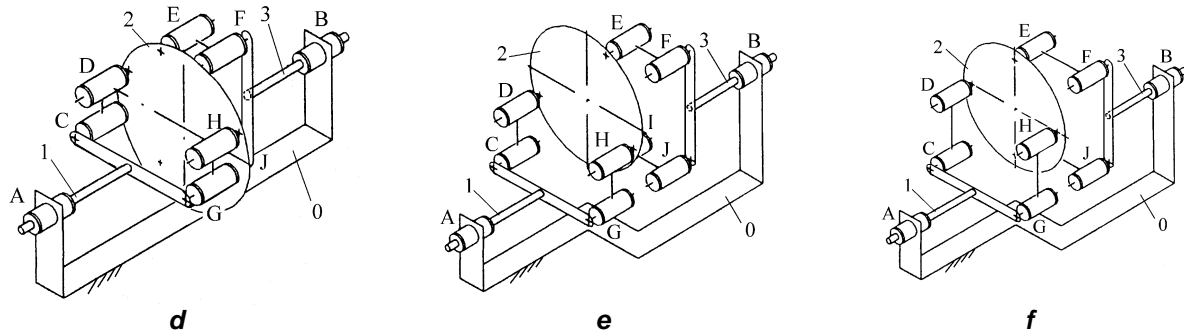


Figure 3. The homokinetic transversal mobile couplings (continued)

4. CONCLUSIONS

In the case of the transversal mobile couplings as multibody systems with three mobile bodies, the homokinetism result in two steps: first $\varphi_1 = \varphi_2$, and then $\varphi_2 = \varphi_3$, according to the conditions relative to the geometry of the bodies. Specifying the coupling constructive conditions are important here, because is useful in coupling's dynamic behavior and in manufacturing process [1, 6].

From this transversal couplings type, with levers, there are known and used the Semiflex, Krawtschenko, Schmidt, Kärger, John couplings. The multibody theory was also useful to discover some new other constructive solutions.

The method can be applied also for other known mobile couplings, as multibody systems with four or five mobile bodies and for other new solutions of mobile couplings, identified previously by the authors in structural analysis with graphs [5].

In the future researches, the author intend to analyze these new particular cases and also to find their optimal shape configuration, in the design process.

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